#### NOTE

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# Initial shapes of stress-strain curve of wood specimen subjected to repeated combined compression and vibration stresses and the piezoelectric behavior

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Abstract This study investigated the relationship between the initial shape of the stress ( $\sigma$ )-strain ( $\varepsilon$ ) curve of a Chamaecyparis obtusa wood specimen subjected to repeated combined compression and vibration stresses at various angles between the fiber direction and load direction and the piezoelectric behavior. The main findings of the study are: (1) the  $\sigma$ - $\varepsilon$  curve became convex initially, and then the stress was proportional to the strain. The  $\sigma$ - $\varepsilon$  curve had almost the same shape during both loading and unloading. (2) The  $\sigma$ -piezoelectric voltage (P) curve was nonlinear, with a maximal point or cusp on the curve, which had almost the same shape during both loading and unloading, as was also observed for the  $\sigma$ - $\varepsilon$  curve. (3) The plot of the first derivative of the stress  $[d\sigma/d\varepsilon (= \sigma')]$  against  $\varepsilon$  was nonlinear. The  $\sigma' - \varepsilon$  and  $P - \varepsilon$  curves at various angles were fairly similar. (4) The stress at the maximal point (or cusp) of the  $\sigma$ -P curve decreased with an increase in the angle between the fiber direction and load direction. The tendency of the stresses was very similar to that of Young's modulus and compression strength calculated from Hook's law and Hankinson's law, respectively.

**Key words** Stress–strain curve · Wood · Piezoelectric voltage · First derivative of the stress · Combined compression and vibration stresses

# Introduction

It is generally thought that the piezoelectric effect of wood results from natural cellulose crystals in the cell wall. However, natural cellulose cannot exist as a molecule in wood.

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H. Yamamoto Nagoya University, Nagoya 464-8601, Japan Many molecular chains of cellulose form fiber structures in bundles with hemicellulose and lignin, i.e., microfibrils. The cell wall can be considered as a frame for these microfibrils. Hence, the increase and decrease of the piezoelectric voltage during the deformation of wood originates from the dynamic deformation of the cell wall.

Many studies of the piezoelectric effect in wood have examined the physical properties of small specimens of wood under a minute load. There is very little research on deformation.<sup>1-4</sup> Therefore, in our previous report,<sup>5</sup> we examined test specimens subjected to combined compression and vibration stresses at a 45-degree angle between the fiber direction and load direction, to clarify the relationship between the piezoelectric voltage and deformation. Following our previous experiment,<sup>4</sup> this study investigated the relationship between the piezoelectric behavior and the initial shape of the stress–strain curve of a wood specimen subjected to combined compression and vibration stresses at various angles between the fiber direction and load direction.

# **Materials and methods**

The specimens used were made of kiln-dried hinoki (Chamaecyparis obtusa Endl.); the external dimensions were  $3.0 \times 3.0 \times 9.0$  cm. The ten specimens were first conditioned in a room at a constant temperature of 20°C and a constant humidity of 60% for 6 months. The average moisture content and density of the specimens before testing were 11.0%  $\pm$  0.1% (mean  $\pm$  standard deviation) and 0.48  $\pm$  $0.02 \,\mathrm{g/cm^3}$ , respectively. We placed the plane electrodes on a radial section in which the angles between the axial direction and the fiber direction of the specimen were 0, 15, 30, 45, 60, 75, and 90 degrees. The electrodes used to detect the piezoelectric voltage were pieces of aluminum foil measuring  $3 \text{ cm} \times 3 \text{ cm} \times 100 \mu \text{m}$  bonded with double-sided tape to the center of opposite sides of the specimen. The lead wires were bonded to the electrodes using electrically conductive paint.

The piezoelectric voltage generated by the wood was detected by the electrodes and amplified. Noise was removed with a 1/3-octave band-pass filter with an input impedance of  $10 M\Omega$  (NEC Sanei), and the signal was measured using a highly sensitive alternating current voltmeter with a built-in AC-DC converter (NF Co.). In this study, the piezoelectric voltage measured with this apparatus was extremely small. The piezoelectric voltage (load, displacement) was stored in a data acquisition controller (NEC Sanei) at 2-s intervals, and input to a personal computer.

A hydraulic testing machine (servo pulser, EHF-UG100kN-20L, full-scale range =  $\pm 100$  kN, Shimadzu) was used to apply the load. Specimens were set in a jig, and static compression with the superimposed minute sinusoidal load (*F*) given by Eq. 1 below, was applied.<sup>6</sup> Loads were applied in the following order: first, a compression load  $F_1$  = 0.2 kN was applied, followed by a sinusoidal load with frequency f = 30 Hz and amplitude a = 0.2 kN. Finally, a static compression load,  $F_n$ , was applied until it reached 40% of the proportional limit compression load of the specimen. A sinusoidal load made it possible for the apparatus to detect the piezoelectric voltage. The combined load, *F*, is given by:

$$F = F_{\rm n} + a \sin[(2\pi f)t] \tag{1}$$

where t is the time, 10s.

In this study, the piezoelectric voltage generated with the load output of the servo pulsar was monitored using a storagescope, as shown in Fig. 1. The setting load imposed



**Fig. 1.** Apparatus used to measure piezoelectric voltage (load, displacement) showing a test specimen ready for measurement: A, servo pulser, which applies a load to the test specimen; B, controller for the servo pulser; C, personal computer for the controller; D, 1/3-octave band-pass filter with an input impedance of  $10M\Omega$ ; E, highly sensitive alternating current voltmeter with a built-in AC-DC converter; F, storagescope; G, data acquisition controller; H, personal computer for recording the output from the data acquisition controller

on the test specimen used for a combined compression and vibration test was determined by the following procedure. At first, ten test pieces with the same shape as those used for this measurement were prepared, and tests were carried out to determine the proportional limit load of test pieces. The average value was calculated and a value of approximately 40% of the average was set as the setting load (F) applied to the test specimen in this study. The zero point of strain was plotted as the starting point of load.

#### **Results and discussion**

In this measurement system, it was impossible to differentiate between the piezoelectric voltage and electric noise at 75 and 90 degrees between the fiber direction and load direction. In these cases, we could not judge to our satisfaction whether the piezoelectric voltage was actually zero or that the measurement limit of the system was being observed. Therefore, in this study, only the experimental results obtained from specimens of which angles between the fiber direction and load direction were 0, 15, 30, 45, and 60 degrees were collated.

Stress-strain curve and stress-piezoelectric voltage curve

As shown in Fig. 2, the stress ( $\sigma$ ) was plotted against the piezoelectric voltage (P) and the strain ( $\varepsilon$ ) for each loading and unloading process. The figure clearly shows that initially most of the  $\sigma$ - $\varepsilon$  curve became convex, and then the stress was proportional to the strain. That is, the  $\sigma$ - $\varepsilon$  curve was nonlinear at a low proportionality level, and had almost the same shape during both loading and unloading. Subsequently, the  $\sigma$ -P curve was nonlinear, with a maximal point or cusp on the curve, and had almost the same shape during both loading as did the  $\sigma$ - $\varepsilon$  curve.

The piezoelectric behavior in the elastic region resulted from electrical and elastic phenomena, which do not show significant hysteresis. From the well-known fact that natural cellulose crystals in the cell wall of wood cause the piezoelectric phenomenon, and the result indicating the nonlinear elasticity of the piezoelectric behavior, we postulated that natural crystalline cellulose in wood shows nonlinear elasticity. However, the possibility of the correspondence of the nonlinearity of the global wood structure until the force is transmitted to the natural cellulose crystal in the wood specimen cannot be abandoned. However, we recently obtained a result indicating nonlinear elasticity of natural cellulose crystals from X-ray stress measurement.<sup>7</sup> Further analysis of this problem will be conducted using more accurate measurements.

Relationship between piezoelectric voltage and first derivative values of stress

The first derivative of the stress  $[d\sigma/d\varepsilon (= \sigma')]$  was calculated, which corresponds to the Young's modulus of the test



**Fig. 2.** Examples of stress–strain  $(\sigma - \varepsilon)$  curves (*left*) and stress–piezoelectric voltage  $(\sigma - P)$  curves (*right*) with repeated combined compression and vibration stresses.  $\theta$ , Angle between the fiber direction and load direction

specimen. Each value of  $\sigma'$  and *P* obtained from the calculation was plotted against the strain, as shown in Fig. 3. The value of  $\sigma'$  was not constant against the strain and the  $\sigma'-\varepsilon$  curve was nonlinear. The  $\sigma'-\varepsilon$  and *P*- $\varepsilon$  curves at various angles were fairly similar. The correlation coefficients for the regressions for both relationships are shown in Table 1. The correlation coefficients, except that for 60 degrees, showed high values for both the loading and unloading processes.

It is known that the change in the diffraction intensity is proportional to the change in microfibril orientation, because the diffraction intensity in a given direction is proportional to the preferred orientation in the reciprocal space.<sup>8</sup> Here, the ratio of the diffraction intensity reflected from the (004) plane under no load,  $I_0$ , to that under a load, I, was plotted against the compression stress in Fig. 4. In this case, the X-ray stress measurement was followed using the method of our earlier report.<sup>9</sup> In this stress range, the ratio



**Fig. 3.** Examples of  $(P-\varepsilon)$  curves and the first derivative of the stressstrain  $[d\sigma/d\varepsilon (= \sigma')] - \varepsilon$  curves for loading (*left*) and unloading (*right*) under combined compression and vibration stresses. *Arrows* show the vertical axis of each curve

**Table 1.** Correlation coefficients for the regressions for relationships

 between the piezoelectric voltage and the first derivative values of

 stress during loading and unloading

θ	Loading	Unloading
 0°	0.90	0.87
15°	0.88	0.97
30°	0.98	0.95
45°	0.91	0.95
60°	0.74	0.37

 $\theta$ , Angle between the fiber direction and load direction

of the peak intensity  $(I/I_0)$  became larger than 1.0 and increased with increase of compression stress as well as in the tension test.<sup>8,10,11</sup> These results support the hypothesis that the uniformity of the orientation of microfibrils increases



**Fig. 4.** Relationship between the surface strain and the ratio  $(I/I_0)$  of the diffracted intensity in the (004) plane under the unloaded  $(I_0)$  and loaded (I) conditions for loading (*filled circles*) and unloading (*shaded circles*)

when compression stress is applied. In the meantime, the experimental fact obtained from this study is that the piezoelectric voltage and Young's modulus of the test specimens increase at the initial stage of compression load. In particular, there seems to be a close relation with the increase of piezoelectric voltage and Young's modulus when the orientation of the cellulose microfibrils is improved by applying a compression load.

Thus, it appears that the nonlinear behavior of the piezoelectric voltage, the nonlinear elasticity of natural crystalline cellulose in wood, and improved of orientation of microfibrils are all complicatedly intertwined.

Relationship between stress at maximal point of  $\sigma$ -*P* curve, Young's modulus, and compression strength

The stress at the maximal point (or cusp) of the  $\sigma$ -*P* curve was plotted against the angle between the fiber direction and load direction (Fig. 5). The maximal stresses were found to decrease with increase in the angle. Here, we adapted Hook's law (Eq. 2) and Hankinson's law (Eq. 3) for calculating Young's modulus and compression strength, respectively.

$$\frac{1}{E_{\theta}} = \frac{1}{E_{x}}\cos^{4}\theta + \frac{1}{E_{y}}\sin^{4}\theta + \left(\frac{1}{G_{xy}} - \frac{2\nu_{xy}}{E_{x}}\right)\cos^{2}\theta\sin^{2}\theta \qquad (2)$$

$$\sigma_{\theta} = \frac{\sigma_0 \sigma_{90}}{\sigma_0 \sin^n \theta + \sigma_{90} \cos^n \theta}$$
(3)

where  $E_x = 9.15$  GPa,  $E_y = 0.71$  GPa,  $G_{xy} = 0.59$  GPa,  $v_{xy} = 0.29$ , and n = 1.5. The tendency of experimental stress data was very similar to that for Young's modulus and compression strength calculated from the above equations. That is, it was clarified that the stress data obtained from the piezo-electric response has anisotropy.



**Fig. 5.** Comparison of relationships between  $\theta$  and stress at the maximal point of the  $\sigma$ -*P* curve ( $\sigma_{n}$ , *filled circles*), Young's modulus (*E*, *solid line*), and compression strength ( $\sigma_{m}$ , *dashed line*) of test specimens. *E* and  $\sigma_{m}$  are calculated from Eqs. 2 and 3. *Arrows* show the vertical axis for each curve

Although we cannot further discuss these experimental results because the generation mechanism of the piezoelectric effect of wood is not proven, we want to investigate engineering relations between them in the future.

## Conclusions

We investigated the relationship between the initial shape of the  $\sigma$ - $\varepsilon$  curve of a wood specimen subjected to combined compression and vibration stresses at various angles between the fiber direction and load direction and the piezoelectric behavior. As a result, most of the  $\sigma$ - $\varepsilon$  curve became convex, and then the stress was proportional to the strain. The  $\sigma$ -P curve was nonlinear, with a maximal point or cusp on that curve, which had almost the same shape during both loading and unloading, as also observed for the  $\sigma$ - $\varepsilon$  curve. The  $\sigma'$ - $\varepsilon$  ( $\sigma'$  corresponds to Young's modulus of the test specimen) and P- $\varepsilon$  curves at various angles were fairly similar. These behaviors of  $\sigma$ , P, and  $\sigma'$  in relation to  $\varepsilon$  indicate that the cellulose crystal in wood or the actual wood shows nonlinear elasticity.

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