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ORIGINAL ARTICLE

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Method for predicting tension capacity of sawn timber considering slope of grain around knots

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Abstract This study introduced a new parameter, the area reduction factor (ARF), to consider the effect of knots on the tension strength of timber. It is an improved version of the knot area ratio (KAR). ARF considers both the projected area of knots and the effect of the slope of grains around the knots. The tension capacity of a tested structural timber was predicted as a product of ARF, clear wood tension strength parallel to the grain, and the area of the cross section. ARF was determined as the minimum value obtained when a knot measurement window of 100 mm was slid along the plank. The prediction method was examined with 11 planks. The average ratio of the predicted capacity to the actual value was 1.11 with a coefficient of variation of 0.26. The average ratio obtained by using a KAR-based parameter, the clear wood area ratio (CWAR), was 2.15 with a coefficient of variation of 0.23. To study the reliability of ARF and CWAR as single parameters, the correlations of ARF and CWAR with the tension strength were determined for 57 planks. The coefficients of determination for ARF were slightly better than those for CWAR, although both of them seemed to be quite poor predictors of tension capacity when used alone. Therefore, a multi-parameter model is preferred and should be a subject for further studies.

Key words Knot area ratio (KAR) · Tension capacity · Structural timber · Slope of grain

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Introduction

It is often considered that knots in structural timber have no load-carrying capacity in tension. Therefore, complicated strength reduction created by knots has been simplified by calculating the knot area ratio (KAR), which is obtained as a projection of knots located within a certain length to the cross section of the plank under concern. KAR has some correlation with the tension strength of timber. For example, Johansson et al.[1] obtained coefficients of determination (R^2) of 0.35 and 0.26 for planks of Nordic spruce with cross sections of 34×145 mm and 58×120 mm, respectively.

It is also known that the slope of the grain decreases the strength of knot-free regions close to knots. Hatayama[2] examined the effect of the slope of a grain around a single knot on both the tension strength and the modulus of elasticity with four different species of timber. They were western hemlock (*Tsuga heterophylla* Sarg.), Amabilis fir [*Abies amabilis* (Dougl.) Farbes], sugi (*Cryptomeria japonica* D. Don), and akamatsu (*Pinus densiflora* Sieb. et Zucc). He developed an empirical connection between the angle of the grain slope on the fracture surface and the coefficients of Hankinson's formula.[3]

According to Hatayama's empirical formula, the slope angle of grains passing around a knot is a function of the knot diameter and horizontal distance from the edge of the knot. Based on his empirical formula and Hankinson's formula, Hatayama calculated load-carrying tension capacities for 55 planks. The ratio of the calculated capacity to the actual tension capacity gave an average of 0.96 with a coefficient of variation of 0.13. The thicknesses of the cross sections examined in Hatayama's research varied from 16 to 20 mm and the widths from 90 to 190 mm. Thus, the specimens were quite thin compared with cross sections of planks used in practical applications. The length of the plank was 400 mm and contained only one face-knot, the axis of which was perpendicular to the horizontal axis of the cross section. However, edge knots, which seem to cause the most severe stress situations according to Foley,[4] were not considered in Hatayama's research.[2]

In this study, the ideas of KAR and Hatayama's empirical formula have been combined and an improved KAR-based factor, namely, the area reduction factor (ARF) has been established to consider the strength reduction due to the slope of grains around knots. An advanced method for predicting tension capacity was created based on ARF. The prediction value is a product of the ARF value, the cross-sectional area, and the tension strength parallel to grains of a clear wood specimen.

Theory

In order to apply Hatayama's empirical formula derived for the sloping grain angle of a thin plank containing a single knot to a thick structural timber containing multiple knots, it was postulated that within a thin circular layer of a structural timber, where the center corresponds to the pith, the slope angle is a function of the knot size and the length of the arc from the edge of the knot to the area considered (Fig. 1). The axis of the layer always crosses the axis of the knot projected to the cross section perpendicularly. Then the modified Hatayama empirical formula can be expressed as follows:

$$\theta = \frac{15 \cdot \sqrt{N^D \cdot D}}{\sqrt{l}} - \frac{D}{2} - 5, \qquad 0 \le \theta \le 90 \tag{1}$$

In Eq. 1, θ is the slope angle of the grain (deg) in a thin circular layer passing through the projected knot area. D is the arc length (cm) of the knot area within the circular layer. N is a constant dependent on species and the type of the knot ($0.9 \le N \le 1.3$) and l is the distance (cm) from the knot edge along the circular layer. If the formula gives a negative value, θ is 0°, if the value is larger than 90, θ is 90°. Referring to Hatayama,[2] the tensile strength at an inclination of θ with the direction of the grain could be expressed by using Hankinson's formula:

$$f_{t.\theta} = \frac{0.1}{\sin^{1.4}\theta + 0.1 \cdot \cos^{1.4}\theta} \cdot f_{t.0} \tag{2}$$

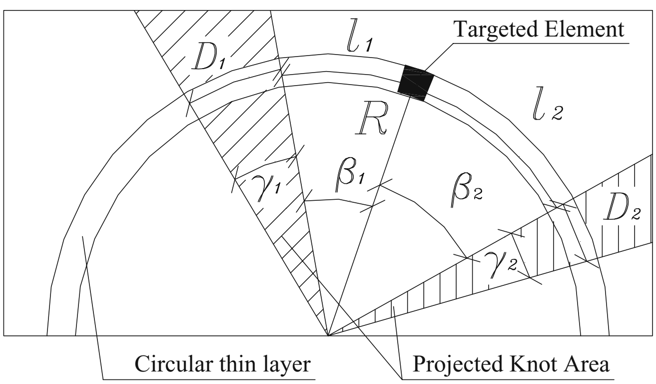

Fig. 1. Example model for considering calculation of area reduction factor (ARF). γ, Angle covered by one projected knot; l, length from the edge of knot to the targeted element; D, knot size; β, angle between the targeted element and edge of knot via origin

In Eq. 2, $f_{t.\theta}$ represents the tensile strength (N/mm^2) where the angle of inclination with the direction of the grain is θ, $f_{t.0}$ is the tension strength parallel to the grain (N/mm^2), 0.1 is the ratio of the tensile strength perpendicular to the grain to that parallel to the grain. The constants 0.1 and 1.4 were experimentally determined by Hatayama.[2]

In this study, the projections of the knots located within a 100-mm-long window are considered. By using polar coordinates with the origin at the pith (Fig. 1), the following equations can be written for the arc distance, l, from the knot edge along the arc and the size of knot, D:

$$l = R\beta \tag{3}$$
$$D = R\gamma \tag{4}$$

In Eq. 3, R is the radial coordinate of the element and β is the angle between the radial coordinate and the edge of the knot. In Eq. 4, γ is the angle covered by one knot.

By substituting l and D into eq. 1, the grain angle at one element, θ, can be obtained. If the grain angle at the targeted element is affected by several knots, the biggest inclination is chosen because the reduction of the tension strength increases as a function of the grain angle. Thus, the predicted tension capacity will be on the safe side. A local area reduction factor (LARF) for each element is determined by just dividing both sides in Eq. 2 with the tension strength parallel to the grain, $f_{t.0}$. The following equation can be written for the LARF of an element:

$$k(\theta) = \frac{0.1}{\sin^{1.4}\theta + 0.1 \cdot \cos^{1.4}\theta} \tag{5}$$

In Eq. 5, $k(\theta)$ is a function of θ and is equal to 0 at the knot area because the knot is assumed to carry no load. The tension capacity of the timber can now be obtained by assuming that the tension strength parallel to the grain, $f_{t.0}$, is constant within a timber and integrating the strength of differential elements over the whole cross section:

$$F_t = \int f_{t.\theta} \mathrm{d}A = \int k(\theta) f_{t.0} \mathrm{d}A = \mathrm{ARF} f_{t.0} A \tag{6}$$

In Eq. 6, F_t is the tension capacity of a plank, A is the cross-sectional area and ARF is the area reduction factor defined as:

$$\mathrm{ARF} = \frac{\int k(\theta) \mathrm{d}A}{A} \tag{7}$$

Equation 6 is the basis of a new method for predicting the tension capacity by means of ARF.

A KAR-based method for predicting the reduction of the tension strength considers the projected knot area as a non-load-carrying part, but ignores the effect of the sloping of grains around knots on the tension strength. The basis of this method is the clear wood area ratio (CWAR), which is a complement value of KAR, leading to the following expression for the strength:

$$F_t = \mathrm{CWAR} f_{t.0} A \tag{8}$$

In other words, CWAR can be also defined with Eq. 7 by letting $k(\theta)$ be one at knot-free part and zero at the

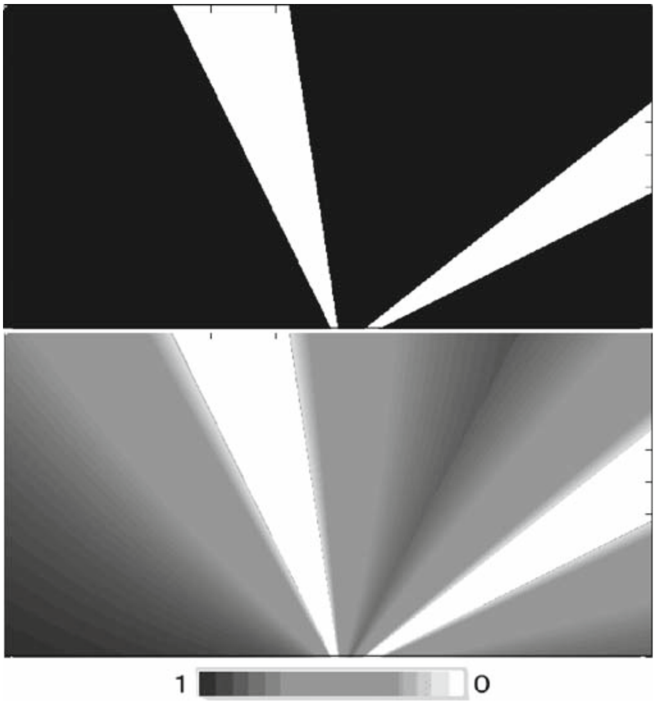

Fig. 2. Images of strength reductions due to clear wood area ratio (CWAR, *upper*) and area reduction factor (ARF, *lower*) of same cross section with projected knots. Knot area is shown in upper figure in white

projected knot part. An example of the visualized strength reduction by CWAR and ARF in one cross section is shown in Fig. 2.

Methodology

Specimens

The timbers tested, Nordic spruce (*Picea abies*), were grown mostly in Finland and some in Russia. The cross sections were 50 × 100 and 50 × 150 mm. The method used in sawing the logs divided them into two symmetrical planks by splitting the pith (2 ex log). One pair of the two symmetrical planks was used for structural-sized tension tests. Small clear wood specimens for determining clear wood tension strength were taken from one plank of each pair of symmetrical planks, although they could not be obtained from all logs (Fig. 3). This is because in most of the cases, the other samples were used for the bending tests relating to another study by Hanhijärvi et al.[5] in which the ratio between the tension and bending strength of structural timbers was studied. The idea of this theory was alighted when bending tests were almost completed and the number of available planks for this study was quite limited. However, it was estimated to be large enough to demonstrate this new idea. All of the tested materials were conditioned in a room so that moisture content of materials can be maintained around 12%.

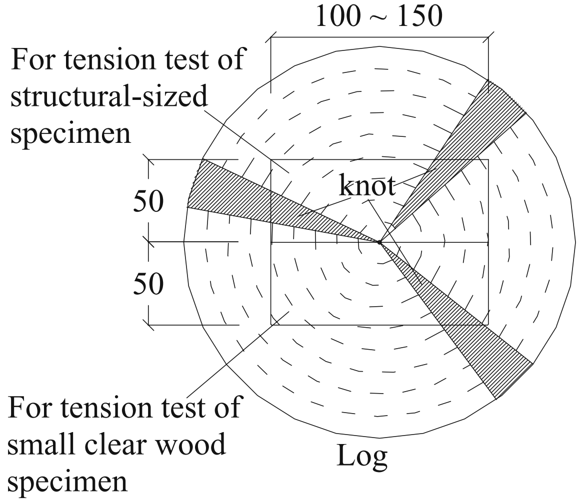

Fig. 3. Sawing type of 2 ex log

Numbering of specimens

In labeling the specimens, the first character, K, indicates Nordic spruce. The second character indicates the origin: V, Vologda in Russia; P, Kainuu in Finland; L, West Finland (Tampere, Seinäjoki, and Rauma); K, East Carelia in Russia; E, East Finland (Kitee); N, Novgorod in Russia. The last character, A or B, distinguishes the two planks sawn 2 ex log. The first of the three numbers represents the dimensions: 2, 50 × 150 mm; 1, 50 × 100 mm. The final two numbers were unique to each timber.

Specimens for structural timber tension test

The structural size tension strength was tested for 79 planks. Cross sections were 50 × 100 or 50 × 150 mm and the number of planks for each cross section was 39 and 40, respectively. The length of the structural-sized specimen was 3600 mm.

Specimens for small clear wood tension test

Small clear wood specimens could be taken from only nine planks with cross sections of 50 × 100 mm and two planks with cross sections of 50 × 150 mm because most of the other samples of sawn symmetrical planks were destroyed in the sister study.[5]

Specimen measurements

Before tension test of structural timber

The pith coordinates were measured at two points in the inner edge of the part clamped with grips.

After tension test of structural timber

Knot measurement was carried out after testing. All documented knots were only those with diameter larger than

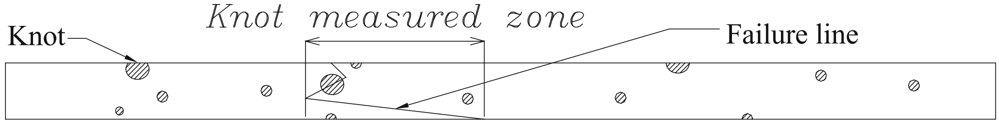

Fig. 4. Example of knot measured zone in structural timber after tension test

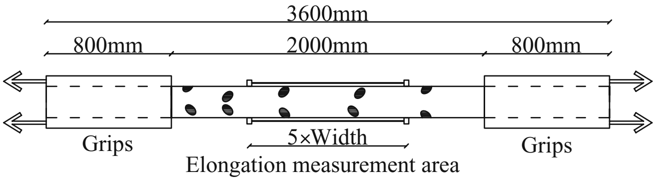

Fig. 5. Test arrangement for measuring tension capacity (F_t) and modulus of elasticity in tension

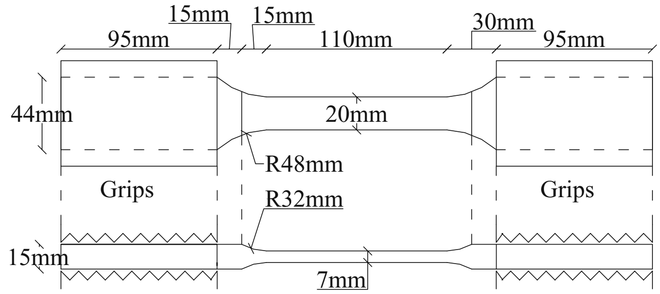

Fig. 6. Test arrangement for measuring tension strength of small clear wood ($f_{t,0}$)

7 mm and only those that were located in the transverse direction of the failure plane regardless of whether they were included in the failure plane (Fig. 4).

All specimens were visually examined to evaluate failure factors. The factors inducing failure were categorized into, "Edge knot," "Face knot," "Grain deviation at edge," "Grain deviation at face," and "Other defects." With the focus of this research being on the effect of knots on the tension strength of structural timber, failures with "Grain deviation at edge," "Grain deviation at face," and "Other defects" were excluded from further analysis. After this evaluation process, 27 out of 40 planks with a cross section of 50 × 100 mm and 30 out of 39 planks with a cross section of 50 × 150 mm were selected for analysis of the relationship between the load-carrying tension capacity and the knot parameters (CWAR and ARF).

Tension test of structural-sized specimens

The tension testing of each structural-sized specimen was performed according to EN408.[6] The distance between the grips was 2000 mm. The grips allowed no rotation of the timber ends. The adjustment of the loading rate (a constant force increase) guaranteed that the failure occurred within 300 ± 120 s. The deformation was measured at both edges. The span of the gauge was five times the width of the sawn timber (Fig. 5).

Tension test of small clear wood specimens

Three to five pieces were obtained and tested from one plank. The tension strengths parallel to the grain of the small clear wood specimens were determined and the average values were calculated. The test arrangement is shown in Fig. 6.

Calculation

Predicted tension capacities by means of both CWAR and ARF were calculated for 11 specimens, for which the tension strengths parallel to the grain were determined by small clear wood specimens.

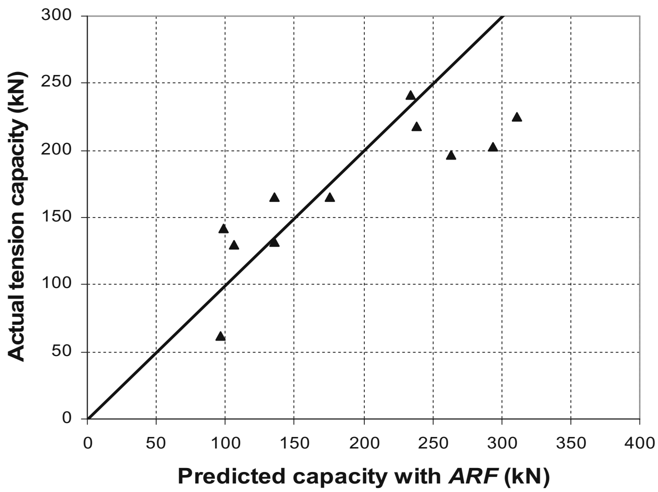

Fig. 7. Relationship between actual tension capacity and tension capacity predicted by means of ARF. *Solid line* is the ideal relation

Fifty-seven specimens were used for calculation of ARF and CWAR. ARF and CWAR were calculated based on the documented worst knot clusters by projecting knots located within 100 mm in the longitudinal direction and obtained as the minimum value within one plank. The pith coordinate used for the calculation was the average value of the pith coordinates measured at two points. To calculate ARF, N = 1.1 was used in Eq. 1 regardless of the quality of a knot.

Results and discussion

The results of tests and calculations made for the analyzed 57 planks are listed in Tables 1 and 2. Only 11 specimens could be used for prediction of the actual tension strength. The tension capacities predicted by ARF were quite close to the actual tension capacity (Fig. 7), whereas the capacities predicted by CWAR were much larger than the actual

Table 1. List of data for planks of 50 × 100 mm

Sample	Density (kg/m^3)	MOE (N/mm^2)	F_t (kN)	$f_{t.0}$ (N/mm^2)	CWAR	ARF	F_t(CWAR) (kN)	F_t(ARF) (kN)	Failure length (cm)
KK143B	458	14185	203	117	0.86	0.50	502	294	3
KE132A	476	13711	241	103	0.84	0.45	433	234	5
KK136A	359	6287	62	58	0.74	0.34	212	97	9
KE110A	368	8117	132	70	0.76	0.38	267	135	15
KE107A	426	10378	165	96	0.76	0.37	364	176	43
KL117B	357	9510	142	58	0.74	0.34	216	98	45
KL137A	462	11232	165	88	0.73	0.31	322	136	45
KL142A	469	12096	130	72	0.69	0.30	247	107	47
KK113B	429	13384	218	93	0.85	0.52	398	252	55
KV102B	425	11479	193	–	0.83	0.47	–	–	3
KV143A	485	11150	179	–	0.79	0.45	–	–	12
KV134A	407	11954	157	–	0.67	0.30	–	–	13
KV127B	376	9374	141	–	0.72	0.32	–	–	16
KP117A	380	9303	129	–	0.77	0.38	–	–	17
KL134A	513	12089	136	–	0.63	0.28	–	–	18
KK116B	418	12240	228	–	0.73	0.44	–	–	23
KP114B	492	10606	180	–	0.73	0.33	–	–	28
KV141A	441	11857	208	–	0.79	0.53	–	–	28
KL141A	395	10112	137	–	0.73	0.42	–	–	31
KE137A	511	14146	234	–	0.77	0.36	–	–	33
KP140B	501	13490	202	–	0.82	0.51	–	–	33
KL127B	436	11087	130	–	0.84	0.51	–	–	38
KL102A	500	15883	192	–	0.64	0.32	–	–	47
KV123B	385	12119	186	–	0.82	0.51	–	–	49
KK123A	438	12215	218	–	0.86	0.57	–	–	68
KE122B	450	13040	213	–	0.65	0.28	–	–	82
KN102B	364	10142	174	–	0.74	0.35	–	–	51

MOE, Modulus of elasticity in tension; F_t, actual tension capacity; $f_{t.0}$, tension strength parallel to grain of small clear wood; CWAR, clear wood area ratio; ARF, area reduction factor; F_t(CWAR), tension capacity predicted by means of CWAR; F_t(ARF), tension capacity predicted by means of ARF

Table 2. List of data planks of 50 × 150 mm

Sample	Density (kg/m^3)	MOE (N/mm^2)	F_t (kN)	$f_{t.0}$ (N/mm^2)	CWAR	ARF	F_t(CWAR) (kN)	F_t(ARF) (kN)	Failure length (cm)
KV213B	403	11390	225	77	0.84	0.54	486	311	13
KE233B	462	10866	197	82	0.77	0.43	476	263	50
KE205B	454	13611	329	–	0.91	0.51	–	–	96
KE211A	460	10762	153	–	0.89	0.58	–	–	22
KE220B	466	14755	330	–	0.89	0.64	–	–	132
KK202A	458	11960	212	–	0.92	0.69	–	–	39
KK216A	425	12006	289	–	0.86	0.58	–	–	72
KK217A	384	9552	155	–	0.84	0.54	–	–	28
KK221A	378	10359	232	–	0.87	0.54	–	–	17
KK237A	408	11036	274	–	0.89	0.63	–	–	9
KL204B	453	12538	327	–	0.86	0.50	–	–	17
KL209A	446	10669	249	–	0.87	0.56	–	–	37
KL223B	469	13968	385	–	0.81	0.45	–	–	39
KL235A	398	10117	139	–	0.91	0.70	–	–	18
KL240B	374	9490	189	–	0.93	0.67	–	–	8
KL241B	470	14280	322	–	0.82	0.51	–	–	31
KN201A	361	9613	205	–	0.95	0.81	–	–	6
KP208A	453	12415	254	–	0.81	0.45	–	–	18
KP210B	447	12621	316	–	0.93	0.72	–	–	74
KP212A	501	14499	375	–	0.87	0.50	–	–	61
KP221B	419	10896	334	–	0.84	0.51	–	–	7
KP223A	412	10129	249	–	0.82	0.46	–	–	6
KP231B	417	11609	311	–	0.86	0.55	–	–	66
KP236A	434	12105	325	–	0.83	0.42	–	–	36
KP238A	460	12590	325	–	0.92	0.69	–	–	122
KV211B	459	14048	357	–	0.92	0.69	–	–	16
KV223B	438	13503	312	–	0.91	0.64	–	–	28
KV226A	404	11366	238	–	0.96	0.87	–	–	83
KV229A	433	11333	273	–	0.82	0.46	–	–	49
KV231A	499	13956	353	–	0.83	0.49	–	–	35

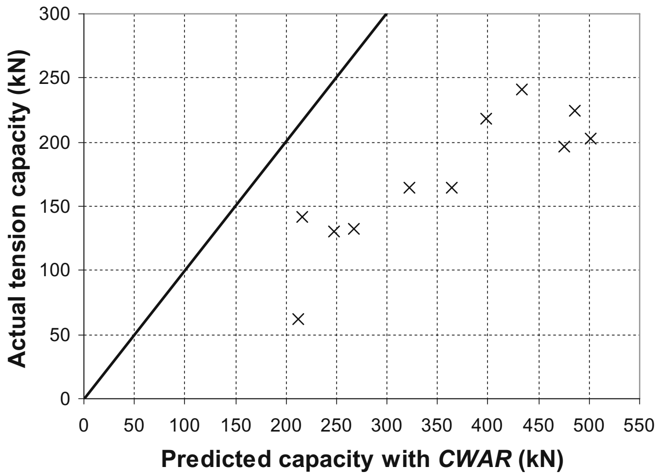

Fig. 8. Relationship between actual tension capacity and tension capacity predicted by means of CWAR

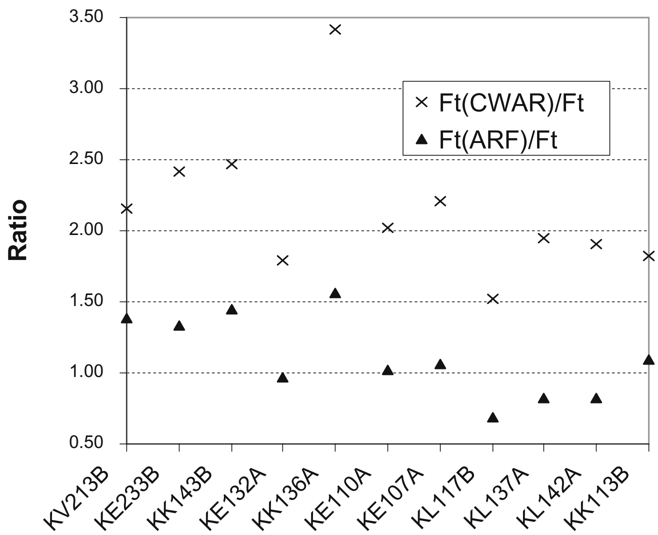

Fig. 9. Ratios of predicted tension capacities to actual tension capacities

tension capacity (Fig. 8). With ARF, the average ratio of predicted strength to actual strength was 1.11 with a coefficient of variation of 0.26. CWAR gave an average ratio of 2.15 with a coefficient of variation of 0.23 (Fig. 9). Correlations of ARF and CWAR with load-carrying tension capacities were also examined. With 27 planks with cross section of 50×100 mm, the coefficients of determination (R^2) were 0.11 for CWAR and 0.19 for ARF. On the other hand, the coefficients of determination (R^2) obtained from 30 planks with cross sections of 50×150 mm were 0.02 for CWAR and 0.07 for ARF. The results showed that ARF had a slightly better correlation than CWAR, even though neither of them is a good predictor of the tension strength as a single parameter. This means that for more accurate prediction, a model combining various parameters with ARF is necessary. The average length of the failure surface calculated for 57 planks was 367 mm with a coefficient of variation of 0.77. However, the knot measurement range is 100 mm. According to Hanhijärvi et al.,[5] KAR obtained with a knot measurement range of 300 or 900 mm showed a lower correlation with the tension strength than a length of 150 mm, while the average failure length received for their specimens was 460 mm. The length of the knot measurement range may not need to be more than 300 mm, but it may be an issue if 100 mm applied in the study is the most appropriate length.

Conclusions

In this research, a KAR-based factor, ARF, was established. Although results for ARF were obtained with a limited number of specimens, the results support the usefulness of ARF.

First, far better results in predicting tension capacity were obtained with ARF than with CWAR. This result suggests that the reduced strength due to knots can be predicted better by ARF than by CWAR when these values are combined with a parameter describing the clear wood strength parallel to the grain.

Second, as a single parameter predictor, ARF showed better correlation than CWAR with the tension strength of planks. However, the coefficient of determination was quite small for both parameters. This supports the development of multiparameter models where other parameters are combined with ARF. An explanation for the low correlation may be that the effect of the knot location cannot be considered in KAR or ARF. In addition, the knot measurement range does not correspond with the failure length.

To confirm the reliability of ARF, further research with a sufficient number of specimens is necessary. Improvement of the presented ARF-based prediction method may be achieved by combining the clear wood tension strength parallel to the grain with some nondestructive parameter. The dependency of the tension capacity from the location of knots and the failure mechanism is also a matter to be studied in developing predicting methods for the tension strength of timber.

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