ORIGINAL ARTICLE

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Analysis of creep of wood during water adsorption based on the excitation-response theory

Abstract The creep behavior of wood during water adsorption was mathematically analyzed based on the excitation-response theory. The creep change $\phi_u(t)$, obtained by subtracting an instantaneous compliance $J_u(0)$ from a creep compliance $J_u(t)$, was linear in terms of moisture content at a steady state of moisture and was separable into two functions of time and moisture content. The creep compliance, however, was nonlinear. The creep change during water adsorption was obtained by applying the excitation-response theory to the creep change in a steady state of moisture. The equation was formally equal to the results reported so far. By using the derived equations, it was theoretically proved that the change in creep compliance during water adsorption from moisture content u_0 to u_1 is always greater than the difference between creep compliance at u_0 and that at u_1 in the steady state.

Key words Creep · Water adsorption · Excitation-response theory · Boltzmann superposition principle · Mechanosorptive

Introduction

For wood the creep during moisture change causes a remarkable deflection. The deflection increases during the first water adsorption and all desorption processes under cyclic moisture changing. This fact suggests that the effect, the so-called mechanosorptive effect, depends on a complex interaction between moisture and wood. The properties were reviewed by Grossman[1] in detail. In a previous report, the author tried to formulate the creep behavior during

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water desorption on the basis of a generalized mechanical model.[2] The creep compliance during water desorption was derived by considering viscosity related to the free volume created during the process. It was approximately equivalent to the empirical equations reported by Takemura[3] and Leicester.[4] This result showed the validity of the Maxwell model proposed by Takemura et al.[5]

The object of this report is to characterize theoretically the creep during water adsorption, a subject already been reported on by many research scientists. Probably an empirical trial alone cannot elucidate it. Both theoretical and empirical trials can be useful and necessary to clarify the mechanism of the mechanosorptive effect. In the present report we discuss creep behavior in a steady state of moisture and then try to describe mathematically the creep during water adsorption on the basis of the excitation-response theory. Creep behaviors in steady and nonsteady states are compared.

Experimental

Samples were obtained from Japanese ash (*Fraxis mandshurica* Rupr.). They had rectangular dimensions of 90(L) × 8(R) × 2(T) mm. They were conditioned to appropriate moisture content of about 0–20% in a desiccator where the prescribed saturated solution was placed.

Creep measurement was carried out in a chamber at 20°C. The span is 65 mm. Samples were wrapped by polyethylene film to keep a constant moisture content. A cantilever bending load of 1.37 N in linear dimension was used.

Results and discussion

Moisture dependence of creep function

Creep behavior of samples with various moisture contents (MCs) were examined to clarified the relation between creep compliance and moisture content. Figure 1 shows the
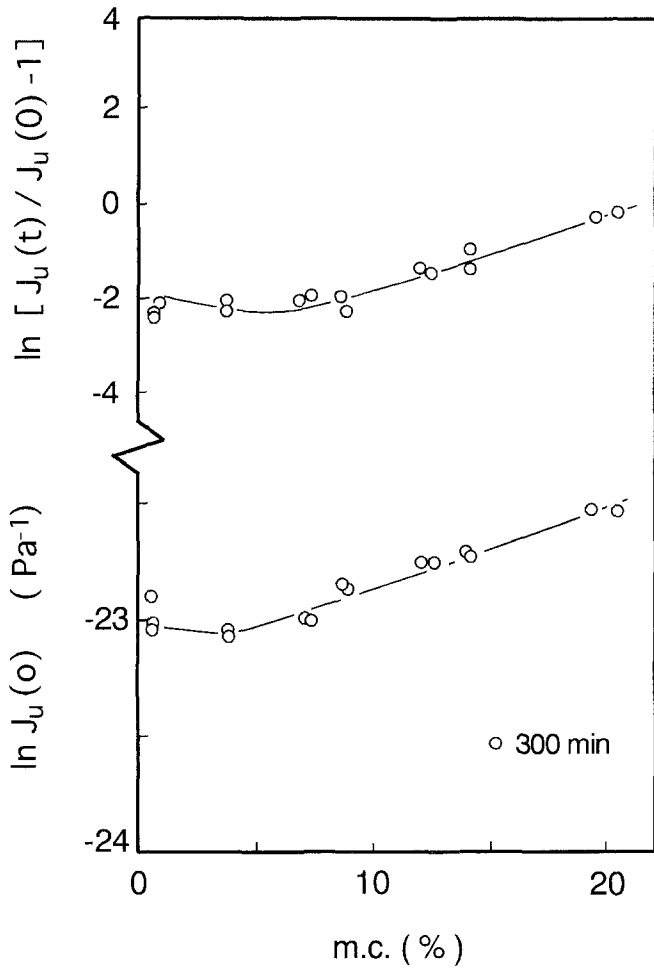
Fig. 1. Dependence of the logarithmic instantaneous compliance and the logarithmic relative creep compliance on moisture content (*m.c.*)

dependence of the logarithmic relative creep compliance $\ln[J_u(t)/J_u(0)-1]$ and the logarithmic instantaneous compliance $\ln[J_u(0)]$ on the moisture constant: $J_u(t)$ is the creep compliance at time t. Both relations are linear at a moisture range of 5%–20% ($0.05 \leq MC \leq 0.20$). There was little dependence of both on moisture content at less than 5%, that is, $J_u(t)$ is nearly equal to $J_d(t)$ in the moisture range. Moreover, the slope of $\ln[J_u(t)/J_u(0)-1]$ versus moisture content related to a time function $h(t)$ as Takemura et al.[5] and Suzuki[6] reported: $h(t)$ is a creep change under the oven-dried condition. Putting $u = MC - 0.05$, we obtain the following equation:

$$J_u(t) = J_u(0)\big[1 + h(t)\exp(mu)\big] \qquad (1)$$

$$J_u(0) = J_d(0)\exp(nu) \qquad (2)$$

Then

$$J_u(t) = J_d(0)\exp(nu)\big[1 + h(t)\exp(mu)\big] \qquad (3)$$

where m and n are constants, $0 \leq u \leq 0.15$. Equation (3) represents creep compliance in a steady state of moisture. We found that the relation between $J_u(t)$ and moisture content is nonlinear, and that moisture content influences both

creep function and instantaneous compliance. The effect depends on two parameters: m and n.

The above result can also be derived from results reported by others. For example, both Suzuki[6] and Takemura et al.[5] reported that a creep deflection obtained by subtracting an instantaneous deflection from a total deflection was separable into time and moisture functions. For example, Takemura et al. showed that the creep compliance of beech at moisture contents of less than 20% was represented by the following equation:

$$J_u(t) = J_u(0)\big[1 + f(t)g(u)\big], \qquad (0 \leq u \leq 0.2) \qquad (4)$$

where $f(t)$ and $g(u)$ are functions of time and moisture content, respectively. In equation (4):

$$g(u) = \exp(m'u) \qquad (m' = \text{constant}) \qquad (5)$$

Kollmann[7] reported that the elastic modulus of spruce and oak slightly increased with increasing moisture content up to 5% and decreased above it. Considering that the elastic modulus is almost constant in the range of 0–5%, we obtain the following equation on the approximation that the compliance as the inverse of the elastic modulus, Ju, is represented by a exponential function of moisture content:

$$J_u = J_d\exp(n'u), \qquad (u = MC - 0.05, n' = \text{constant}) \qquad (6)$$

where J_d is a compliance at oven-dried condition. Equation (6) describes the results by Kollmann[7] if n' is appropriately selected. Here, we can expect the following equation when J_d is regarded as an instantaneous compliance $J_d(0)$:

$$J_u(0) = J_d(0)\exp(n'n). \quad (u = MC - 0.05, n' = \text{constant}) \qquad (7)$$

Accordingly, from Eqs. (4), (5), and (7):

$$J_u(t) = J_d(0)\exp(n'u)\big[1 + f(t)\exp(m'u)\big] \qquad (8)$$

Equation (8) is equal to Eq. (3).

Linearity of creep function

The creep change subtracted an instantaneous compliance from a creep compliance in Eqs. (2) and (3) represented by

$$J_u(t) - J_u(0) = h(t)J_d(0)\exp[(m + n)u] \qquad (9)$$

Putting

$$\phi_u(t) = J_u(t) - J_u(0) \qquad (10)$$

$$\chi(u) = J_d(0)\exp(ku) \qquad (k = m + n) \qquad (11)$$

we obtain

$$\phi_u(t) = h(t)\chi(u) \qquad (12)$$

That is, the creep change $\phi_u(t)$ is separable into two parts: $h(t)$ and $\chi(u)$. Therefore, $\phi_u(t)$ is a linear function of $\chi(u)$. This supports the results reported by Suzuki[6] and Takemura et al.[5] In this connection, Dart and Guth,[8] Guth and Dart,[9] and Kitahara and Okabe[10] reported that the stress relaxation function is separable into a time function and a temperature function.

The above fact suggests the probability of analysis based on the excitation-response theory. Now, let us consider the moisture change as excitation and the creep deflection as response. The excitation-response theory consists of two requirements: one the law of causality and the other the Boltzmann superposition principle. The former relates to the direction of time flow and is generally valid for the physical macrophenomena. It is independent of linearity. On the other hand, the latter holds in the system where one of a pair of physical quantities is linear in terms of the other. Considering Eq. (12), we apply the theory to creep during water adsorption on the assumption that both of the above requirements are valid for creep. If the application is valid, the following equation should be formed, which is the empirical result reported so far:

$$\Phi_u(t) = KM(t) \tag{13}$$

where $\Phi_u(t)$ is the creep change during water adsorption, K is a constant, and $M(t)$ is a function of moisture change.

Application of the Boltzmann superposition principle

We examined the change of $\phi_u(t)$ during water adsorption. Let us consider the $\phi_u(t)$ term of Eq. (12) when the change of $\Delta\chi$ with time is $\Delta\chi_0, \Delta\chi_1, \Delta\chi_2, \ldots \Delta\chi_i, \ldots$ at time $t_0, t_1, t_2, \ldots, t_i, \ldots$, respectively. Then, the following equation holds if the Boltzmann superposition principle is valid in this case:

$$\Phi_u(t) = \sum_i^\infty h(t - t_i)\Delta\chi i \tag{14}$$

Putting $\Delta\chi_i \to 0$

$$\Phi_u(t) = \int_{-\infty}^t h(t - t')\frac{d\chi(t')}{dt'}dt' \tag{15}$$

Here, $\chi(u)$ is replaced by $\chi(t)$, considering u of Eq. (11) as a time function. After integration by parts, Eq. (15) is reduced to

$$\Phi_u(t) = \left[h(t - t')\chi(t')\right]_{-\infty}^t - \int_{-\infty}^t \frac{dh(t - t')}{dt'}\chi(t')\,dt' \tag{16}$$

Analysis of Eq. (16) requires the form of $u(t)$, as

$$\chi(t) = J_d(0)\exp\left[ku(t)\right] \tag{17}$$

Thus, we assume that $u(t)$ is a straight line in the narrow region of moisture change during water adsorption, as the change is monotonic in many cases:

$$u(t) = u(0) + ct \qquad (c = \text{constant} > 0) \tag{18}$$

where $u(t) = mc(t) - 0.05$ $(0.05 \le mc(t) \le 0.20)$: $mc(t)$ is the moisture content as a function of time. On the other hand, putting $u = 0$ in Eq. (6),

$$h(t) = \frac{J_d(t) - J_d(0)}{J_d(0)} \tag{19}$$

Moreover, considering $J_d(t)$ as a creep compliance at oven-dried condition

$$
\begin{aligned}
J_d(t) &= \int_0^\infty L(\lambda)\left[1 - \exp\left(-\frac{t}{\lambda}\right)\right]d\lambda + J_d(0) \\
&= \int_{-\infty}^\infty L(\ln\lambda)\left[1 - \exp\left(-\frac{t}{\lambda}\right)\right]d\ln\lambda + J_d(0)
\end{aligned} \tag{20}
$$

where λ is the retardation time, and $L(\lambda)$ and $L(\ln\lambda)$ are the distribution functions of retardation times and the retardation spectrum, respectively.

From Eqs. (16) to (20), we obtain

$$\Phi_u(t) = -\exp\left[ku(0)\right]\int_0^\infty L(\lambda)d\lambda + \exp\left[ku(t)\right]\int_0^\infty \frac{L(\lambda)}{1 + kc\lambda}d\lambda \tag{21}$$

This equation represents the creep change during water adsorption as a function of both moisture content and time.

Approximation

Equation (21) cannot be simply rearranged. Thus, we try to calculate the approximate equation from Eq. (21).

The retardation spectrum of oven-dried wood specimens $L(\ln\lambda)$ is represented by,

$$L(\ln\lambda) = \begin{bmatrix} L_0 & (\ln p \le \ln\lambda \le \ln q) \\ 0 & (\ln\lambda \le \ln p,\ \ln q \le \ln\lambda), \end{bmatrix} \tag{22}$$

as the spectrum is approximated by the box-type spectrum. The lower limit value lnp is a sufficiently small value, whereas the upper limit lnq is much greater than the measurement time of creep during water adsorption. This is valid from many empirical results. Substituting Eq. (22) into Eq. (21), we have

$$
\begin{aligned}
\Phi_u(t) = L_0\left(\ln\frac{q}{p} - \ln\frac{1 + kcq}{1 + kcp}\right)\exp\left[ku(t)\right] - L_0\ln\frac{q}{p}\exp\left[ku(0)\right].
\end{aligned} \tag{23}
$$

Considering $0 < kc$ and $0 < p << q$, we find $(p + kcpq) << (q + kcpq)$, so $q/p >> (1 + kcq)/(1 + kcp)$. Then, we obtain

$$\ln\frac{q}{p} - \ln\frac{1 + kcq}{1 + kcp} \approx \ln\frac{q}{p} \tag{24}$$

From the above approximation, Eq. (23) is reduced to

$$\Phi_u(t) = L_0\ln\frac{q}{p}\left\{\exp\left[ku(t)\right] - \exp\left[ku(0)\right]\right\} \tag{25}$$

This is approximately Eq. (21). The simple form is due to the box-type spectrum by which creep behavior of oven-dried wood is characterized.

Equation (25) becomes simpler, if $kct < 1$. In many cases $kct < 1$ is valid: $t < 10^4$ to 10^7, $c \approx 10^{-5}$ (Takemura et al.[5]) to 10^{-6} (Hunt[11]), and $k \approx 10^{-1}$ to 10 (Norimoto et al.[12]). Then, $\{\exp[ku(t)] - \exp[ku(0)]\}$ is nearly equal to

$k\{\exp[ku(0)]\}\{u(t) - u(0)\}$, considering $u(t) = u(0) + ct$ and $kct < 1$. Accordingly, we have

$$\Phi_u(t) = KM(t) \tag{26}$$

where

$$K = k\exp[ku(0)]L_0\ln\frac{q}{p} \tag{27}$$

$$M(t) = u(t) - u(0). \tag{28}$$

Equation (26) describes the creep behavior during water desorption.[12] The equation (i.e., the term of creep change during water adsorption) formally is the same as that during water desorption in the equations reported by Takemura,[3] Leicester,[4] and Nakano.[2]

The above derivation shows that the creep change during water adsorption can be derived on the basis of the excitation-response theory. This means that the superposition principle holds for the effect of moisture on creep behavior.

Comparison of steady state and nonsteady state of moisture

Experimental results show that the creep during water adsorption, before which any desorption process is not present, is always greater than that in a steady state. We compared the difference after time t between the creep change at moisture content u_0 and that at u_1, Δ_s, with that during water adsorption from u_0 to u_1 after time t, Δ_{ns}: experimental result is $\Delta_s < \Delta_{ns}$.

From Eqs. (10) to (12), we have the following as an approximate equation in a steady state of moisture:

$$\phi_u(t) = h(t)J_d(0)\exp(ku)$$
$$= [J_d(t) - J_d(0)]\exp(ku) \tag{29}$$

Moreover, from Eqs. (20) and (22)

$$J_d(t) - J_d(0) = \int_{\ln p}^{\ln q} L_0\left[1 - \exp\left(-\frac{t}{\lambda}\right)\right]d\ln\lambda$$
$$= L_0\ln\frac{q}{p} - \int_{\ln p}^{\ln q} L_0\exp\left(-\frac{t}{\lambda}\right)d\ln\lambda \tag{30}$$

The second term in Eq. (30) is rearranged using the properties of the following function:

$$E_i(-x) = \int_{\infty}^{x}\frac{1}{t}\exp(-t)dt \tag{31}$$

Then

$$J_d(t) - J_d(0) = \begin{bmatrix} 0 & (t \ll p) \\ L_0\ln(1.781t/p) & (p \ll t \ll q) \\ L_0\ln(q/p) & (q \ll t) \end{bmatrix} \tag{32}$$

We obtain the following as $\phi_u(t)$ using Eqs. (29) and (32). Considering that the measurement time is $p < t < q$ in general

$$\phi_u(t) = L_0\ln\left(\frac{1.781t}{p}\right)\exp(ku) \tag{33}$$

This is the approximate equation in a steady state of moisture.

Here, let us compare Δ_s with Δ_{ns} using Eqs. (25) and (33). Using Eq. (33), Δ_s is represented by

$$\Delta_s = L_0\ln\frac{1.781t}{p}\left[\exp(ku_1) - \exp(ku_0)\right] \tag{34}$$

On the other hand, from Eq. (25), Δ_{ns} is

$$\Delta_{ns} = L_0\ln\frac{q}{p}\left[\exp(ku_1) - \exp(ku_0)\right] \tag{35}$$

Then, the ratio of Δ_{ns} to Δ_s, R, is represented by

$$R = \frac{\Delta_{ns}}{\Delta_s} = \frac{\ln\dfrac{q}{p}}{\ln\dfrac{1.781t}{p}} > 1 \tag{36}$$

as the upper limit time q is far greater than the measurement time. Consequently, the creep change during water adsorption is necessarily greater than that in a steady state.

Conclusions

The creep compliance in a steady state of moisture as a function of both moisture content and time was represented by

$$J_u(t) = J_d(0)\exp(nu)[1 + h(t)\exp(mu)]$$
$$(u = MC - 0.05,\ 0.05 < MC < 0.20)$$

where m and n are constant, and $J_d(0)$ and $h(t)$ are the instantaneous compliance and creep function at the oven-dried condition, respectively. The creep change is the instantaneous compliance subtracted from the creep compliance, $\phi_u(t)$, was

$$\phi_u(t) = h(t)\chi(u),$$

where $\chi(u)$ is a function of moisture content. That is, $\phi_u(t)$ is linear in terms of $\chi(u)$.

Using this relation, the creep change during water adsorption, $\Phi_u(t)$, was calculated based on the excitation-response theory, and the following equation was obtained;

$$\Phi_u(t) = KM(t)$$

where

$$K = k\exp[ku(0)]L_0\ln\frac{q}{p} \quad (p, q, \text{ and } k = \text{constant},$$
$$p \ll t \ll p)$$

$$M(t) = u(t) - u(0) \quad (u(t) = mc(t) - 0.05,$$
$$0.05 < mc(t) < 0.20)$$

That is, $\Phi_u(t)$ was formally equal to the equation reported so far.

When the difference after time t between the creep change at moisture content u_0 and that at u_1, Δ_s, was compared with the creep change during water adsorption after t, Δ_{ns}, $\Delta_s < \Delta_{ns}$ was the result using the derived equations. It was proved that the creep during water adsorption is necessarily greater than that at the steady state.

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