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ORIGINAL ARTICLE

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Fatigue of structural plywood under cyclic shear through thickness II: a new method for fatigue life prediction

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Abstract For plywood specimens under shear through the thickness, a fatigue life prediction method based on strain energy has been newly developed with the fatigue process and failure criterion applicable to various loading conditions. Once the fatigue process and failure criterion of the plywood specimen were determined by the fatigue data measured under a loading condition other than the square loading waveform, the fatigue life of a specimen under various loading conditions could be predicted easily and accurately by the first cycle loading test. The relationship between stress level and the predicted fatigue life was also similar to that between stress level and the experimentally determined fatigue life. The fatigue life prediction method proposed may be widely applicable to the prediction of the fatigue life of solid wood and wood composites.

Key words Fatigue life prediction · Strain energy · Shear through thickness · Plywood

Introduction

Many studies of the fatigue of wood and wood composites have already been conducted considering several influencing factors. One such factor is the loading conditions that include the loading waveform and loading frequency. Many studies have been conducted on the separate effects of loading waveform and loading frequency on the fatigue of wood and wood composites, and it has been established that both loading waveform and loading frequency affect the fatigue life of wood and wood composites.[1–10] However, systematic explanations that commonly apply to various loading conditions have not been proposed for the fatigue behavior of wood and wood composites.

In Part 1 of this study,[11] the fatigue test on the plywood specimen under shear through the thickness was conducted under three loading conditions: a square waveform at a loading frequency of 0.5 Hz, a triangular waveform at 0.5 Hz, and a triangular waveform at 5.0 Hz. The fatigue process and failure criterion independent of loading condition could be experimentally obtained on the basis of strain energy. The purpose of the present study was to establish an approach for predicting the fatigue life of plywood specimens under cyclic shear through the thickness on the basis of the fatigue process and failure criterion obtained in Part 1 of our study.[11] Here, we propose a new method of predicting the fatigue life of solid wood and wood-based materials under cyclic load with various loading conditions.

Materials and methods

Fatigue life prediction method based on strain energy

The relationship between stress level (SL) and the number of loading cycles to failure, that is, fatigue life (N_f), was found to be highly dependent on the loading waveform and frequency in the fatigue of plywood specimens under shear through the thickness (Fig. 1), as described in Part 1.[11] This is because the deformation behavior of the plywood specimen at certain stress levels depends on the loading condition. Thus, the strain energy per cycle was analyzed in detail in Part 1, because strain energy per cycle reflects the deformation behavior of the plywood specimen during loading from zero to the peak stress level.

It became clear in Part 1 that the fatigue process of a plywood specimen under cyclic shear through the thickness could be expressed based on strain energy:

$$V_{ac} = V_{1st}N^{b} \quad (1)$$

where V_{ac} is the cumulative strain energy from the first cycle to the N^{th} loading cycle for each specimen and is considered to be the index of fatigue progression, and V_{1st} is the strain

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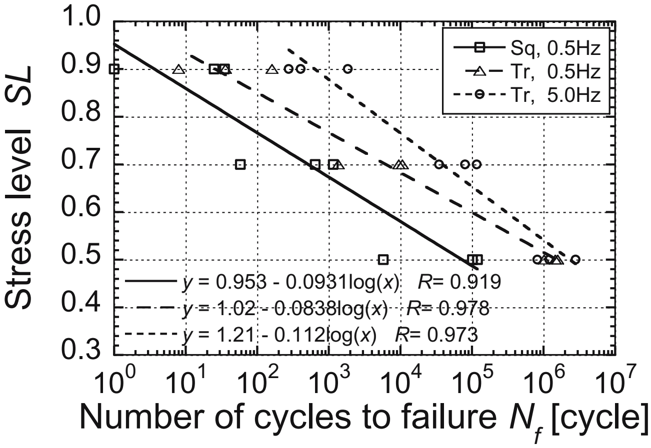

Fig. 1. Relationship between stress level (*SL*) and number of loading cycles to failure (N_f). *Sq*, square waveform; *Tr*, triangular waveform

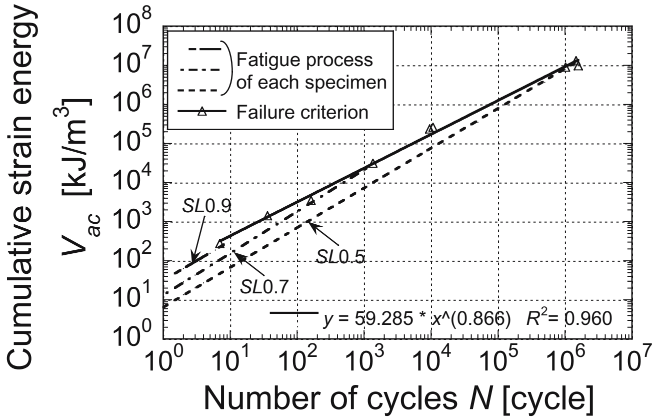

Fig. 2. Fatigue process and failure criterion based on strain energy for triangular waveform at loading frequency of 0.5 Hz

Table 1. Fatigue process and failure criterion experimentally obtained under three loading conditions

Loading condition	Fatigue process ($V_{ac} = V_{1st}\, N^b$)	Failure criterion ($V_{acf} = c\, N_f^d$)
Sq, 0.5 Hz	$V_{ac} = V_{1st}\, N^{1.05}$	$V_{acf} = 69.764\, N_f^{0.826}$
Tr, 0.5 Hz	$V_{ac} = V_{1st}\, N^{1.03}$	$V_{acf} = 59.285\, N_f^{0.866}$
Tr, 5.0 Hz	$V_{ac} = V_{1st}\, N^{1.03}$	$V_{acf} = 64.902\, N_f^{0.847}$

Sq, square waveform; Tr, triangular waveform; V_{ac}, cumulative strain energy from first cycle to N^{th} loading cycle; V_{acf}, cumulative strain energy from first cycle to fatigue life (N_f); V_{1st}, strain energy at first cycle

energy at the first loading cycle (V_{1st}) for each specimen. Strain energy at the first loading cycle (V_{1st}) varies with the loading condition and applied stress level. On the other hand, coefficient *b* was found to be almost the same among all the specimens regardless of loading condition and stress level, as listed in Table 1. As such, the coefficient *b* determined under a specific loading condition is applicable to various loading conditions. Therefore, the fatigue process of each specimen under various loading conditions and stress levels can be expressed by Eq. 1 when the strain energy at the first loading cycle (V_{1st}) is determined.

The fatigue failure criterion of a plywood specimen under cyclic shear through the thickness could also be expressed based on strain energy:

$$V_{acf} = cN_f^{\,d} \tag{2}$$

where V_{acf} is the cumulative strain energy from the first cycle to fatigue life (N_f). The equations obtained under the three loading conditions are almost the same (Table 1), and thus the fatigue failure criterion obtained under one loading condition could also be used for other loading conditions.

Typical examples of the fatigue process of a plywood specimen and the failure criterion obtained for a triangular waveform at a loading frequency of 0.5 Hz are shown in Fig. 2. The cumulative strain energy of the specimens increased with the number of loading cycles. Then the specimens fractured when the lines representing the fatigue process and the failure criterion crossed. That is to say, the number of loading cycles for the intersection of the lines representing the failure criterion and the fatigue process of a specimen is its fatigue life. Therefore, the fatigue process and failure criterion based on strain energy might enable the prediction of fatigue life of plywood specimens.

A new procedure for prediction of fatigue life is proposed in Fig. 3. First, fatigue test of the specimen is conducted under one loading condition to obtain the base data for the determination of the fatigue process and failure criterion. As described above, the fatigue process and failure criterion determined under one loading condition can be applied to fatigue under other loading conditions. When the fatigue life of a specimen is predicted under a prescribed loading condition and stress level, the first cycle loading test of the specimen is conducted under the prescribed loading condition and stress level to obtain the strain energy at the first loading cycle (V_{1st}). The intersection of the lines representing the fatigue process of the specimen and the failure criterion gives the predicted fatigue life of the specimen. The strain energy at the first loading cycle (V_{1st}) is considered to be the determining factor of fatigue life because it depends on loading condition and stress level. The advantage of this fatigue life prediction method is that the fatigue process and failure criterion are obtained in the fatigue test under one loading condition, after which the fatigue life of a specimen under various loading conditions can be easily predicted by the first cycle loading test rather than by the fatigue test.

Trial and verification of fatigue life prediction method

To examine the fatigue life prediction method for its validity, the fatigue life of plywood specimens was predicted with the data obtained in Part 1.[11] The fatigue test of plywood specimens under cyclic shear through the thickness was conducted under three loading conditions: a square waveform at 0.5 Hz, a triangular waveform at 0.5 Hz, and a triangular waveform at 5.0 Hz. Three stages equivalent to stress levels (*SL*s) of 0.5, 0.7, and 0.9 were determined ac-

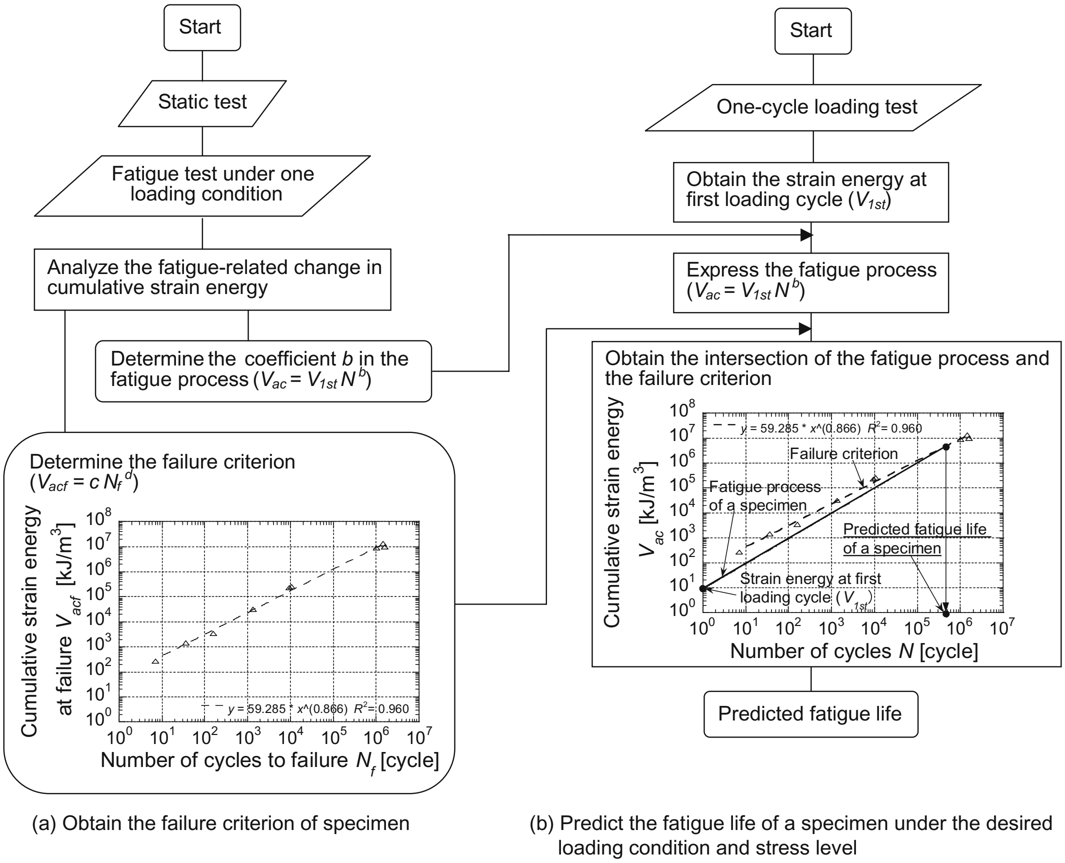

Fig. 3. Flowchart for newly developed method of fatigue life prediction

cording to the ratio of the preset peak stress to the mean static strength in shear through the thickness. Three specimens were used at each stress level under the respective loading conditions. First, the data obtained in a triangular waveform at 0.5 Hz was used as the base data for the determination of the fatigue process and failure criterion. The fatigue lives of specimens with a square waveform at 0.5 Hz and a triangular waveform at 5.0 Hz were estimated according to the fatigue life prediction method proposed in Fig. 3. The predicted fatigue life was compared with that experimentally obtained in the fatigue test. The effects of the base data for the determination of the fatigue process and failure criterion on the fatigue life prediction are discussed below.

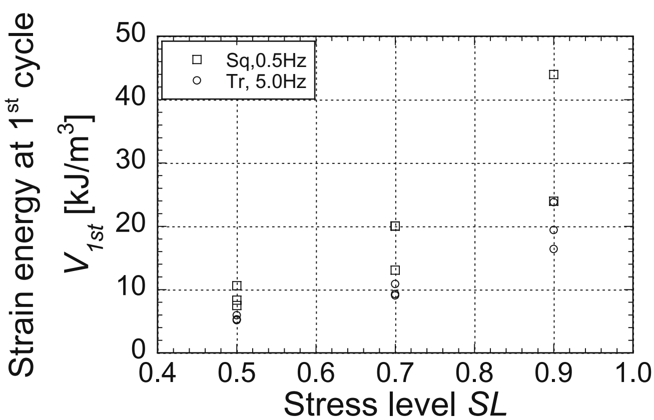

Fig. 4. Strain energy at first loading cycle (V_{1st}) obtained experimentally for a square waveform at a loading frequency of 0.5 Hz and a triangular waveform at 5.0 Hz

Results and discussion

Fatigue life prediction

The fatigue process and failure criterion determined with the data obtained for a triangular waveform at a loading frequency of 0.5 Hz are listed in Table 1. They were used for the fatigue life prediction for a square waveform at 0.5 Hz and a triangular waveform at 5.0 Hz. Strain energy at the first loading cycle (V_{1st}) for all the specimens under these two loading conditions is shown in Fig. 4. Strain energy at the first loading cycle was found to be dependent on the loading condition and stress level. Then the fatigue life of each specimen was predicted by the intersection of the lines representing the fatigue process and the failure criterion obtained for a triangular waveform at 0.5 Hz, as shown in Fig. 3.

The results of the predicted and experimentally obtained fatigue lives were plotted against stress level for a square waveform at 0.5 Hz (Fig. 5a) and a triangular waveform at 5.0 Hz (Fig. 5b), respectively. The predicted fatigue lives were found to be similar to the experimentally obtained fatigue lives under both loading conditions, although the predicted fatigue lives appeared to vary a little from the experimental fatigue lives for the square waveform at 0.5 Hz. Each regression line between stress level and fatigue life was statistically significant at the 1% level under both loading conditions. The regression lines for the predicted and experimentally obtained fatigue lives were also similar for a square waveform at 0.5 Hz and a triangular waveform at 5.0 Hz. They were statistically compared by analysis of covariance (ANCOVA) with checking of the assumption of parallelism,[12] in which found no significant difference between the predicted and experimentally obtained regression lines at the 5% level (Table 2). ANCOVA showed that the adjusted means of the predicted and experimentally obtained regression lines were not significantly different at the 5% level under both loading conditions (Table 3). These results indicate that the fatigue life prediction proposed in this study could reasonably give the relationship between stress level and fatigue life for various loading conditions. This relationship is essential for the design of timber structures considering the fatigue of structural members. There-

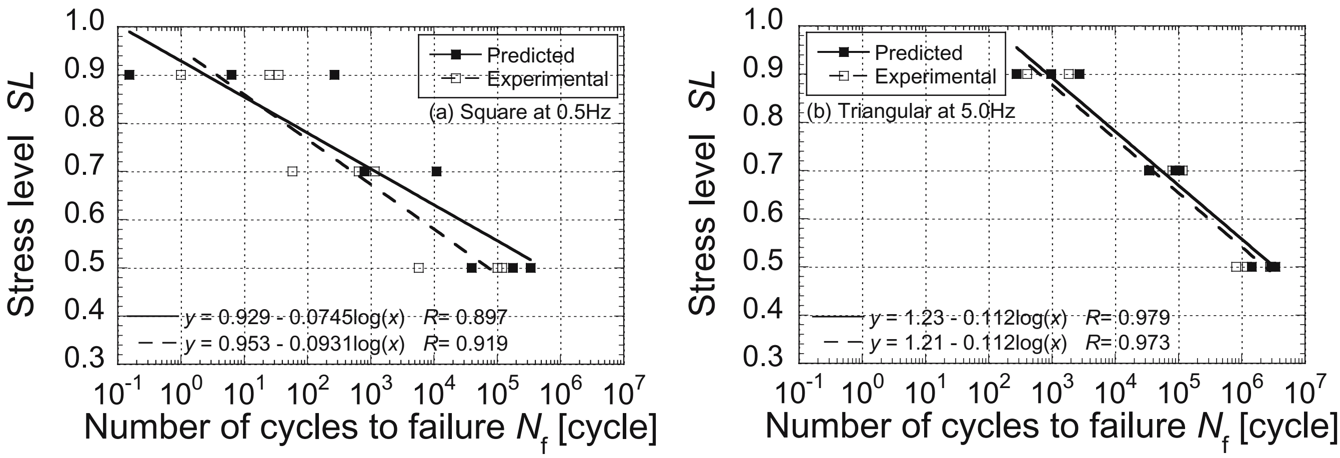

Fig. 5a, b. Relationship between stress level (SL) and fatigue life (N_f) obtained experimentally and predicted for **a** square waveform at loading frequency of 0.5 Hz and **b** triangular waveform at 5.0 Hz

Table 2. Test for assumption of constant slope in experimental and predicted lines

Source	DF	Sum of squares	Mean square	F	Probability > F
Square at 0.5 Hz[a]					
Regression	1	0.005	0.005	0.801	0.386
Error	14	0.084	0.006		
Corrected total	15	0.089	0.006		
Triangular at 5.0 Hz[b]					
Regression	1	0.000	0.000	0.001	0.972
Error	14	0.023	0.002		
Corrected total	15	0.023	0.002		

DF, degrees of freedom
[a] Square waveform at loading frequency of 0.5 Hz
[b] Triangular waveform at 5.0 Hz

Table 3. Analysis of covariance for experimental and predicted regression lines

Source	DF	Sum of squares	Mean square	F	Probability > F
Square at 0.5 Hz[a]					
Adjusted mean	1	0.004	0.004	0.645	0.434
Adjusted error	15	0.089	0.006		
Adjusted total	16	0.092	0.006		
Triangular at 5.0 Hz[b]					
Adjusted mean	1	0.001	0.001	0.719	0.410
Adjusted error	15	0.022	0.002		
Adjusted total	16	0.024	0.001		

[a] Square waveform at loading frequency of 0.5 Hz
[b] Triangular waveform at 5.0 Hz

fore, the ability to easily obtain the relationship between stress level and fatigue life under various loading conditions is very significant.

Figure 6 shows the cumulative frequency of the predicted and experimentally obtained fatigue lives for both a square waveform at 0.5 Hz and a triangular waveform at 5.0 Hz. The predicted and experimentally obtained fatigue lives showed almost the same distribution. These results also support the validity of the estimation of fatigue life by the prediction method proposed in this article.

Effect of base data on fatigue life prediction

In the fatigue life prediction described above, the data obtained for a triangular waveform at 0.5 Hz was used as the base data for determination of the fatigue process and failure criterion. Then the data obtained under two other loading conditions was used as the base data, and the effect of base data on the fatigue life prediction was investigated.

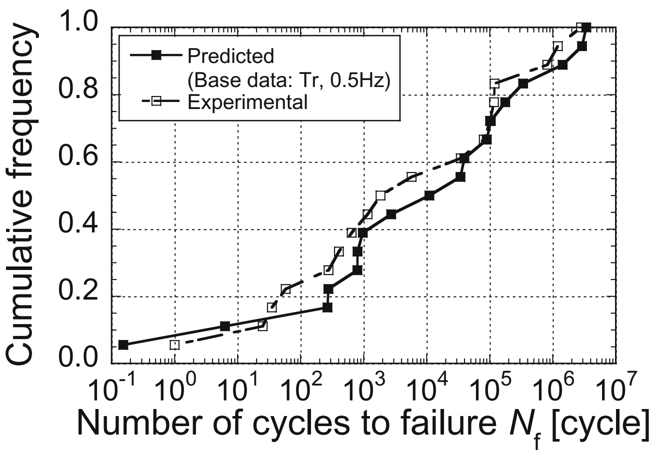

Fig. 6. Cumulative frequency of fatigue life obtained experimentally and predicted for both square waveform at loading frequency of 0.5 Hz and triangular waveform at 5.0 Hz

At first the data obtained for a square waveform at 0.5 Hz was used as the base data to determine the fatigue process and failure criterion, and the fatigue life of a plywood specimen was predicted for a triangular waveform at 0.5 Hz and a triangular waveform at 5.0 Hz. The cumulative frequency of the predicted and experimentally obtained fatigue lives under these two loading conditions is shown in Fig. 7a. The distribution of the predicted fatigue life was found to be different from that of the experimentally obtained fatigue life. As shown in Table 1, the coefficient *b* of the fatigue process obtained for a square waveform at 0.5 Hz was slightly larger than that for other loading conditions. The coefficient *b* is an exponent in the equation for the fatigue process (Eq. 1). Therefore, slight difference in coefficient *b* has considerable influence on the fatigue process. This is one of the reasons why the fatigue life was not correctly estimated with the base data obtained for a square waveform at 0.5 Hz. Figure 7b shows the cumulative frequency of the predicted and experimentally obtained fatigue lives for both a square waveform at 0.5 Hz and a triangular waveform at 0.5 Hz for the case in which the data obtained in a triangular waveform at 5.0 Hz was regarded as the base data. The distributions of the predicted and experimentally obtained fatigue lives were similar, as is the case with Fig. 6. These results indicate that the fatigue data obtained for a square waveform is not suitable as the base data to determine the fatigue process and failure criterion in the fatigue life prediction method. The load in a square waveform generates the characteristic stress–strain curve because peak stress is constantly applied during most of the loading phase in one cycle, as shown in Part 1 of this study.[11] This loading characteristic of the square waveform has some adverse effect on the fatigue life prediction proposed in this article. All other loading conditions, with the exception of the square waveform, can be adopted to obtain the data for the determination of the fatigue process and failure criterion.

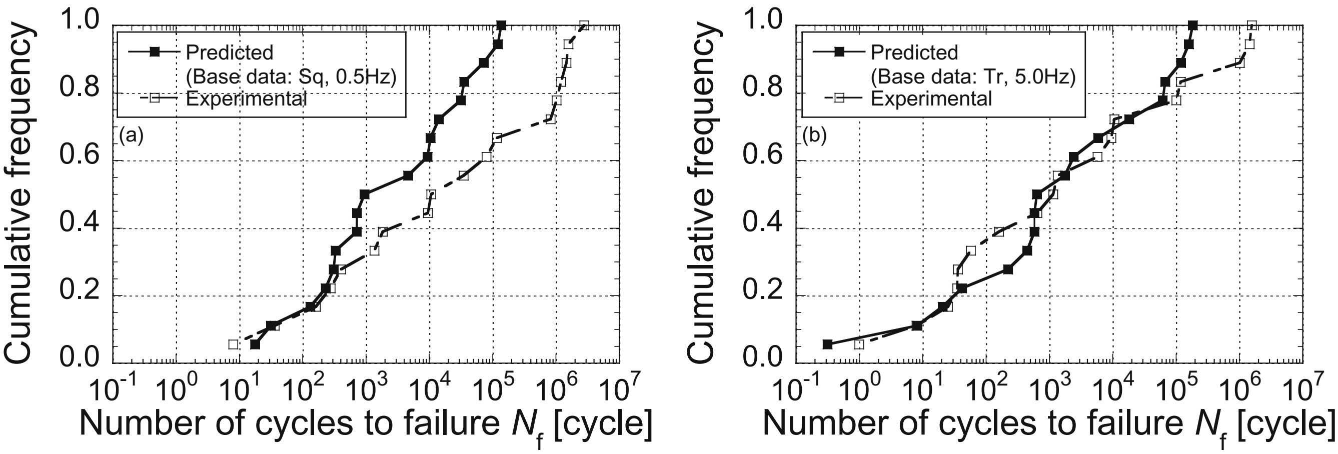

Fig. 7a, b. Cumulative frequency of fatigue life obtained experimentally and predicted for **a** triangular waveform at 0.5 and 5.0 Hz, and **b** square and triangular waveforms at 0.5 Hz

Conclusions

In this study, an approach to predict the fatigue life of a plywood specimen under cyclic shear through the thickness was investigated on the basis of strain energy. Once the fatigue process and failure criterion of the plywood specimen are determined by the fatigue data obtained under one loading condition, the fatigue life of a specimen under various loading conditions can be predicted easily and accurately by the first cycle loading test. The relationship between stress level and the predicted fatigue life was similar to that obtained experimentally. We note that the fatigue data obtained for a square waveform is inappropriate for the determination of the fatigue process and failure criterion. The new fatigue life prediction method proposed in this article may be applicable for prediction of the fatigue life of solid wood and wood composites under other loading modes such as tension, compression, and bending with various loading profiles. This method will be verified by applying it to the prediction of the fatigue life of solid wood under tension and torsion in subsequent studies.

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