J Wood Sci (2000) 46:159–163

NOTES

Hiroshi Yoshihara · Masamitsu Ohta

Estimation of the shear strength of wood by uniaxial-tension tests of off-axis specimens

Received: September 7, 1998 / Accepted: May 14, 1999

Abstract To determine shear strength we conducted uniaxial-tension tests of off-axis specimens and examined the proper off-axis angles. Sitka spruce (*Picea sitchensis* Carr.) and katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) were used for the studies. Uniaxial tension tests of the specimens with various off-axis angles were conducted, and the shear stress at failure was obtained. Independent of the tension tests, torsion tests were conducted, and the shear strengths were obtained. Comparing the data of the uniaxial tension and torsion tests, we examined the validity of estimating shear strength by the off-axis tension test. The shear strengths obtained from the tension tests coincided well with those measured by the torsion tests when the specimen had an off-axis angle of 15°–30°. In this off-axis angle range, the tensile stress perpendicular to the grain might have a serious influence on the shear strength, and we thought that the shear strength predicted by uniaxial tension tests should be treated as an approximate value despite the simplicity of the tension test. Other test methods should be adopted to obtain the precise shear strength of wood.

Key words Shear strength · Uniaxial-tension test · Off-axis specimen · Torsion test

Introduction

In several studies the shear strengths of wood were derived by torsion tests of rectangular bars.[1-4] In torsion tests, however, the shear stress does not distribute in the specimen homogeneously, and it is impossible to determine the shear strength from the torsion test data without supposing the extension of the shear stress during torsional loading. Hence, it would take a complex procedure to obtain the shear strength by torsion.[3,4]

In contrast, uniaxial-tension tests of off-axis specimens is easier than these methods, although the pure shear stress condition cannot be expected. When the stress condition is close to the pure shear stress condition, however, we believe that the shear strength obtained can be used as a proper approximation. Here, we performed uniaxial-tension tests of specimens with various grain orientations and attempted to determine the shear strength by comparing the uniaxial tension testing data with those obtained from the torsion tests.

Theory

In this study we defined the grain direction and the direction perpendicular to the grain as x and y, respectively. When a material with a grain orientation of ϕ is placed under tension and the uniaxial tensile stress σ_ϕ occurs in the material, the tensile stresses in the x- and y-directions (σ_x and σ_y, respectively) and the shear stress in the xy-plane, (τ_{xy}) can be written as follows:

$$\sigma_x = \sigma_\phi \cos^2\phi \quad (1a)$$
$$\sigma_y = \sigma_\phi \sin^2\phi \quad (1b)$$
$$\tau_{xy} = \sigma_\phi \cos\phi \sin\phi \quad (1c)$$

The tensile strengths parallel and perpendicular to the grain are measured at the angle $\phi = 0°$ and 90°, respectively, where the normal stresses σ_x and σ_y reach their maximal at these angles. The shear stress τ_{xy} is close to the shear strength of orthotropic symmetry at the angle where the value of τ_{xy} is maximal at a certain off-axis angle because the shear stress dominates the failure near the off-axis angle range. Daniel and Liber proposed that the off-axis angle of 10° is proper for determining the shear properties of fiber composites.[5] After their proposal, several studies were

H. Yoshihara (✉)
Faculty of Science and Engineering, Shimane University, 1060 Nishikawazu-cho, Matsue, Shimane 690-8504, Japan
Tel. +81-852-32-6508; Fax +81-852-32-6598
e-mail: yosihara@riko.shimane-u.ac.jp

Masamitsu Ohta
Graduate School of Agricultural and Life Sciences, The University of Tokyo, Tokyo 113-8657, Japan

conducted, and the validity of 10° off-axis tests has been accepted.[6–10]

Experiment

Specimen

Sitka spruce (*Picea sitchensis* Carr.) and katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) were used in this experiment. Specimens were conditioned at 20°C and 65% relative humidity (RH) before and during the tests.

Uniaxial-tension tests of off-axis specimens

Tensile strengths of off-axis specimens were measured by uniaxial tension tests. The shape and dimensions of the specimens are shown in Fig. 1. Plates made of kamba (Japanese birch, *Betula* sp.) were bonded to each end of the strip to avoid stress concentration at the grips. The strengths of the longitudinal direction, S_L, and the radial direction, S_R, were determined as the average of five test specimens. Specimens with grain angles of 0° to 30° at intervals of 5° and of 30° to 90° at intervals of 15° were prepared. A load was applied at the deformation velocity of 1 mm/min. The tensile strength corresponding to each grain orientation was obtained as the average of five test specimens for each orientation. From Eq. (1), the tensile strength was transformed to the stress components of orthotropic symmetry.

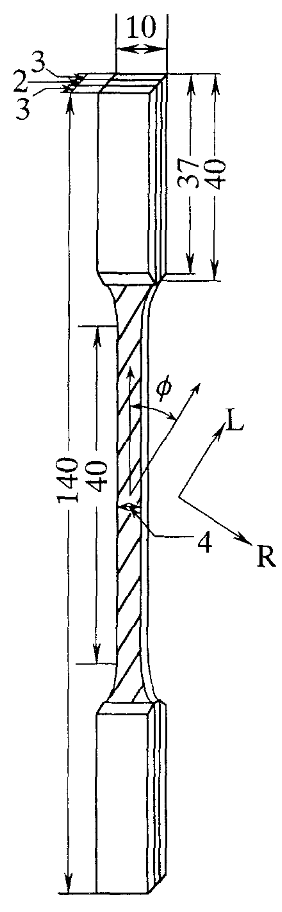


Fig. 1. Specimen for uniaxial tension test (unit: millimeters)

Torsion tests

Figure 2 shows the torsion test specimens. To avoid the stress concentrations imposed by the grips, specimens were cut to dog-bone shapes, and the longitudinal-radial (LR) or longitudinal-tangantial (LT) surfaces were wider than the others to ensure the occurrence of shear failure at the wider surface. We defined the specimen with the wider surfaces on the LR planes as "LR-type" and the others as "LT-type." For the LR-type, the x-, y-, and z-axes coincided with the radial, tangential, and longitudinal directions, respectively, whereas the axes coincided with the tangential, radial, and longitudinal directions for the LT-type. These specimens were twisted around the longitudinal direction (z-axis), and the torsional moment–torsional angle relations were obtained. The shear moduli in the LR and LT planes, G_{LR} and G_{LT}, were obtained from the following equation:

$$\begin{cases} G_{LR} = \left(\dfrac{M}{\theta}\right)_{LR} \cdot \left[a^3 b \left\{ \dfrac{1}{3} - \dfrac{2a}{b}\sqrt{\dfrac{G_{LR}}{G_{LT}}} \left(\dfrac{2}{\pi}\right)^5 \sum_{n=1}^{\infty} \dfrac{1}{(2n-1)^5} \cdot \tanh \dfrac{(2n-1)\pi b}{2a} \sqrt{\dfrac{G_{LT}}{G_{LR}}} \right\} \right]^{-1} \\ G_{LT} = \left(\dfrac{M}{\theta}\right)_{LT} \cdot \left[a^3 b \left\{ \dfrac{1}{3} - \dfrac{2a}{b}\sqrt{\dfrac{G_{LT}}{G_{LR}}} \left(\dfrac{2}{\pi}\right)^5 \sum_{n=1}^{\infty} \dfrac{1}{(2n-1)^5} \cdot \tanh \dfrac{(2n-1)\pi b}{2a} \sqrt{\dfrac{G_{LR}}{G_{LT}}} \right\} \right]^{-1} \end{cases} \quad (2)$$

where a and b are the width and thickness of the specimens, respectively, and $(M/\theta)_{LR}$ and $(M/\theta)_{LT}$ are the initial inclinations of torsional moment-torsional angle relationships of the LR and LT specimens, respectively. The shear moduli were obtained as the result of convergence by the successive approximation method.

The shear stress was obtained from the torsional moment–torsional angle relation. Originally, the shear stress on the surface of isotropic circular bar was derived by Nadai.[11] We converted his equation into that applicable for the orthotropic bar with a rectangular cross section. From the converted equation, the shear stress at the center of the LR plane is written as follows.[4]

$$\tau_{LR} = p_{LR} \left\{ M + 0.2 \cdot \frac{a^2 + b^2}{ab} \cdot \theta^2 \frac{d}{d\theta}\left(\frac{M}{\theta}\right) \right\} \quad (3)$$

where

$$p_{LR} = \frac{1}{a^2 bk} \cdot \left[\frac{-8}{\pi^2} \sqrt{\frac{G_{LR}}{G_{LT}}} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \cdot \tanh \frac{(2n-1)\pi b}{2a} \sqrt{\frac{G_{LR}}{G_{LT}}} \right] \tag{4}$$

and

$$k = \frac{1}{3} - \frac{2a}{b} \sqrt{\frac{G_{LT}}{G_{LR}}} \left(\frac{2}{\pi} \right)^5 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \cdot \tanh \frac{(2n-1)\pi b}{2a} \sqrt{\frac{G_{LR}}{G_{LT}}} \tag{5}$$

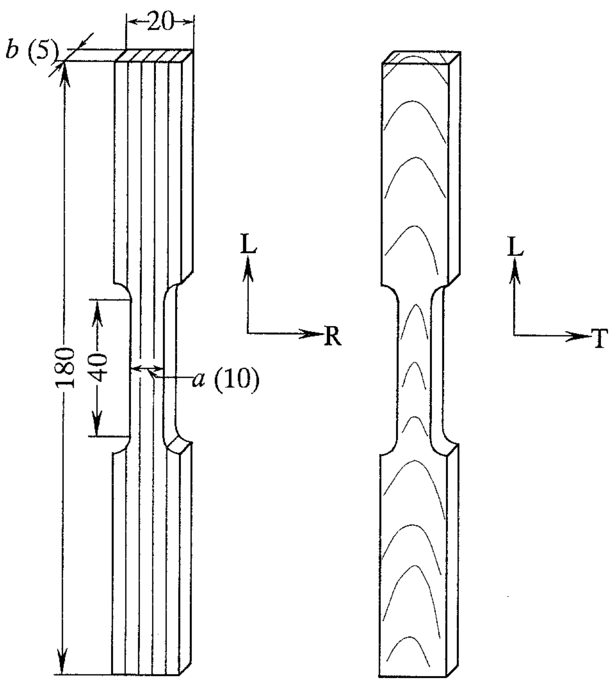


Fig. 2. Specimen for torsion test (unit: millimeters) **Left** LR-type. **Right** LT-type. LR, longitudinal-radial; LT, longitudinal-tangential

The shear strength, S_{LR}, was obtained by substituting the torsional moment and the torsional angle at failure into Eq. (3).

Results and discussion

Table 1 shows the shear modulus G_{LR} and shear strength S_{LR} on the longitudinal-radial (LR) plane obtained from the torsion tests. When the shear stresses obtained by the transformation of uniaxial tensile strength is close to these values, we may discuss the validity of uniaxial tensile tests of off-axis specimens for measuring shear strength.

Figure 3 shows the tensile strength σ_ϕ corresponding to the off-axis angle ϕ. Substituting σ_ϕ into Eq. (1), the maximum stress components in the orthotropic symmetry are shown in Fig. 4. As for the maximum shear stress as in Fig. 4a, the variance analyses were conducted, and we could not find significance among the data of ϕ = 10°–30° for spruce and ϕ = 15°–30° for katsura. The values for the shear stress in these off-axis angle ranges are close to the shear strengths obtained by the torsion tests.

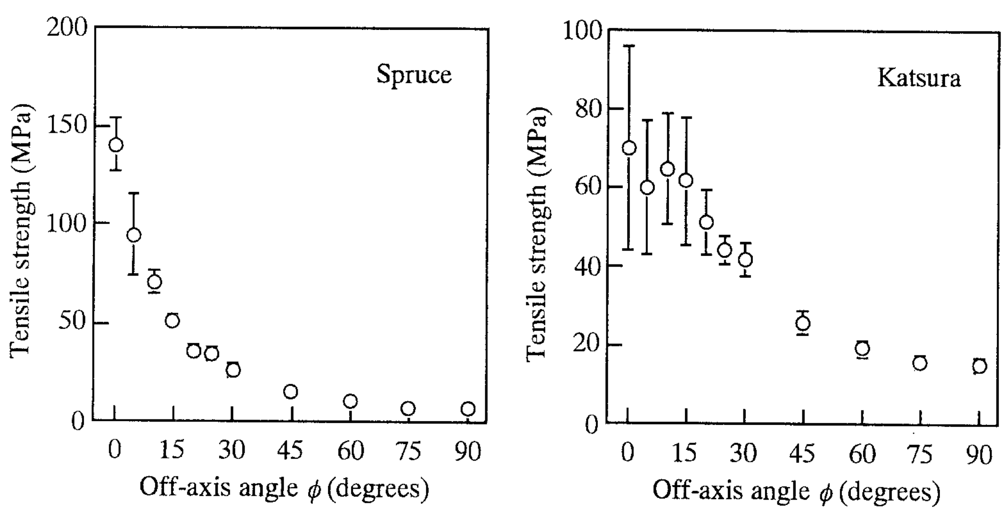
For the shearing characterization of composite materials, as previously mentioned, an off-axis angle of 10° is recommended.[5–9] We think that most of the composite materials examined there had shear strengths much smaller than the

Table 1. Shear modulus G_{LR} and shear strength S_{LR} obtained by the torsion tests

Species	G_{LR} (GPa)	S_{LR} (MPa)
Spruce		
Mean	0.60	11.3
COV (%)	5.1	4.1
Katsura		
Mean	0.81	15.9
COV (%)	11.2	12.2

COV, coefficient of variation. Five LR- and LT-type specimens were used for determining the shear modulus and shear strength
LR and LT, Same as in Fig. 2

Fig. 3. Tensile strength corresponding to the off-axis angle. *Circles* and *bars* are the average and standard deviations, respectively. Five specimens were used for determining the tensile strength corresponding to the off-axis angle



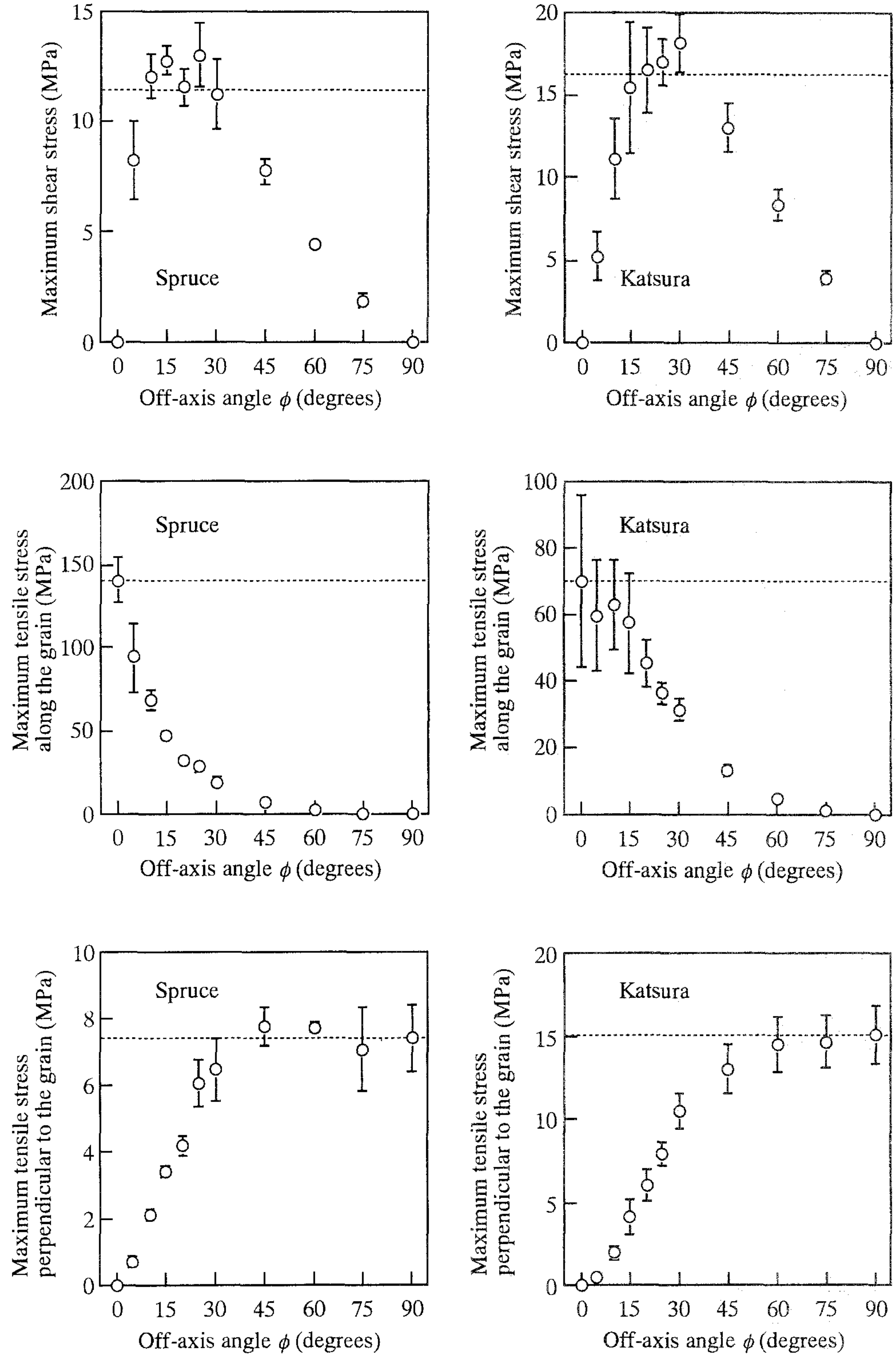


Fig. 4. Stress components in the orthotropic axis at the occurrence of failure corresponding to the off-axis angle. *Circles* and *bars*, same as in Fig. 3; *dashed line*, strength in the orthotropic symmetry

tensile strengths along the grain. When the difference between the tensile strength along the grain and the shear strength of orthotropic symmetry is small, the off-axis angle range where the shear stress dominates the failure would be small. Table 2 shows tensile strengths in the longitudinal and radial directions, S_L and S_R, respectively. As for spruce, which shows a large difference between the values of S_L and S_{LR}, the shear failure predominantly occurred at an angle of 10°. In contrast, the failure at 10° for katsura was dominated by the tensile stress along the grain because of the small difference between S_L and S_{LR}.

Figure 4 shows the tensile stress perpendicular to the grain at failure, σ_y corresponding to the off-axis angle ϕ. Although the shear stress τ_{xy} is stable in the off-axis angle

Table 2. Tensile strengths in the longitudinal and radial directions (S_L and S_R, respectively)

Species	S_L (MPa)	S_R (MPa)
Spruce		
Mean	141	7.4
COV	9.6	13.4
Katsura		
Mean	70	15.1
COV	36.8	11.7

COV, same as in Table 1. Five specimens were used for determining the tensile strengths

range of 15°–30°, the value of σ_y cannot be ignored in this angle range. This σ_y may have a serious influence on the measurement of shear strength.

Considering the experimental results mentioned above, we believe that the shear strength predicted by uniaxial tension tests should be treat as an approximate value despite the simplicity of the tension test. Other test methods should be adopted to obtain the precise shear strength of wood.

Conclusion

We tried to measure the shear strength of wood by uniaxial tension tests on off-axis specimens and obtained the following results. The shear strengths obtained from the tension tests coincided well with those measured by the torsion tests when the specimen had an off-axis angle of 15°–30°, which is larger than the angle recommended for fiber composites (10°). In this off-axis angle range, the tensile stress perpendicular to the grain might have a serious influence on shear strength, and we concluded that the shear strength predicted by uniaxial tension tests should be treated as an approximate value despite the simplicity of the tension test. Other test methods should be adopted to obtain the precise shear strength of wood.

References

1. Minami Y (1953) Torsion tests on wood. Mokuzai Kogyo 8:32–34
2. Suzuki N, Okohira Y (1982) On the measurement of shearing strength by means of the torsion test of wood sticks with rectangular cross section. Bull Mie Univ Dept Agric 65:46–49
3. Okusa K (1977) On the prismatical bar torsion of wood as elastic and plastic materials with orthogonal anisotropy. Mokuzai Gakkaishi 23:217–227
4. Yoshihara H, Ohta M (1997) Analysis of the shear stress/shear strain relationships in wood obtained by the torsion tests. Mokuzai Gakkaishi 47:457–463
5. Daniel IM, Liber T (1975) Lamination residual stresses in fiber composites. NASA CR-134826
6. Chamis CC, Sinclair JH (1977) Ten-degree off-axis test for shear properties in fiber composites. Exp Mech 17:339–346
7. Pindera MJ, Herakovich CT (1986) Shear characterization of unidirectional composites with the off-axis tension test. Exp Mech 26:103–112
8. Sun CT, Berreth SP (1988) A new end tab design for off-axis tension test of composite materials. J Comp Mater 22:766–779
9. Cron SM, Palazotto AN, Sandhu RS (1988) The improvement of end-boundary conditions for off-axis tension specimen use. Exp Mech 28:14–19
10. Pindera MJ, Choski G, Hidde JS, Herakovich CT (1987) A methodology for accurate shear characterization of unidirectional composites. J Comp Mater 21:1164–1184
11. Hill R (1950) Mathematical theory of plasticity. Oxford, London, p 94