ORIGINAL ARTICLE

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Fixed–fixed flexural vibration testing method of beams for timber guardrails

Abstract Resonance frequencies of beams with various types of end supports were examined for flexural vibration. Rectangular beams with dimensions of 300 (L) × 25 (R) × 5 or 10 mm (T) were used as the test specimens. Various compressing stresses were applied to the parts around both ends of a test beam and flexural vibration tests were conducted. The measured resonance frequency started to increase from the resonance frequency of a beam with simply supported ends and was stable around the resonance frequency of a beam with fixed ends as the compressing stress increased. The stable resonance frequency was lower than the theoretical value because perfect fixation of a beam to a post was difficult. From these results, the temporal change in resonance frequency itself, rather than the stable resonance frequency, is effective to examine whether a beam has enough strength as a guardrail.

Key words Boundary conditions · Fixed ends · Flexural vibration test · Resonance frequency · Timber guardrail

Introduction

In 1998, timber guardrails were approved for use in national and prefectural roads if they could pass a crash test with cars and trucks. Various timber guardrails have been subsequently developed and more than 50 km of timber guardrails are in use today.[1,2]

A method to estimate deterioration of wood used for guardrails, however, has not been established. Consequently, it is difficult to decide when the guardrails should be replaced.

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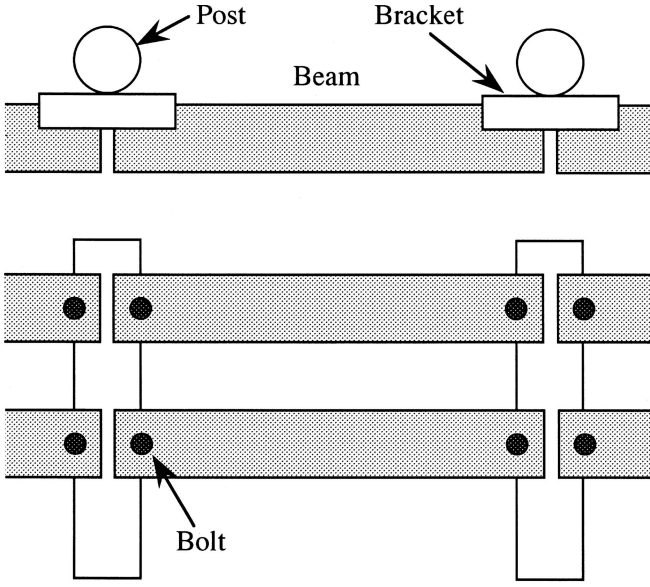Hokkaido Forest Products Research Institute, Asahikawa 071-0198, Japan

For this purpose, obtaining the changes in strength over time of the wood for the timber guardrail is important. Although the strength can be measured accurately by removing the wood from a guardrail, this method is difficult to adopt because it takes much time. Hence, a testing method by which the appropriate strength can be measured in the lumber fixed to a post of a guardrail is needed. Naturally, the test should be conducted without rupturing the specimen.

Young's modulus is one of the properties that is related to the strength and can be obtained without damaging the specimen. One method for measuring Young's modulus is a flexural vibration test.

The flexural vibration test has been widely used because of its simplicity. There have been several studies on its testing method: for example, the supporting position of a specimen, the distance between a specimen and a head of a vibration generator,[3] and gripping of a cantilever-type specimen have been examined. However, these studies do not assume measurements in actual end conditions of a beam used as a guardrail but rather for typical end conditions: hinged, clamped, sliding, and free.

A beam for a timber guardrail is fixed to a post by a bolt through a bracket (Fig. 1). Therefore, the end conditions of the beams used as timber guardrails are thought to be between those of simply supported ends and fixed ones. In actual structures, however, the resonance frequency of beams with simply supported ends is increased by 20% when taking the restriction by joints and the effect of floor joists into account.[4–6]

On the other hand, the resonance frequency of a beam with fixed ends is about twice as large as that of a beam with simply supported ends, according to the Euler-Bernoulli elementary theory on bending. Hence, end conditions of a beam must be understood to accurately measure the Young's modulus of wood lumber in actual structures. We investigated matters that require attention in conducting the fixed–fixed flexural vibration test.

Fig. 1. Timber guardrail

Fig. 2. A test beam supported elastically

Theory

An elastically supported beam

According to the Euler-Bernoulli theory, the differential equation for the bending of a beam is

$$EI = \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0, \tag{1}$$

where E, I, y, x, ρ, A, and t are Young's modulus, moment of inertia of a cross section, lateral deflection, distance along the beam, density, cross-sectional area, and time, respectively.
Solving Eq. 1 gives us

$$y = C_1 \cos \lambda_n x + C_2 \sin \lambda_n x + C_3 \cosh \lambda_n x + C_4 \sinh \lambda_n x, \tag{2}$$

where C_1–C_4 are constants, and

$$\lambda_n^4 = \frac{(2\pi f_n)^2 \rho A}{EI}, \tag{3}$$

where f_n and n are resonance frequency and resonance mode number, respectively. Then, resonance frequency is

$$f_n = \frac{m_n^2}{2\pi l^2} \sqrt{\frac{EI}{\rho A}}, \tag{4}$$

where $m_n = \lambda_n l$ and l is length.
The first resonance mode is discussed in this study. For symmetric modes, $C_2 = C_4 = 0$.
When the ends of a beam are restricted by springs for vibration in the y-direction (k_1 and k_3 in Fig. 2) and rotation (k_2 and k_4 in Fig. 2), the boundary conditions are
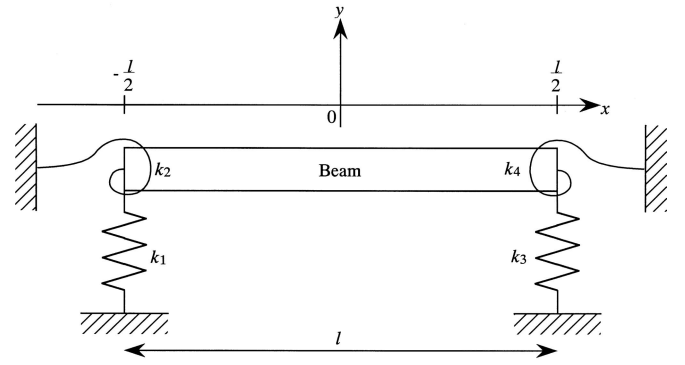
$$\begin{cases} x = -\dfrac{l}{2} : EI \dfrac{\partial^3 y}{\partial x^3} = -k_1 y, \ EI \dfrac{\partial^2 y}{\partial x^2} = k_2 \dfrac{\partial y}{\partial x} \\[2mm] x = \dfrac{l}{2} : EI \dfrac{\partial^3 y}{\partial x^3} = k_3 y, \ EI \dfrac{\partial^2 y}{\partial x^2} = -k_4 \dfrac{\partial y}{\partial x} \end{cases} \tag{5^{7,8}}$$

Substituting Eq. 2 into Eq. 5 under conditions of $k_1 = k_3$ and $k_2 = k_4$ gives

$$\begin{bmatrix} EI\lambda_n^3 \sin \dfrac{\lambda_n l}{2} - k_1 \cos \dfrac{\lambda_n l}{2} & EI\lambda_n^3 \sinh \dfrac{\lambda_n l}{2} - k_1 \cosh \dfrac{\lambda_n l}{2} \\[2mm] -EI\lambda_n \cos \dfrac{\lambda_n l}{2} - k_2 \sin \dfrac{\lambda_n l}{2} & EI\lambda_n \cosh \dfrac{\lambda_n l}{2} + k_2 \sinh \dfrac{\lambda_n l}{2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{6}$$

When the solution of Eq. 6 is not $C_1 = C_3 = 0$, the determinant of the matrix is 0. Then, the following equation is obtained.

$$\left(EI\lambda_n^3 \tan \frac{\lambda_n l}{2} - k_1 \right)\left(EI\lambda_n + k_2 \tanh \frac{\lambda_n l}{2} \right)$$
$$+ \left(EI\lambda_n^3 \tanh \frac{\lambda_n l}{2} - k_1 \right)\left(EI\lambda_n + k_2 \tan \frac{\lambda_n l}{2} \right) = 0. \tag{7}$$

With both end parts of a specimen being supported by posts in our experiment, k_1 is assumed to be ∞. Consequently, Eq. 7 is

$$\tan \frac{m_n}{2} + \tanh \frac{m_n}{2} = -\frac{2EI}{k_2 l} m_n. \tag{8}$$

If $k_2 = 0$, Eq. 8 becomes

$$\tan \frac{m_n}{2} + \tanh \frac{m_n}{2} = \infty. \tag{9}$$

Hence, m_1 is π. This is the value for a simply supported beam.
If $k_2 = \infty$, Eq. 8 becomes

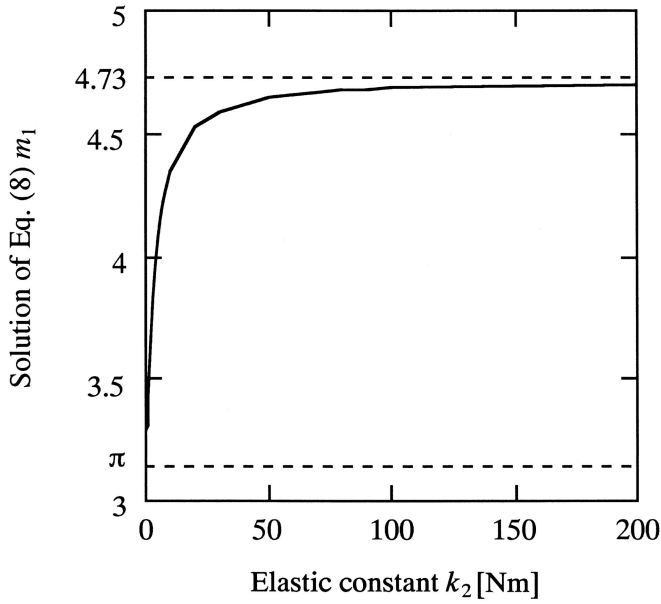$$\tan \frac{m_n}{2} + \tanh \frac{m_n}{2} = 0. \tag{10}$$

Fig. 3. Solutions of Eq. 8 for various k_2. Parameters used: Young's modulus 12 GPa, width 25 mm, thickness 5 mm, length 300 mm

Hence, m_1 is 4.730. This is the value for a beam with fixed ends.

The m_1 of Eq. 8 is plotted to k_2 as shown in Fig. 3.

Timoshenko theory of bending

Flexural vibration is influenced by shear and rotary inertia. Timoshenko added these terms to the Euler-Bernoulli elementary theory of bending and developed the following differential equation of bending:[9]

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} - \rho I\left(1 + \frac{sE}{G}\right)\frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{s\rho^2 I}{G}\frac{\partial^4 y}{\partial t^4} = 0, \quad (11)$$

where G is shear modulus and $s = 1.18$ is the shear deflection coefficient.[10]

When Eq. 11 is solved under the fixed–fixed condition, the resonance frequency f_n can be written as follows:

$$f_n = \frac{p_n^2}{2\pi l^2}\sqrt{\frac{EI}{\rho A}}. \quad (12)$$

The value of p_n in Eq. 11 is obtained by the following transcendental equations:

$$\frac{\tanh\frac{p_n}{2}\sqrt{\sqrt{B_t^2 p_n^4 + 1} - A_t p_n^2}}{\tan\frac{p_n}{2}\sqrt{\sqrt{B_t^2 p_n^4 + 1} + A_t p_n^2}} = -\frac{\sqrt{\sqrt{B_t^2 p_n^4 + 1} + A_t p_n^2}}{\sqrt{\sqrt{B_t^2 p_n^4 + 1} - A_t p_n^2}} \quad (13)$$

for symmetric modes, where

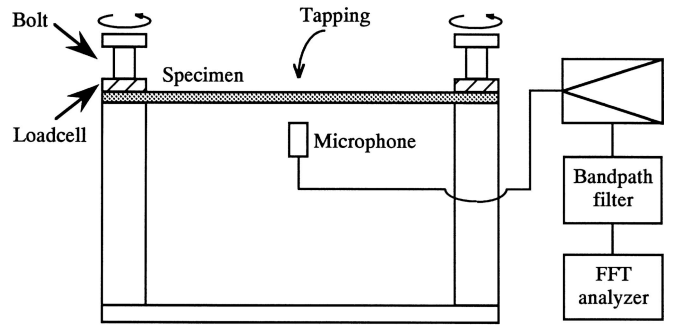$$A_t = \frac{I}{2Al_2}\left(1 + \frac{sE}{G}\right), \quad (14)$$

Fig. 4. The vibration test under various end conditions

and

$$B_t = \frac{I}{2Al^2}\left(-1 + \frac{sE}{G}\right). \quad (15)$$

Experimental

Specimens

To investigate wood species with various density, kiri (paulownia, *Paulownia tomentosa* Steud.), sugi (Japanese cedar, *Cryptomeria japonica* D. Don), akaezomatsu (Sakhalin spruce, *Picea glehnii* Mast.), shioji (ash, *Fraxinus spaethiana* Lingelsh.), and shirakashi (white oak, *Quercus myrsinaefolia* Blume) were used as specimens. The dimensions of each specimen were 300 mm (longitudinal, L) in length, 25 mm (radial, R) in width, and 5 or 10 mm (tangential, T) in thickness.

The specimens were conditioned at 20°C and 65% relative humidity for several months. The tests were conducted under the same conditions.

Vibration test

To obtain the resonance frequency of the first mode by bending, flexural vibration tests were conducted by the following procedure. An apparatus (End condition controller KS-200 Takachiho Seiki) shown in Fig. 4 was used to provide various end conditions. The regions of 25 (L) × 25 mm (R) from both ends were supported by the posts of the apparatus, the cross section of which was 25 × 25 mm. By screwing a bolt attached to a load cell, the test beam was compressed. The compressing load was measured by the load cell and recorded by a data logger. The vibration was excited in the direction of the thickness at the center part by a hammer. Motion of the beam was detected by a microphone in the center part. The signal was processed through a fast Fourier transform (FFT) digital signal analyzer to yield high-resolution resonance frequencies.

A free–free flexural vibration test was also undertaken to measure the Young's and shear moduli. A test beam was
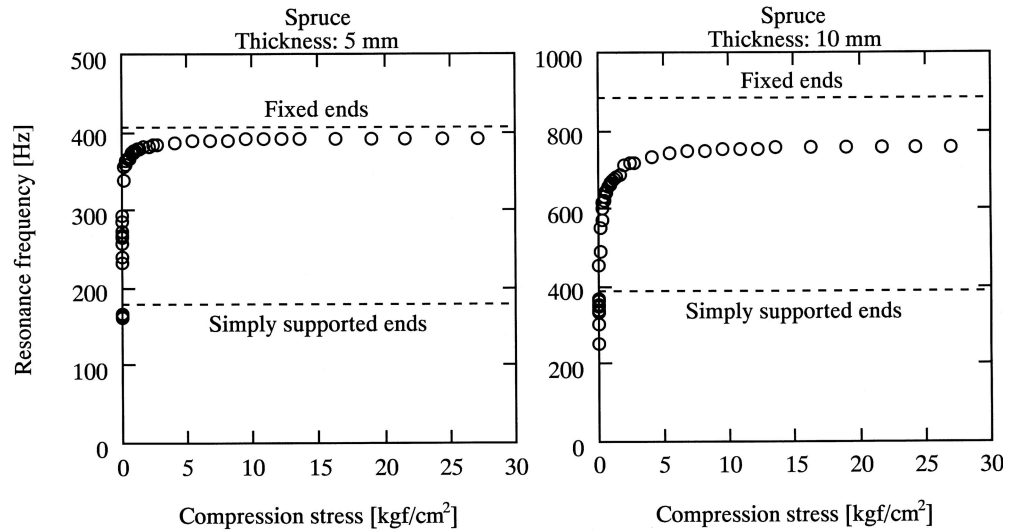
suspended by two threads at the nodal positions of the free–free vibration corresponding to its resonance mode. The vibration was excited and recorded by the above-described method. The Young's and shear moduli were calculated with the following regression method, which is called the "TGH method" in this article.

Goens[11] approximated the solution of the Timoshenko differential equation (Eq. 11) by a Taylor series. Hearmon[12] regarded the approximated equation of the solution as a linear equation. Using the Young's moduli based on the Euler-Bernoulli elementary theory of bending of several resonance modes, the regression is made and the Young's and shear moduli can be obtained from the slope and intercept.[12] By the TGH method, Young's modulus (E_{TGH}) and shear modulus can be calculated at the same time using only the flexural vibration test.

Substituting E_{TGH} and $m_n = 4.730$ (the frequency equation of a beam with free ends is the same as Eq. 10) into Eq. 4, the resonance frequency without the shear and rotary inertia effects (f_0) was obtained. On the other hand, the resonance frequency taking the shear and rotary inertia effects into account (f_S) was calculated by solving Eq. 13 with "Mathematica Ver. 3.0" software (Wolfram Research) using E_{TGH} and the shear modulus, and then substituting E_{TGH} and the obtained p_n into Eq. 12.

Results and discussion

For all the wood species used for the experiments, the measured resonance frequency increased rapidly early in the increasing compression load process and approached the value of the fixed ends (Fig. 5). This trend was similar to the m_1–k_2 relationship shown in Fig. 3. These results show that the measured resonance frequency, and consequently the specific Young's modulus, ran the risk of being smaller than the true value if the wood was not fixed completely to the post of a guardrail. It is difficult to place and remove a load cell or a torque meter on site. Therefore, the stable reso-

Table 1. Ratio of measured resonance frequency to estimated frequency

Wood type	f_M/f_0	f_S/f_0
Thickness: 5 mm		
Paulownia	0.957	0.997
Cedar	0.959	0.998
Spruce	0.952	0.997
Ash	0.940	0.996
Oak	0.941	0.997
Thickness: 10 mm		
Paulownia	0.856	0.990
Cedar	0.885	0.992
Spruce	0.841	0.987
Ash	0.803	0.984
Oak	0.867	0.991

f_M, The measured resonance frequency at the maximum compression stress; f_S, the resonance frequency taking the effect of shear deformation and rotary inertia into account estimated by substituting E_{TGH} and p_n into Eq. 12; f_0, the resonance frequency without the effect of shear deformation and rotary inertia estimated by substituting E_{TGH} and $m_n = 4.73$ into Eq. 4

nance frequency should be recorded by increasing the load for fixing it.

However, it should be noted that the measured resonance frequency that was stable at larger compression stress was smaller than f_0: for example, the resonance frequency at the maximum compression stress (f_M) was about 95% (thickness: 5 mm) and 85% (thickness: 10 mm) of f_0 (Table 1). These discrepancies cannot be ignored, especially in the case of thickness = 10 mm: the specific Young's modulus is $0.85^2 = 0.72$ of the true value.

The resonance frequency of the part of a beam without compression stress is decreased by the shear and rotary inertia effects in flexural vibrations when a beam is thick and resonance mode number is large.[9] Thus, such effects were examined. The fact that f_S decreased by only 0.3%–1% and less than f_M (Table 1) means that the shear and rotary inertia effects in the part without the compression stress were not serious.
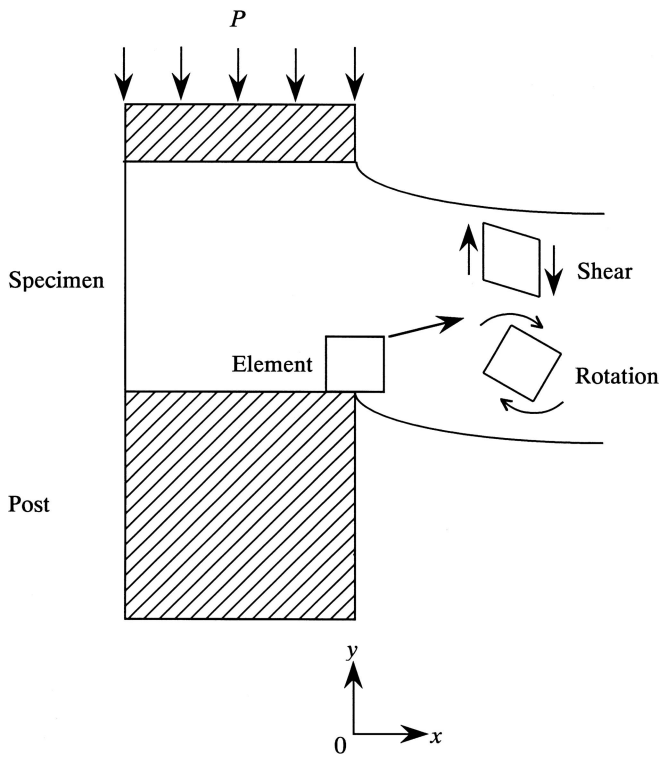
Fig. 6. Deformation of an element at compressed parts

Fig. 7. Method for solving Eq. 8

Next, the shear deformation and rotation at the compressed part was examined to investigate the poor fixation. Although the part of a specimen around a post apparently did not vibrate in the thickness direction because of compression, it would be difficult to restrict the shear deformation and rotation perfectly: dividing a test specimen into many elements like a finite element method (FEM) simulation, the shear deformation and rotation of elements around $x = 0$ in Fig. 6 will not be restricted sufficiently. The length–thickness plane of such an element will be shear-deformed and rotated easily during vibration in the thickness direction. In this case, the slope around the compressed part is not 0 and k_2 will not be ∞.

According to the theory described above, under the condition of $k_1 = \infty$ and $k_2 \neq \infty$, $\frac{m_n}{2} + \tanh\frac{m_n}{2} \neq 0$ in Eq. 10. Hence, m_1 is smaller than the value for a beam with fixed ends (Fig. 3), thus, the resonance frequency decreases.

Assuming that the size of the element is the same, there are more elements in the thickness direction in a thick specimen than in a thin one. Therefore, the resonance frequency decreased more in the case of thickness = 10mm than 5mm. When $k_2 \neq \infty$, m_n is also influenced by E, I, and l from Eq. 8 (see Fig. 7).

Although it is difficult to obtain the resonance frequency accurately from the fixed–fixed flexural vibration test mentioned above, this testing method is nevertheless effective to maintain and control qualities of timber guardrails. This testing method is useful for the 100% inspection of wooden beams for timber guardrails and the results dominate mechanical properties.
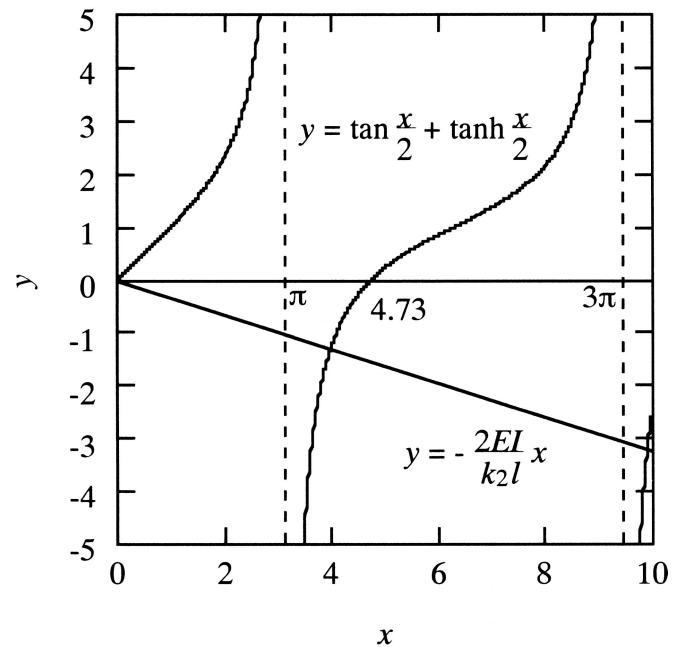
Members of metal guardrails are manufactured in factories so that their qualities meet the requirements of Ministry of Land, Infrastructure, and Transport of the Government of Japan, which means that all members are tested. Similarly, the 100% inspection will have to be conducted in the case of timber guardrails. Testing all wooden beams directly after constructing timber guardrails enables the initial properties and location of each beam to be obtained simultaneously. Consequently, periodic inspections of timber guardrails will be simplified. For example, inspections can be concentrated on beams with lower initial resonance frequency.

A visual inspection is also important and can be performed simply, but the vibration test has particular advantages over it. The resonance frequency reflects the mechanical properties of the inner layers, unlike the the visual inspection which can only provide information on the surface. Results of the visual inspection may also be different from worker to worker. Even if a wooden beam looks deteriorated, it may not be necessary to replace a beam if the mechanical qualities fulfill the requirements.

Conclusions

The resonance frequency in timber guardrail beams between the simply supported condition and fixed condition was obtained from flexural vibration tests. The results were as follows:

1. The measured resonance frequency started to increase from the resonance frequency of a beam with simply supported ends and approached that of fixed ends as the

compressing stress increased. This trend was similar to the m_1–k_2 relationship in Eq. 8.

2. The stable resonance frequency was lower than the theoretical value because perfect fixation of a beam to a post was difficult: it would be difficult to perfectly restrict the shear deformation and rotation of the part of a specimen around a post.

3. Temporal change in the stable resonance frequency will be effective to estimate the deterioration of wooden beams used for timber guardrails.

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