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Prediction of bending properties for structural glulam using optimized distributions of knot characteristics and laminar MOE

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Abstract This study established a prediction model for bending properties of glued-laminated timber (glulam) using optimized knot and modulus of elasticity (MOE) distributions of lumber laminate as the main input variables. For this purpose, knot and MOE data were investigated for all pieces of lumber that were prepared for glulam manufacturing, and statistical distributions of knot size, knot number in one lumber, and MOE of each laminate were optimized as distribution functions. These knot and MOE data were used as input variables in the prediction model for bending properties, and were also used in generating virtual glulam using the inverse transform method. Prediction of bending properties for glulam was carried out using the transformed section method, which is partially provided in ASTM D 3737 (Annex A4). Predicted values were compared with those from full-scale four-point bending tests for 60 six-layered glulams with 10 different laminar combinations. Finally, the allowable bending properties of glulam for each specific laminate combination were determined by calculating the fifth percentile of the modulus of rupture and the average modulus of elasticity from virtual test results of more than 1000 virtual glulams. From the results of this study, predicted bending properties for glulam and their distributions could be used for structural design in both allowable stress design and limit state design.

Key words Bending properties of glulam · Knot characteristics · Optimized distribution · Inverse transform method · Transformed section method

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Introduction

Structural glued-laminated timber (glulam) is one of the oldest engineered wood products.[1] Compared with sawn timbers as well as other structural materials, glulam has several distinct advantages in size capability, architectural effects, seasoning, variation of cross sections, grades, and effect on the environment.[2] Especially in countries, such as Korea, that have difficulty in cultivating larger-diameter trees, glulam has been thought to be the best alternative material for larger structural members, which can be manufactured by laminating small-sized lumbers. Therefore, the establishment of design values for structural glulam manufactured with available lumber is very important.[3–5]

In general, the allowable stress design method (ASD), which has been one of the dominant methods in timber structures, assumes that allowable properties of a specific material are provided.[6] For most materials, these allowable properties could be determined by numerous full-scale tests. In the case of glulam, however, it is nearly impossible to perform a large number of full-scale failure tests to give the statistical values for a specific laminate combination. Therefore, much research has focused on prediction modeling.

There have been many attempts to model the performance of glulam, which could be divided into empirical I_k/I_g modeling, finite element modeling, and transformed section modeling.[7]

The first efforts to predict glulam beam performance were made by using an empirical method, refered to as the I_k/I_g method. This method accounts for the strength-reducing influence of knots as a function of their moments of inertia. This method determines allowable design stresses in bending for glulam members by multiplying bending stress indices by strength ratios for each lumber grade, and is the basis for the current American standard, ASTM D 3737.[8] However, statistical distributions of glulam beam strength are not predicted with this method.[9]

Because of the recent shift to reliability-based design, the major focus of glulam research has been to model statistical distributions accurately using finite element modeling.[10–13] In this approach, the laminations of the glulam beams were divided into elements, or cells, and input required the generation of clear wood densities and knot sizes and assigning them to each cell. The probability distributions of beam strength and stiffness were characterized using Monte Carlo simulation.

Another approach taken to analyze glulam beams was the transformed section method.[14] This method transforms the composite glulam cross section into that with the same properties so that simple elastic formulas can be applied, which were incorporated into the ASTM Standard D 3737 Annex 4. A transformed section analysis was performed to calculate the allowable load-bearing capacity of the beam.

Although much research has been carried out as stated above, there remains the need to obtain the design values for the glulam strength and stiffness using more accurate, practical, and statistical methods without the critical problems that have existed in the three models mentioned above.

For this reason, this research was performed to develop a new prediction model for the glulam strength and stiffness using the transformed section method and inverse transform method, which incorporate the simulation method.

The virtual glulams were manufactured by the inverse transform method, which generated the input values from the optimized distribution for knot, modulus of elasticity (MOE), and modulus of rupture (MOR) data. These input values were based on the actually measured data for knot and MOE, and regressed data for MOR. Virtual bending tests were then performed by the transformed section method, which was partially based on ASTM D 3737. More than 1000 virtual bending tests were used to produce statistical distributions of bending MOR and MOE for any specific glulam, and allowable design values were determined.

Theoretical background

Inverse transform method

The inverse transform method provides the most direct route to generating a random sample.[15] This method was used for generating virtual glulam with a specific laminate combination.[16] As stated above, prediction of bending properties for single glulam using the transformed section method needs knot size data, the MOE, and MOR for each lumber laminate. Through the analysis of actually measured knot and MOE data, optimized distributions for both variables were determined for each grade. Using the inverse transform method, a simulation model was developed to produce a certain lumber laminate, which conformed to selected distribution properties.

For example, the following equation would be applied in the case in which the Weibull distribution was the optimum one.[17]

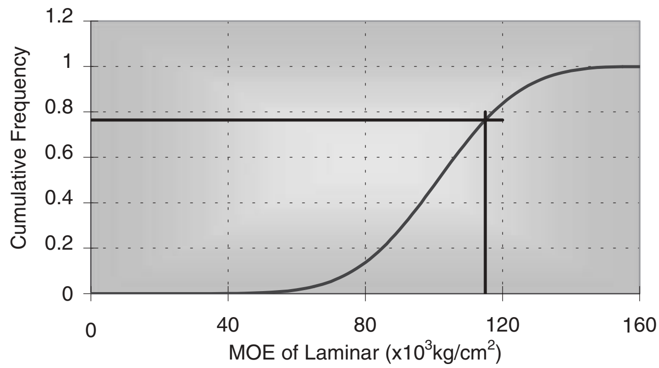

Fig. 1. An example of application of the inverse transform method to determine the modulus of elasticity (MOE)

$$\begin{aligned} F(x) &= 1 - e^{(x/\beta)^{\alpha}} && (\text{for } x > 0) \\ F(x) &= 0 && (\text{for } x \le 0) \end{aligned} \tag{1}$$

The inverse of this function was arranged as follows.

$$X = \beta(-\ln U)^{1/\alpha} \tag{2}$$

where, U is a random variable ranging from 0 to 1.

In practice, if the optimized cumulative distribution of MOE for a certain lumber grade is provided as in Fig. 1, any randomly selected y-axis value between 0 and 1 can be used to give the corresponding x-axis value as an input value.

Transformed section method

In order to predict the bending strength and stiffness of glulam, a computer program was developed based on the transformed section method, which is partially presented in ASTM D 3737 (Annex A4). Basically, sections with different properties were converted into transformed sections with same properties in this method.

In ASTM D 3737 (Annex A4), analysis of a glulam beam with three stiffness zones was provided as follows. For the three-zone beam depicted in Fig. 2, the transformed section moment of inertia factor, T_i, can be expressed as

$$T_i = \frac{E_1 d_1^3 - d_2^3(E_1 - E_2) - d_3^3(E_2 - E_3)}{E_1 d_1^3} \tag{3}$$

where E_1, E_2, and E_3 are the moduli of elasticity for the zones shown in Fig. 2, and d_1, d_2, and d_3 are the depths shown in Fig. 2.

The calculation for the I_k/I_G ratio becomes

$$\begin{aligned} R = \frac{1}{\sum_0^{n_1} Z} \Bigg\{ & x_1 \sum_{n_2}^{n_1} Z + \left(\frac{E_2}{E_1}\right) x_2 \sum_{n_3}^{n_2} Z + \left(\frac{E_3}{E_1}\right) x_3 \sum_{0}^{n_3} Z \\ & + \left[h_1^2 \sum_{n_2}^{n_1} Z^2 + \left(\frac{E_2}{E_1} h_2\right)^2 \sum_{n_3}^{n_2} Z^2 + \left(\frac{E_3}{E_1} h_3\right)^2 \sum_{0}^{n_3} Z^2 \right]^{1/2} \Bigg\} \end{aligned} \tag{4}$$

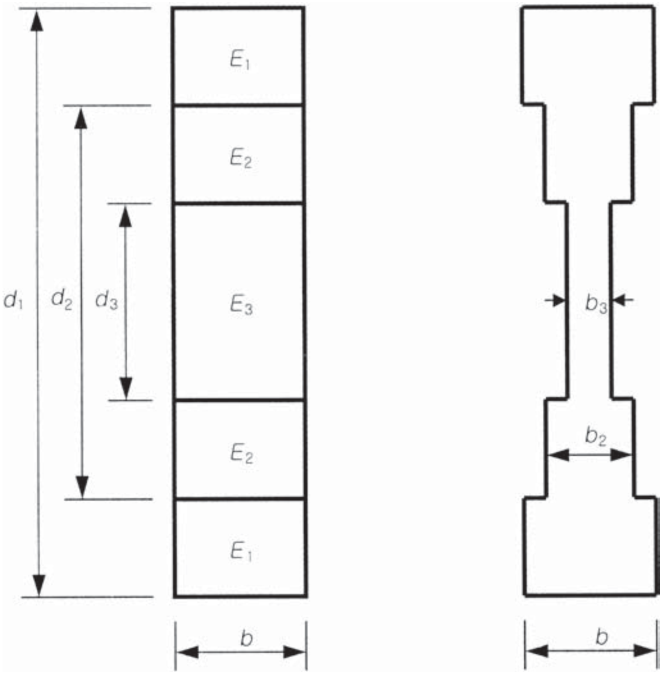

Fig. 2. Example of the transformed section method

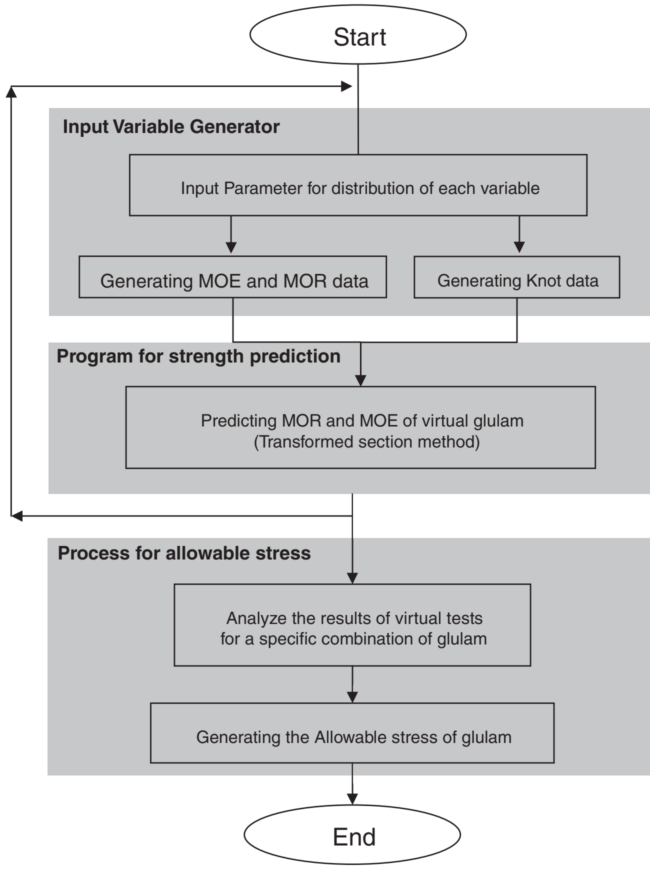

Fig. 3. Flow chart for the prediction model to produce the allowable stress of glulam. *MOR*, modulus of rupture

where $R = I_k/I_G$, x_1, x_2, and x_3 are the average knot sizes expressed as decimal fractions of the width for each virtually generated grade of lumber with average stiffness values of E_1, E_2, and E_3, respectively, h_1, h_2, and h_3 are the differences between the 99.5 percentile and average knot size as decimal fractions of the width of the respective grades, Z, Z^2 are the weighting factors, and n_1, n_2, and n_3 are the number of laminations in d_1, d_2, and d_3, respectively.

A corresponding bending stress modification factor, $\mathrm{SMF_b}$, can be determined by

$$\mathrm{SMF_b} = (1 + 3R)(1 - R)^3(1 - R/2)$$

Multiplication of $\mathrm{SMF_b}$ by the stress index results in an allowable bending stress.

As shown above, the input variables for the computer model using this transformed section method were knot data such as the average and 99.5 percentile knot sizes, and MOE and MOR data for each grade. In this study, however, the virtually generated values from actually measured data for each lumber laminate, except for MOR, were used as input values for more accurate prediction. Therefore, sizes and locations of knots that existed in each laminate were measured and recorded in order to obtain the input knot data.

MOE input data were obtained from the results of machine stress rating tests that were performed before the glulam manufacturing process. Consequently, the optimum distributions for knot size, knot number in each piece of lumber, and MOE for each grade were evaluated and determined for the purpose of generating virtual glulam in the computer modeling stage. These optimized distributions were determined using the Kolmogorov–Smirnov test on normal, lognormal, and Weibull distributions. Unfortunately, the MOR of each laminate could not be achieved from the measurement. Therefore, the MOR of each laminate was predicted using a regression relation between MOE and MOR, which was obtained from the results of small clear specimens for the same species.

After determining the optimum distribution types of input variables used in the transformed section method, many virtual glulam beams were manufactured using the inverse transform method, and these glulam were tested by the prediction model using the transformed section method. This whole procedure was simulated to produce the allowable bending properties of glulam using a computer program, and is summarized in Fig. 3.

Materials and methods

Materials

In this study, Japanese larch lumbers, produced in Korea, were used to manufacture the structural glulam. Dimension of the cross section of the laminate was 38 × 140 mm, and 3600 mm long. All lumbers for laminating were oven-dried to about 15% moisture content (MC). The average

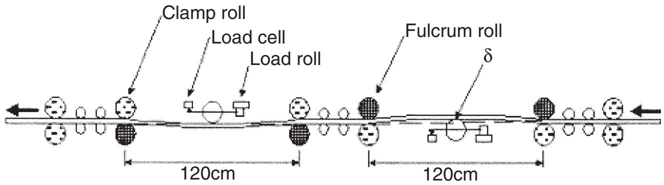

Fig. 4. Schematic diagram of machine stress rating method

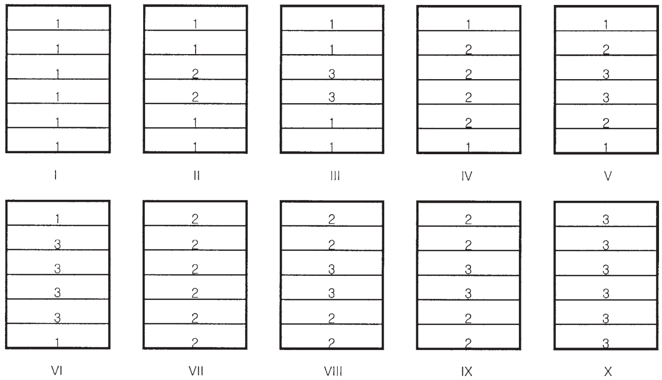

Fig. 5. Ten different laminate combinations applied to glulam manufacture

oven-dry specific gravity was 0.47 g/cm^3 and the average MC was 13.7%. A total of 300 small clear specimens of 10 × 10 × 300 mm were prepared from randomly selected larch lumbers.

Methods

Bending test for small clear specimens

All 300 small clear specimens were tested using the ASTM D 143 standard to obtain the relationship between the MOE and MOR of larch.

Investigation of knot data

Sizes and locations of all knots observed on the surface of laminar that were larger than 6 mm were measured and recorded for visual grading and investigating the knot data. For the purpose of continuous input and database efficiency, a web-posting program was developed and used. On the basis of the results of the knot data, lumbers were divided into the grades of No. 1, No. 2, No. 3, and off-grade.

Machine stress rating

The grading machine used to measure the MOE of laminates and for machine stress rating is illustrated schematically in Fig. 4. A total 692 larch lumbers were tested. As shown in Fig. 4, as each laminate passed through the grading machine, the MOE was calculated automatically from the displacement of the arranged rollers and the load data obtained by load cell.

Glulam manufacture

Lumber laminates, upon completion of visual grading, investigation of knot data, and machine stress rating, were planed to a thickness of 33 mm for spreading of adhesive. Six pieces of lumber were laminated and the final dimensions were about 200 × 140 × 3600 mm. A total of ten laminate combinations were manufactured (Fig. 5). For each combination, four to eight glulams were manufactured.

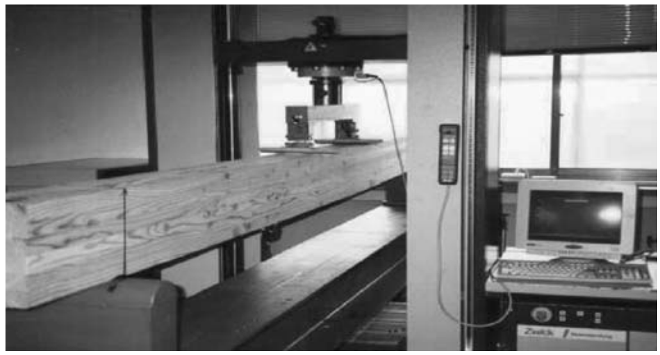

Fig. 6. Full-scale four-point bending test setup

Full-scale bending tests

Full-scale four-point bending tests were carried out for the manufactured glulam in accordance with ASTM D 198. The total span was 3.0 m, load span was 1.0 m, and the loading speed was 10 mm/min (Fig. 6).

Results and discussion

Regression relation between MOE and MOR

The MOR of each lumber laminate was predicted from the regression relation between MOE and MOR of the small clear specimens. The result is shown in Fig. 7.

From the results of the bending tests for 300 small clear specimens, it was considered that MOR was related significantly to MOE, showing an R^2 value of 0.53. Therefore, the regression equation was adopted for predicting the laminar MOR.

The dashed line shown in Fig. 7 is the regression discribed by Green and Kretschmann[18] for four species of softwood (Sitka spruce, western hemlock, Douglas fir, and southern pine). This regression was achieved from full-scale bending tests and showed lower MOR values than the those obtained with the regression of small clear specimens. This

Table 1. Statistical values of knot size distribution for 38 × 140 mm lumber

Grade	Average (%)	99.5 percentile (%)	Standard deviation	Optimum distribution	Average of log X	Standard deviation of log X
No. 1	10.3	26.2	4.23	Lognormal	2.26	0.35
No. 2	12.2	35.9	6.04	Lognormal	2.41	0.42
No. 3	17.1	49.7	9.92	Lognormal	2.69	0.54

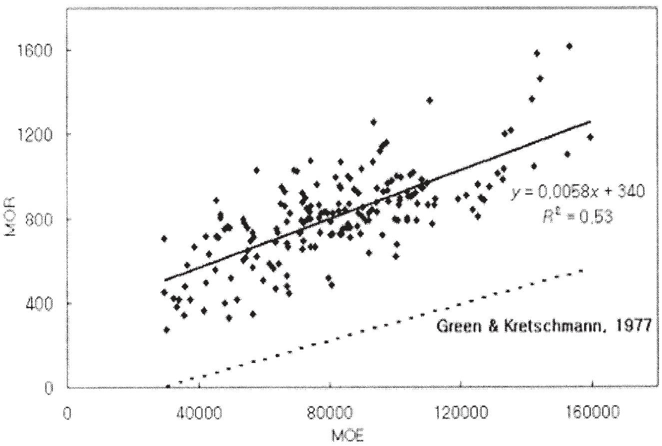

Fig. 7. Relationship between MOE and MOR for small clear specimens (kgf/cm^2). *Dotted line* shows the regression of Green and Kretschmann[18]

difference was caused by several defects such as knots. In this study, however, the effect of knots was included in the prediction model. It was concluded that the regression equation from small clear specimens could be applicable for predicting MOR of each lumber laminate.

Knot data

Input variables used to predict the strength properties of glulam are the average size of knots and the 99.5 percentile value of knot size for each lumber laminate. In order to generate the virtual glulam, the optimum distribution for knot size and optimum distribution for knot number within one laminate were determined using the Kolmogorov–Smirnov test.

Knot size

All knots not less than 6 mm were measured and recorded in accordance with ASTM D 3737 (Annex A6), and all measured knot sizes were included in the data-base. A total of 35 000 knots were analyzed using a visual basic program. In Fig. 8, distributions of knot size for larch (38 × 140 mm) lumber are presented for each grade. As shown in Fig. 8, the optimum distribution for knot size was determined as a lognormal distribution, and statistical distribution values for each grade are presented in Table 1.

Table 2. Statistical values of knot number in one laminate for 38 × 140 mm lumber

Grade	Optimum distribution	Average	Standard deviation
No. 1	Normal	40.9	8.1
No. 2	Normal	44.2	8.8
No. 3	Normal	35.2	11.3

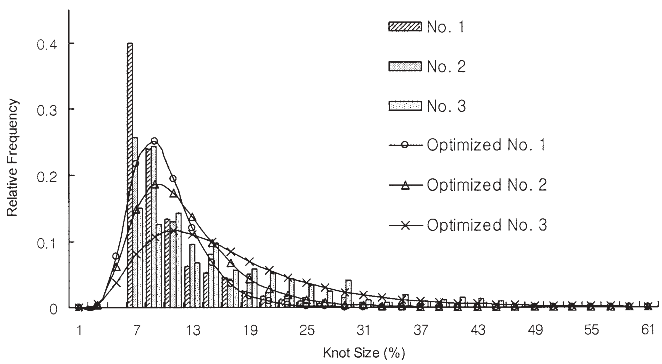

Fig. 8. Distribution of knot size for 38 × 140 mm lumber

Number of knots in one laminate

Investigation of the number of knots in each laminate indicated that there was little difference between each grade: the average value was 40.9 and the standard deviation was 8.08. In Fig. 9, the distributions for the number of knots in one piece of larch lumber for each grade are presented. As shown in this figure, the optimum distribution for the number of knots in one laminate was determined as a normal distribution. Distribution properties of the number of knots in one laminate for each grade are presented in Table 2.

Modulus of elasticity of laminates

All lumbers, upon completion of visual grading and investigation of knot data, were machine stress rated, and the distribution of MOE for the laminates was analyzed. In Fig. 10, the distributions of MOE for larch (38 × 140 mm) lumber measured by machine stress rating tests are presented for each grade. Optimized distribution and statistical parameters of MOE for laminates are given in Table 3.

Table 3. Optimized distributions and parameters of modulus of elasticity (MOE) for 38 × 140 mm lumber

Grade	Optimum distribution	Average ($\times 10^3$ kgf/cm^2)	Standard deviation ($\times 10^3$ kgf/cm^2)
No. 1	Normal	109	29.5
No. 2	Normal	106	25.9
No. 3	Normal	101	27.9

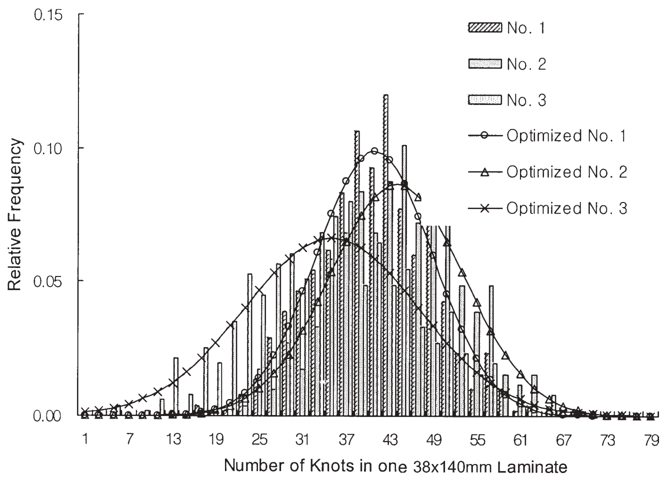

Fig. 9. Distribution of the number of knots in one laminate for 38 × 140 mm lumber

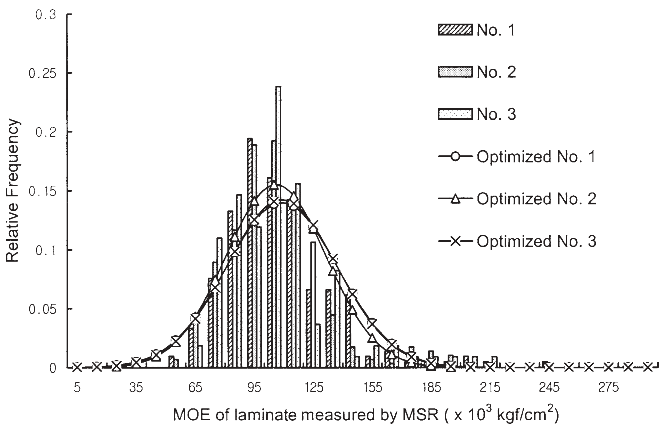

Fig. 10. Distribution of MOE for 38 × 140 mm lumber measured by machine stress rating

Comparison between predicted and measured bending properties

To verify the accuracy of the prediction model for bending strength and stiffness using the inverse transform and the transformed section method, comparisons of the bending MOR and MOE were made between real failure tests and simulation results. Very good agreement was observed (Figs. 11, 12). Bending MOE showed a more significant relationship between predicted and measured values than MOR.

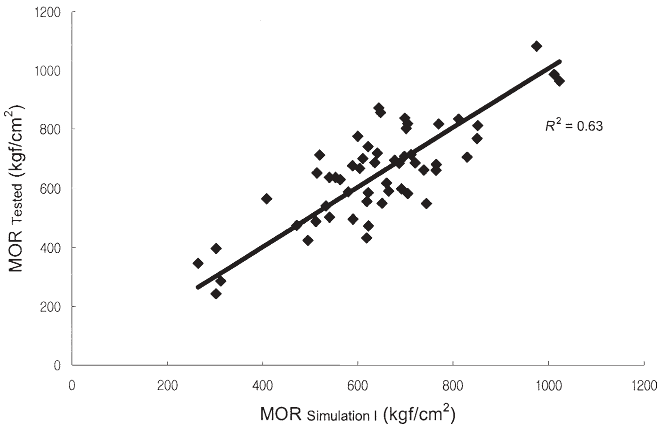

Fig. 11. Relationship between predicted (*simulation I*) and measured bending MOR

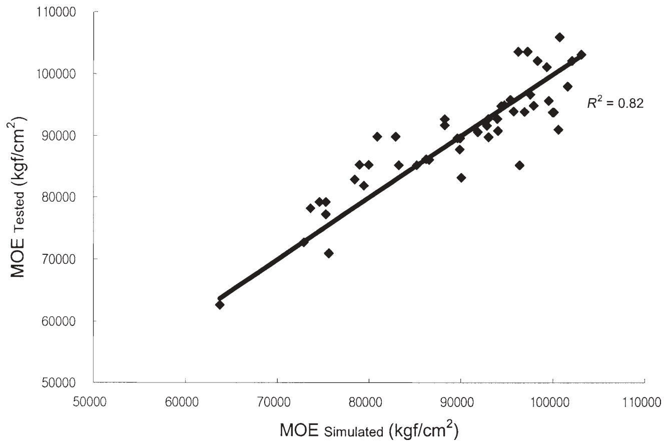

Fig. 12. Relationship between predicted and measured bending MOE

Statistical distribution of bending properties for glulam

In order to generate the virtual laminate, a simulation program was developed to generate the input variables, which were suitable for each optimum distribution, using the inverse transform method. Knot size, number of knots in one laminate, and MOE of laminate produced from this program were each confirmed with the respective measured optimum distributions. One data set of knot number and MOE for each laminate was generated for each process of the program, and knot size for each knot was generated at the same time. Finally, one process of the program resulted in one set of average and 99.5 percentile knot size, and MOE of one laminate. This process was repeated several thousand times for each grade. Among these virtually generated lumbers, six pieces of lumber were selected and laminated to give the proposed laminate combination, followed by virtual testing using the simulation program introduced above. More than 1000 virtual failure tests for glulam were carried out for a specific combination. Through the analysis of these virtual test results, the distributions of MOR and MOE for a specific combination of glulam could be produced. MOR and MOE for ten combinations of glulam are presented in Tables 4 and 5.

Table 4. Predicted bending modulus of rupture (MOR) of glulam

Combination	I	II	III	IV	V	VI	VII	VIII	IX	X
Average	885	868	868	831	828	789	759	740	698	605
Standard deviation	36.6	38.8	35.9	39.5	38.1	36.7	40.0	42.2	42.5	42.4
5th percentile value	833	824	825	782	785	749	703	690	651	548

Data given in units of kgf/cm^2

Table 5. Predicted bending MOE of glulam

Combination	I	II	III	IV	V	VI	VII	VIII	IX	X
Average	104.4	101.5	101.7	97.0	96.7	92.5	85.8	83.5	79.2	67.9
Standard deviation	4.54	3.93	3.61	3.33	3.14	2.85	3.75	3.60	2.95	3.52

Data given in units of $10^3 kgf/cm^2$

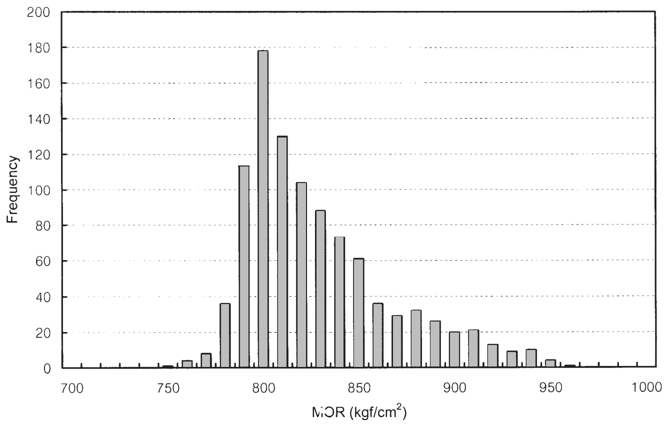

Fig. 13. The simulated distribution of MOR for type V glulam

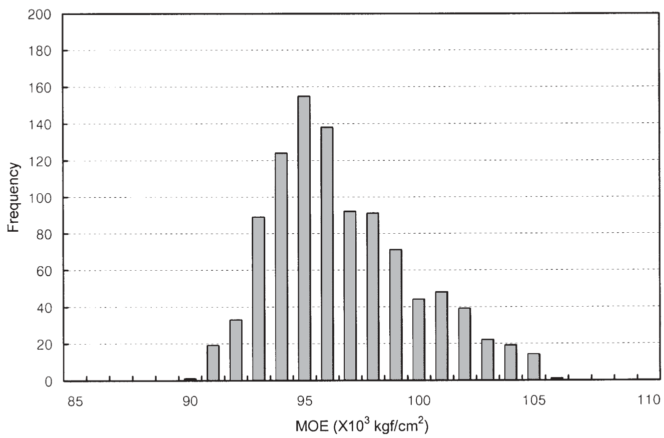

Fig. 14. The simulated distribution of MOE for type V glulam

In Figs. 13 and 14, the results of the simulation program developed in this study are presented for type V glulam. In case of MOR, the simulated distribution shows bias toward the left (lower MOR), which generally occurred in the strength distribution of full-scale lumbers. In all types of glulam, the Weibull distribution was determined as the optimum distribution among normal, lognormal, and Weibull probability distributions. For MOE, the normal distribution was determined as the optimum one.

Conclusions

In this study, MOR and MOE for glulam with a specific laminate combination were predicted by a simulation program. Optimized distributions of knot and MOE data were used as input variables for generating virtual glulam using the inverse transform method. A prediction model for bending properties of glulam was also developed using the transformed section method. Predicted values were compared with those from full-scale four-point bending tests for 60 six-layered glulams that had ten different types of laminate combinations. Finally, the distributions of MOR and MOE for glulam were determined from virtual test results of more than 1000 virtual glulams.

The conclusions are summarized as:

1. Optimized distributions of knot number and MOE for each laminate were determined as normal distributions, while knot size distribution showed a lognormal distribution.
2. Comparison between predicted and tested MOR of glulam indicated that the laminate MOR, one of the input variable, which was obtained from the regression relation between MOE and MOR of small clear specimens, produced reasonable results.
3. The prediction model developed in this study showed very good agreement with test results in both MOR and MOE of glulam.
4. The simulated distributions of MOR and MOE for all types of glulam showed Weibull and normal distributions as the optimum distribution, respectively.

From the results of this study, predicted bending properties for glulam and their distrisbutions could be used for structural design in both allowable stress design and limit state design.

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