J Wood Sci (2001) 47:269–274

ORIGINAL ARTICLE

Jianjun Zhu · Asae Kudo · Takashi Takeda
Morihiko Tokumoto

Methods to estimate the length effect on tensile strength parallel to the grain in Japanese larch

Received: April 7, 2000 / Accepted: August 21, 2000

Abstract To find a desirable method for estimating the length effect on tensile strength (σ_t), we used three methods to analyze the σ_t data from a Japanese larch (*Larix kaempferi*) small, clear specimen. These methods included a nonparametric method, the projection method of Hayashi, and a proposed method. The estimated length effect parameters (g) by the nonparametric method were 0.0237 and 0.0626 for 50th and 5th percentile σ_t distributions, respectively. The projection method requires a standard E_f level (E*: dynamic Young's modulus), arbitrarily chosen for calculating the g value. The g values from the projection method were 0.1122 for low E*, 0.0898 for average E*, and 0.0759 for high E*. The estimated g values by the proposed method using selected σ_t data were 0.1020 and 0.1838 for the 50th and 5th percentiles, respectively. Among the three methods, the nonparametric method did not consider the different distribution of Young's modulus among specimens, and the estimated length effect parameters (g) by this method were small. The projection method reduced the influence of Young's modulus, but the length effect parameters varied with the E* level. The proposed method minimized the dependence on E_f distributions among specimens. We believe the latter method is desirable for estimating the length effect on tensile strength.

Key words Static tensile test · Dynamic Young's modulus · 50th Percentiles · 5th Percentiles · Japanese larch

Introduction

When evaluating strength properties of structural lumber used as engineered wood,[1] tensile strength is usually an important factor. To estimate the tensile strength of structural lumber, the size effect in the length direction must be considered, as there is a decreasing trend of tensile strength with increasing length of the specimen. Up to now, many researchers have focused on length effects on tensile strength for structural lumber. For example, the American Society for Testing and Materials (ASTM)[2] gives a characteristic value (0.14) equivalent to the length effect parameter for adjusting the ultimate tensile stress parallel to the grain in visually graded dimension lumber. There are few reports about length effects on tensile strength for small clear specimens. Here we attempted to investigate length effects on tensile strength for small clear specimens.

In Japan, testing methods for wood, including the method to test tension parallel to grain, are specified by Japanese industrial standards (JIS).[3] Because the test methods have prescribed the particular dimensions and shapes of specimens, we can easily compare various test data obtained by the same method. This is why the size effect had not been encountered before, as the standard test methods called for constant configuration, as Madsen[4] described. There are few reports concerning the size effect on strength in small clear specimens. Among them, Madsen[5] reported the size effects on bending strength in defect-free Douglas fir; and Masuda and Okohira[6] explored the size effect on bending strength in western hemlock. Okohira et al.[7,8] investigated the size effect on compressive strength in hinoki (Japanese cypress) and hemlock, as well as the width effect on the tensile strength of western hemlock. In this research, we focused on the length effect, although there are some empirical studies relative to failure theories.[9]

It is known that there are at least three methods[4] for obtaining the length effect parameter: slope method, shape parameters, and fracture position. The slope method is based on the relation between the logarithm of strength (x) and the logarithm of length (L) (or depth or volume); the length effect parameter g is calculated by $g = -\{\ln[x(L_2)] - \ln[x(L_1)]\}/\{\ln(L_2) - \ln(L_1)\}$. This equation can be rewritten as $x(L_2) = x(L_1) \times (L_2/L_1)^{-g}$. With the shape parameter method the length effect parameter is given as an inverse of the shape parameter (k) of the fitted two-parameter Weibull distribution (2P-Weibull). Size effects have commonly been expressed as a function of k. According to

J. Zhu · A. Kudo · T. Takeda (✉) · M. Tokumoto
Faculty of Agriculture, Shinshu University, Minamiminowa, Nagano 399-4598, Japan
Tel. +81-265-77-1508; Fax +81-265-72-5259
e-mail: takeda@gipmc.shinshu-u.ac.jp

Madsen,[10] the important feature should be $1/k$ ($= g$), not k, as one wants to use these size effects for design purposes. The last method is applied for a bending test with a concentrated load in the center of the span.

In this study, we used the slope method to calculate length effect parameters and then three other methods to analyze them: the nonparametric method, the projection method proposed by Hayashi et al.,[11] and a proposed method. The objective of this study was to find a desirable method for estimating the length effect.

Experiment

Materials

Materials for tests were cut from six 87-year-old Japanese larch (*Larix kaempferi*) trees harvested at Togakushi in Nagano, Japan. The range of diameters at breast height was 32–43 cm. We cut 2 m long logs from the sampled trees and obtained 25 mm thick and 150 mm wide samples from the logs. Details of the materials can be found in the report by Zhu et al.[12] Then 360 mm (A), 390 mm (B), or 460 mm (C) long sticks were randomly cut from the air-dried lumber. All obtained samples were planed to a size that was 25 mm wide (radial direction) and 15 mm thick (tangential direction). For each sample we determined the annual ring width (ARW), density, and dynamic Young's modulus (E_f) by the longitudinal vibration method.[13]

Tensile test

Tensile test specimens were prepared from the samples according to JIS.[3] There were three types of specimen: short (a), normal (b), and long (c); "normal" means the same shape as that specified in JIS.[3] The numbers of specimens for groups a, b, and c were 60, 56, and 63, respectively; and all of them were processed with the A, B, and C samples. Figure 1 shows the shapes and dimensions for each type. The cross section of the thinnest part for all specimens was 5 mm thick and 25 mm wide; and the lengths of this part for a, b, and c specimens were 30, 60, and 120 mm, respectively. There are called spans in the following discussion. Using these specimens, we conducted static tensile tests parallel to the grain and determined the tensile strength (σ_t) for each specimen. The test was conducted using the universal testing machine, and the loading speed was kept at less than 200 kg/min. All specimens failed within the span. Tensile strength was calculated from the maximum load attained and the actual cross-sectional dimensions of the specimens.

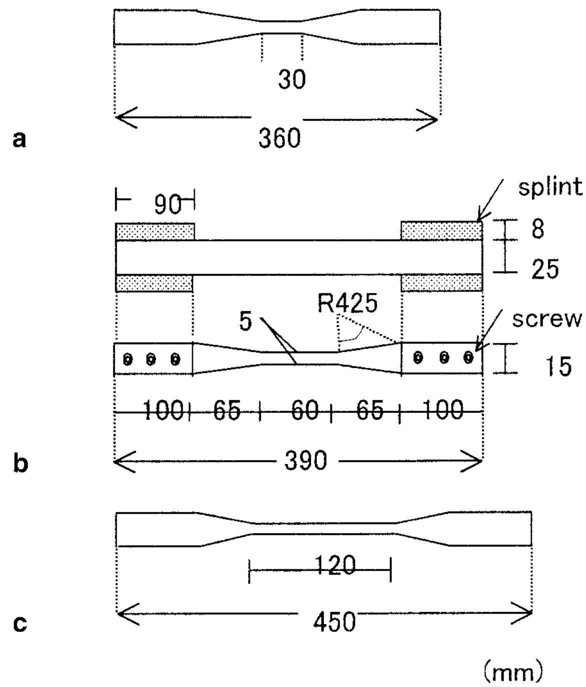

Fig. 1. Specimens. **a** Short type, 30 mm span. **b** Normal type according to JIS, 60 mm span. **c** Long type, 120 mm span; R425, 425 mm radius

Results and discussion

Tensile strength test

The dimensions, test spans, and properties of specimens are shown in Table 1. These values were obtained by measuring the samples before processing the specimens into the shape needed for the tensile tests. There were small differences in annual ring width or density among the specimens. Based on the t-test, there was no significant difference between specimens b and c in terms of the dynamic Young's modulus (E_f) at the 5% significance level ($|t| = 0.153 < 1.982$); there were significant differences between specimens a and b ($|t| = 3.041 > 1.981$) and specimens a and c ($|t| = 2.814 > 1.980$).

Table 2 shows the tensile strength tests for each specimen. The distributions of σ_t are shown in Fig. 2. There were few differences among the A, B, and C specimens. Table 2 also shows the correlation coefficients between σ_t and other wood properties. Among them, the coefficients between σ_t and E_f are the highest.

Generally, there should be a decreasing trend of tensile strength with increasing length of the specimens, but there seems to be no significant length effect based on above results. Based on the weakest link theory, we thought that the different distribution of E_f among specimens might influence the results. Hence we tried to estimate the length effect with several methods.

Nonparametric method

We attempted to estimate the length effect on σ_t by the slope method. First, the 50th and 5th percentiles of the tensile strength distributions for each specimen were obtained by the nonparametric method according to ASTM standard D2915-94.[14] Next, the length effect parameters were obtained by fitting regression curves, as shown in

Table 1. Dimension, test span, and properties of specimens

Specimen	No.	Width (mm)	Thickness (mm)	Length (mm)	Test span (mm)	ARW (mm)	Density (g/cm^3)	E_f (GPa)
a	60	25	15	360	30	2.6 (29.6%)	0.526 (11.2%)	13.7 (20.2%)
b	56	25	15	390	60	2.4 (35.8%)	0.538 (11.7%)	14.6 (22.1%)
c	63	25	15	450	120	2.5 (33.3%)	0.531 (11.5%)	14.4 (18.5%)
Total	179					2.5 (32.8%)	0.531 (11.4%)	14.2 (20.3%)

ARW, annual ring width; E_f, Young's modulus measured by the longitudinal vibration method; a, b, c: short, normal, and long specimens, respectively
Values in parentheses are coefficients of variation
These values were measured before processing specimens into the shape needed for tensile tests

Table 2. Basic statistics for tensile strength (σ_t) and the correlation between it and other properties

Specimen	No.	Tensile strength (σ_t)			Correlation coefficients		
		Mean (MPa)	SD (MPa)	CV (%)	ARW	Density	E_f
a	60	111.4	32.2	29.6	−0.571	0.553	0.762
b	56	114.3	39.7	35.8	−0.742	0.680	0.793
c	63	103.3	27.5	26.7	−0.683	0.728	0.840
Total	179	109.4	33.4	32.8	−0.661	0.642	0.777

SD, standard deviation; CV, coefficient of variation

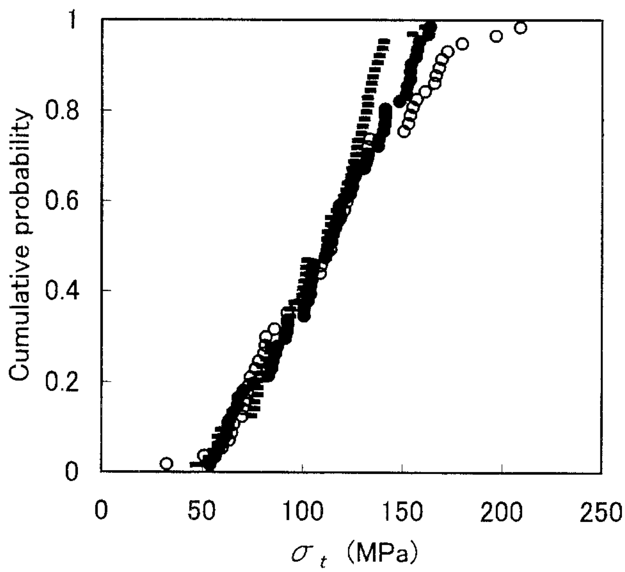

Fig. 2. Distributions of tensile strength (σ_t). *Filled circles*, *open circles*, and *horizontal bars*, specimens a, b, and c, respectively

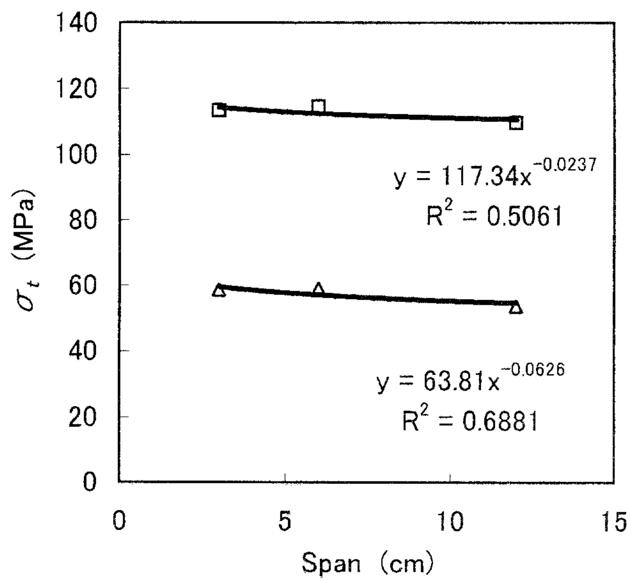

Fig. 3. Length effects by the nonparametric percentile point estimate. *Squares*, 50th percentiles; *triangles*, 5th percentiles

Fig. 3. The obtained parameters were 0.0237 and 0.0626 for the 50th and 5th percentiles, respectively. From above results, it could be found that the length effect seemed minimal by this method.

Projection method

It is difficult to estimate precisely the size effect by comparing the obtained σ_t distributions for some specimens of various lengths when their distributions of Young's modulus do not coincide well with each other. To solve this problem, Hayashi et al.[11] proposed a method in which the length effect parameter was estimated using adjusted σ_t data for Young's modulus. First, we calculated the regression line using Young's modulus (MOE) and tensile strength (TS): TS = $a \times$ MOE + b (where a and b are constants). Then we selected one MOE value as standard (MOE*). We then calculated [TS − $a \times$ (MOE − MOE*)] and used this value as the adjusted TS. The length effect parameter can then be

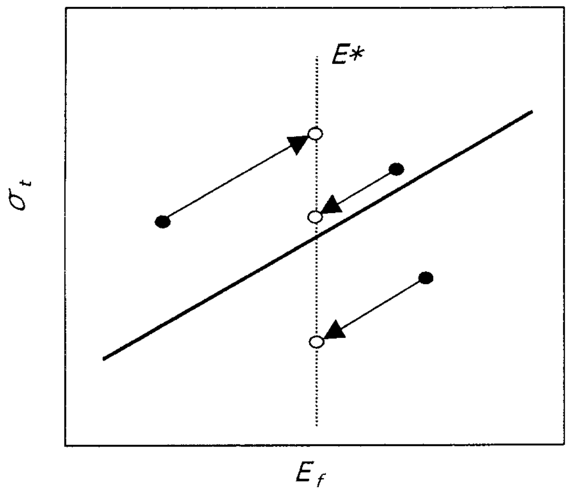

Fig. 4. Adjustment of σ_t data for standard Young's modulus (E^*) level. (Each adjustment σ_t was obtained by sliding the original σ_t onto the standard E level parallel to the regression line.) *Thick line*, regression line; *broken line*, E^*; *filled circles*, original σ_t data; *open circles*, adjusted σ_t data

Table 3. Distributions of adjusted σ_t data for each standard E (E^*) level

Specimen	Mean (MPa)	CV (%)	2P-Weibul	
			k	m
Low E^* level (11.3 GPa)				
a	90.2	23.2	4.83	98.4
b	82.9	29.2	3.88	91.5
c	76.6	19.5	5.72	82.6
Average E^* level (14.2 GPa)				
a	115.7	18.0	6.16	124.4
b	111.2	21.7	5.21	120.8
c	101.6	14.7	7.49	108.0
High E^* level (17.1 GPa)				
a	141.2	14.8	7.46	150.2
b	139.6	17.3	6.50	149.6
c	126.7	11.8	9.23	133.2

k and m, shape and scale parameters of 2P-Weibull, respectively

obtained as the slope (with a negative sign) of the linear regression line of the logarithm of the adjusted TS on the logarithm of the length. The concept of this method is summarized in Fig. 4. We might call this the projection method by setting a standard E level, as all σ_t data were projected on the standard E_f (E^*) level as a screen with lights parallel to the regression line.

Figure 5 shows the relation between E_f and σ_t and the regression lines. To examine the effect of the E^* level on the length effect parameter, we established three E^* levels: low (mean minus SD of E_f distribution of all specimens); average (mean value); and high (mean plus SD). The adjustment of σ_t data for each specimen was carried out with the above method; and distributions of the adjusted σ_t data are shown in Table 3. The parameters of 2P-Weibull in Table 3 were determined by the maximum likelihood method. Using the adjusted σ_t data, we obtained

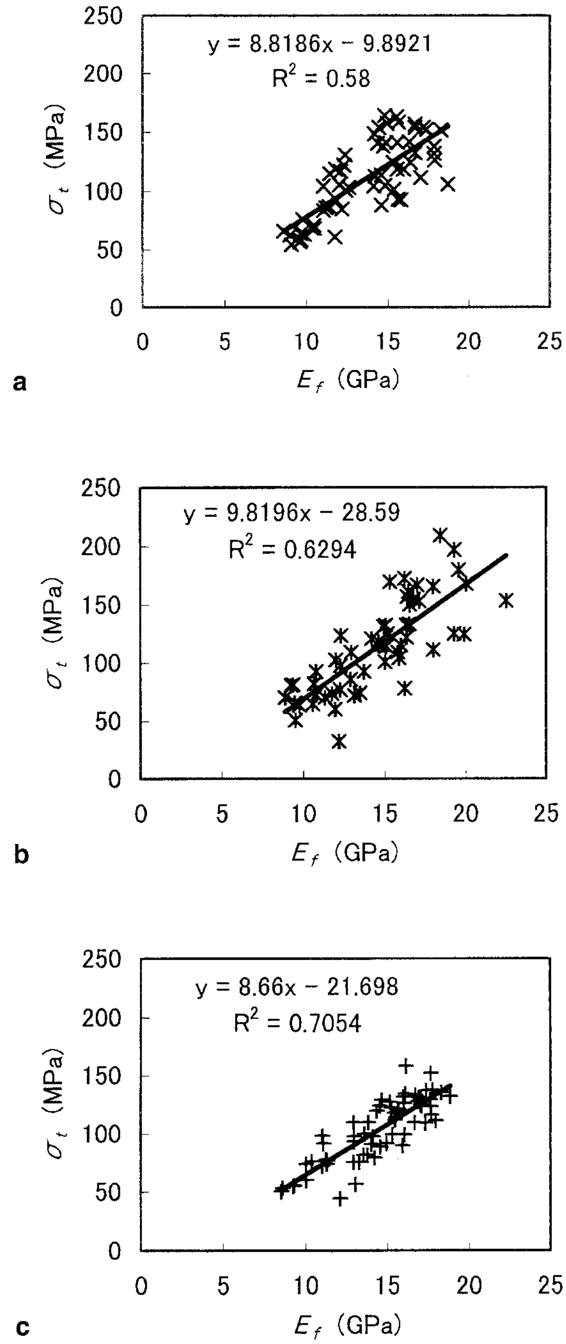

Fig. 5. Relation between E_f and σ_t for different test spans: 30 mm (**a**), 60 mm (**b**), 120 mm (**c**)

linear regression lines by the least-squares method for each E^* level.

$$\text{Low } (E^* = 11.3\text{ GPa}):\quad \ln(\sigma_t^*) = -0.1122\ln(L) + \ln(98.28) \quad (r^2 = 0.0555^*) \tag{1}$$

$$\text{Average } (E^* = 14.2\text{ GPa}):\quad \ln(\sigma_t^*) = -0.0898\ln(L) + \ln(126.15) \quad (r^2 = 0.0678^*) \tag{2}$$

$$\text{High } (E^* = 17.1\text{ GPa}):\quad \ln(\sigma_t^*) = -0.0759\ln(L) + \ln(153.59) \quad (r^2 = 0.0762^*) \tag{3}$$

where σ_t^* is adjusted σ_t data (MPa), L is the span (cm), r^2 is the determination coefficient, and ln means natural loga-

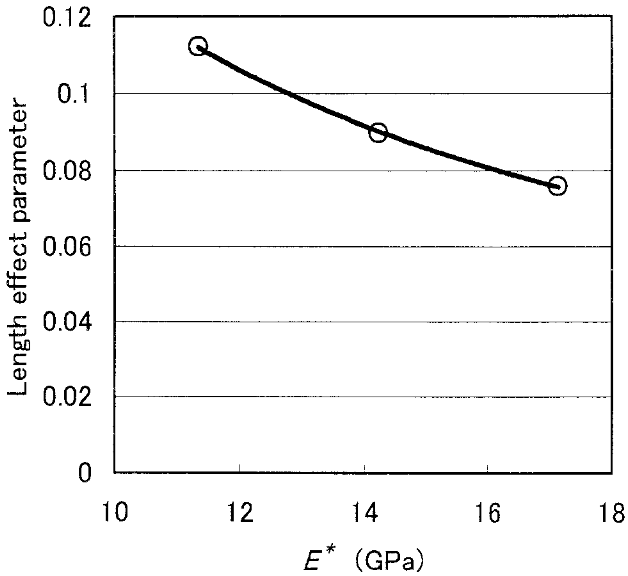

Fig. 6. Length effect parameter estimated using adjusted σ_t data for various standard Young's moduli (E^*)

rithm. It was found that the negative correlations were significant at the 5% level. The length effect parameters were obtained by multiplying by −1 and the coefficients of ln(L). The estimated length effect parameters by the projection method were 0.1122 for low E^*, 0.0898 for average E^*, and 0.0759 for high E^*. The relation between E^* and the length effect parameters by this method are shown in Fig. 6. With increasing E^* level, the length effect parameters showed a decreasing trend.

From the above discussion, it is clear that the estimated length effect parameters depend on the E^* level. If the mean Young's modulus values were always selected as E^*, the variation in the length effect parameters might be small. All these values were larger than the value for the 50th percentile (0.0237) by the nonparametric method. The differences of length effect parameters might be caused by the differences in E_f distributions among specimens.

Proposed method

We attempted another method for estimating length effect parameters to minimize the dependence of the differences in E_f distributions among specimens. Each σ_t is selected if the coupled E_f value is within a particular range, and the length effect parameter is estimated using the distribution of the selected σ_t data set. In this study, the range of E_f was set at 11.0–14.0 GPa for each specimen to compare it to the length effect of structural lumber reported in the literature.[15] 2P-Weibull was fit to the selected σ_t data sets, and the 50th and 5th percentiles were obtained using the 2P-Weibull distribution function for each specimen. The obtained parameters of 2P-Weibull are shown in Table 4. Means and standard deviations for each specimen are also in Table 4.

In Fig. 7 the 50th and 5th percentiles of σ_t, obtained from the parameters of 2P-Weibull, were plotted versus span

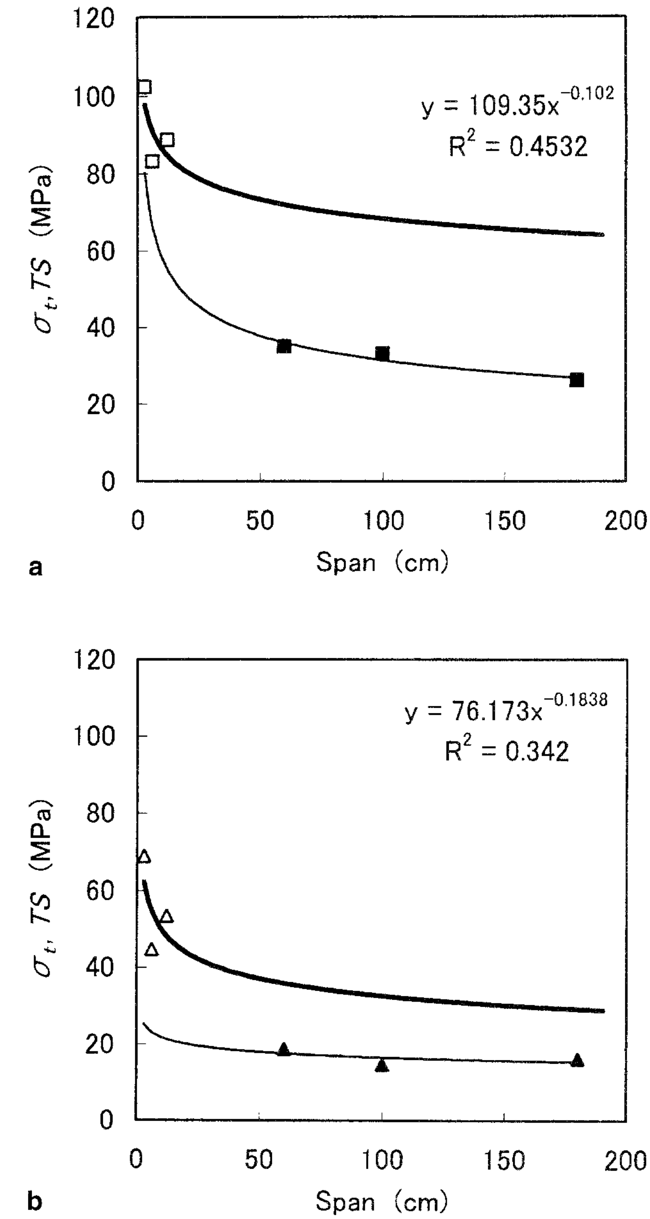

Fig. 7. Length effect using selected σ_t data compared to the referred tensile strength (TS) of structural lumber. **a** 50th Percentiles. *Open squares*, σ_t; *filled squares*, TS. **b** 5th Percentiles. *Open triangles*, σ_t; *filled triangles*, TS

Table 4. Distributions of selected σ_t data

Specimen	Mean (MPa)	CV (%)	2P-Weibul	
			k	m
a	100.7	18.4	6.60	108.1
b	82.6	28.3	4.18	90.9
c	87.7	23.0	5.13	95.3

with regression curves of selected σ_t data. Figure 7 also shows the tensile strength (TS) of structural lumber noted in the literature.[15] The estimated g values using selected σ_t data were 0.1020 and 0.1838 for the 50th and 5th percentiles, respectively. The decreasing tendency with span for 50th

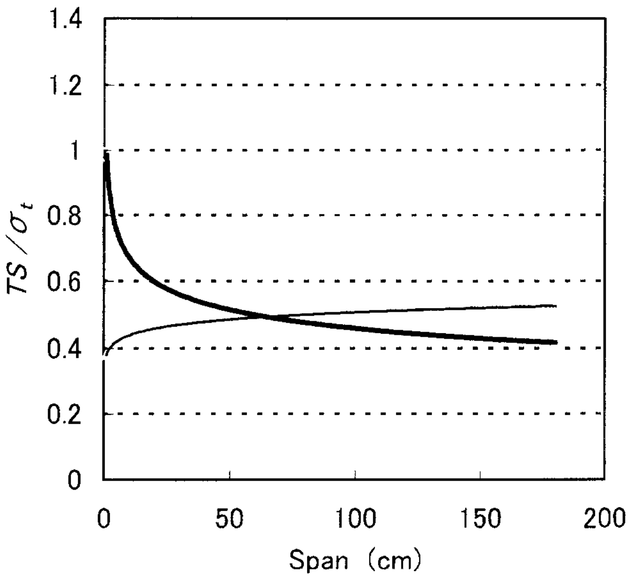

Fig. 8. Ratio of TS to σ_t. *Thick curve*, 50th percentile; *thin curve*, 5th percentile

percentiles was weaker than for the 5th percentiles in small clear specimens, in contrast to structural lumber. To clarify the relation between defect-free wood and structural lumber for practical use, the TS/σ_t was obtained using regression curves for the 50th and 5th percentiles. Figure 8 shows that the ratio for the 50th percentile distinctly decreased as the span increased below 50cm, whereas a slight increase was observed for the 5th percentiles. Above a 50-cm span, the variation was relatively small; and both were roughly 0.5. The strength ratio, defined as the ratio of strength in structural lumber to that in a small clear specimen, had been given 0.39 and 0.47 for tension[16] in ordinary-grade and high-grade lumber, respectively. It may be necessary to take into account the length effect when calculating the strength ratio.

Conclusions

An experimental study was done to estimate the length effect on tensile strength (σ_t) in a Japanese larch small clear specimen. The estimated length effect parameters (g) by the nonparametric method were 0.0237 and 0.0626 for the 50th and 5th percentiles, respectively. The g values from the projection method, based on establishing a standard E_f (E^*) level, were 0.1122 for low E^*, 0.0898 for average E^*, and 0.0759 for high E^*. The estimated g values by the proposed method using selected σ_t data were 0.1020 and 0.1838 for the 50th and 5th percentiles, respectively. Among the three methods, the nonparametric technique did not consider the different distributions of Young's modulus among specimens, and the estimated length effect parameters (g) by this method were small. The projection method reduced the influence of Young's modulus, but the length effect parameters varied with the E^* level. The proposed method minimized the dependence on E_f distributions among specimens. We think the latter is a desirable method for estimating the length effect on tensile strength.

References

1. Hayashi T (1998) Engineered wood. Nihon Mokuzai Shinbunsha, Tokyo
2. American Society for Testing and Materials (1995) Standard practice for establishing allowable properties for visually-graded dimension lumber from in-grade tests of full-size specimens. ASTM D1990–95, ASTM
3. Japanese Industrial Standards Committee (1994) Methods of test for woods: JIS Z 2101 (in Japanese). Japanese Standards Association, Tokyo
4. Madsen B (1992) Structural behaviour of timber. Timber Engineering, North Vancouver, pp 237–247
5. Madsen B (1990) Size effects in defect-free Douglas fir. Can J Civil Eng 17:238–242
6. Masuda M, Okohira Y (1985) Size effect of wood in bending (in Japanese). Bull Faculty Agric Mie Univ 71:61–69
7. Okohira Y, Masuda M, Suzuki N (1989) The size effect of compressive strength of wood (in Japanese). Bull Faculty Bioresources Mie Univ 2:13–21
8. Okohira Y, Masuda M, Suzuki N (1988) The size effect of tensile strength on western hemlock (in Japanese). Bull Faculty Bioresources Mie Univ 1:1–6
9. Saito I, Goto C, Fujita K, Zushi T, Komatsu K, Iijima Y (2000) Discussion on tensile strength of laminae used for glulam. In: Abstracts of the 50th annual meeting of the Japan Wood Research Society, Kyoto, p 125
10. Madsen B (1990) Length effects in 38mm spruce-pine–fir dimension lumber. Can J Civil Eng 17:226–237
11. Hayashi T, Miyatake A, Miyahara H (1997) Size effect on tensile strength of sugi laminae (in Japanese). Mokuzai Kogyo 52:15–19
12. Zhu J, Nakano T, Tokumoto M, Takeda T (2000) Variation of tensile strength with annual rings for lumber of Japanese larch. J Wood Sci 46:284–288
13. Sobue N (1986) Measurement of Young's modulus by the transient longitudinal vibration of wooden beams using a fast Fourier transformation spectrum analyzer. Mokuzai Gakkaishi 32:744–747
14. American Society for Testing and Materials (1994) Standard practice for evaluating allowable properties for grades of structural lumber. ASTM D2915–94
15. Takeda T, Hashizume T (1999) Differences of tensile strength distribution between mechanically high-grade and low-grade Japanese larch lumber. I. Effect of length on the strength of lumber. J Wood Sci 45:200–206
16. Sugiyama H (1976) Design of timber engineering structure (in Japanese). Maruzen, Tokyo, pp 80–85