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ORIGINAL ARTICLE

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Determining Young's modulus of timber on the basis of a strength database and stress wave propagation velocity II: effect of the reference distribution database on the determination

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Abstract A method of determining the Young's modulus of timber using the stress wave propagation velocity without knowing the timber density was developed in our previous study. This method enables the estimation of Young's modulus by Monte Carlo simulation using an existing database of the Young's modulus versus density relationship as reference. Here, in Part II, we consider the effect of the reference distribution database on the accuracy of the estimated Young's modulus by the developed method. Twelve different reference distribution databases were used in this study, containing Young's modulus versus density data for more than 13 000 real-size timber specimens of ten different species. We obtained the following results: (1) the distribution of Young's modulus estimated using an arbitrary stress wave propagation velocity depends on the reference distribution database employed, (2) the most important factor is not that the reference database has data on the same species as the timber in the test, but rather that the reference distribution database covers the foreseeable range of timber densities within the test, and (3) the estimation accuracy is higher than about 80% when the database covers many species and has wide ranges of densities and Young's moduli. This estimation method was developed in order to measure the Young's modulus of timber whose density cannot be measured. Considering that the quality of lumber has a large variation, such estimation accuracy will be useful for practical applications.

Key words Strength database · Stress wave propagation velocity · Young's modulus · Estimation · Monte Carlo simulation

Introduction

In our studies, we have strived to establish a nondestructive inspection method of evaluating timber strength for use in repairing or remodeling wooden structures. In Part I of this study, we proposed a method of estimating Young's modulus by measurement of stress wave propagation velocity only, without knowing the timber density, because the proposed estimation method uses an existing reference distribution database, namely, the relationship between Young's modulus and density, to estimate Young's modulus by Monte Carlo simulation.[1] Moreover, we demonstrated that the Young's modulus estimated using the proposed method (MOE-e) agrees well with the Young's moduli obtained using a bending test (MOE-b) and from the measured density and stress wave propagation velocity (MOE-d) of the timber, and that it was proportional to both of the observed Young's moduli with a high level of correlation.

The Young's modulus estimated by our method may depend on the type of reference distribution database used in the simulation. Here, in Part II of our study, the effect of reference distribution databases on the estimated Young's modulus is investigated to determine the type of reference distribution database that will yield the best estimation accuracy.

Measurement equipment for stress wave propagation velocity

Figure 1a shows the equipment, which is portable and easy to operate, used to measure stress wave propagation time, and hence the velocity. In previous research studies, this equipment has been used to investigate the effects of the inhomogeneity of laminated timber and the effects of cross-

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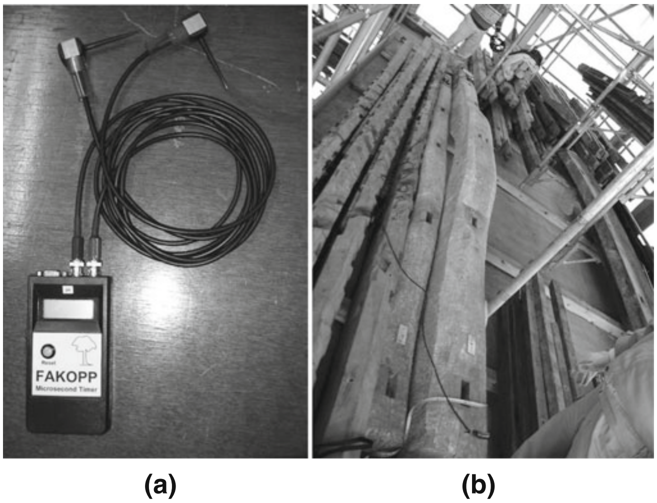


Fig. 1a.b. Measurement of stress wave propagation time. **a** Stress wave timer, **b** field measurement

sectional changes of a board specimen on acoustic velocity.[2,3] For example, to measure longitudinal Young's modulus, two sensors were placed on the ends of the specimen, and one of the sensors was hit with a hammer. The stress wave due to the impact propagated to the other sensor and the propagation time was measured by the equipment (FAKOPP). This measurement can be performed by either removing the specimen, which is shown in Fig. 1b, or by leaving the specimen as part of the structure. The advantage of employing a stress wave in a nondestructive inspection method is that the measurement can be conducted while the specimen remains part of a structure. In other words, the structure does not need to be dismantled for the measurement to be done.

Estimation method for Young's modulus

The relationship between stress wave propagation velocity (v) and Young's modulus (E) is given by $E = \rho v^2$, where ρ is the density. However, it is difficult to measure the density of a timber that remains within a structure or an old timber whose cross section is irregular. Hence, a method using Monte Carlo simulation to obtain Young's modulus from the stress wave propagation velocity only, without measuring the density of a specimen, was proposed in our previous report, which makes up Part I of this study. In that report, a detailed description of this estimation method was given. With our estimation method, it is possible for multiple values to be accepted as the estimated Young's modulus when only one stress wave propagation velocity is measured, and they are called the accepted E_i data in this report. The average accepted E_i data is used as the representative estimated Young's modulus (MOE-e).[1]

In our estimation method, the existing reference distribution database plays a crucial role. Here, in Part II, we discuss the effect of the reference distribution database on the estimated Young's modulus. Table 1 shows a list of the reference distribution databases used in this study, which are sorted in ascending order of average density. These databases on density and Young's modulus were carefully collected by inspection and research organizations across Japan. All the databases, except database 4 and 5, were grouped according to tree species, i.e., todomatsu (*Abies sachalinensis*), Japanese cedar (*Cryptomeria japonica* D. Don), Jezo spruce (*Picea jezoensis*), hiba (*Thujopsis dolabrata*), hemlock (*Tsuga heterophylla*), larch (*Larix kaempferi*), Douglas fir (*Pseudotsuga menziesii*), Japanese cypress (*Chamaecyparis obtusa* Endl.), Japanese red pine (*Pinus densiflora*), and Siberian larch (*Larix sibirica* Ledeb.), whereas database 4 contained all ten species of trees. Database 5 was an excerpt of the mean for each species from the database of the strength performance of commercial lumber (mechanical classification data) published by the Forestry and Forest Products Research Institute.[4] Table 1 also shows major features of the density and Young's modulus for each database. These data are corrected to those corresponding to a moisture content of 15%, as determined by ASTM D-1900.[5] Our estimation method used the distribution of density (ρ) as well as the coefficients of regression lines (a and b) and the residual standard deviation in Young's modulus (SD_i) shown in Table 1. As for the distribution of density, the goodness-of-fit test for the normal distribution, log normal distribution, 2P Weibull distribution, and 3P Weibull distribution were examined for each database. Cumulative distribution functions $[F(\rho)]$ of these distributions are expressed by the following equations,[6,7] in which $\mu, \sigma, \lambda, \varphi, \eta, m$, and γ are parameters given with the average (μ_p) and the standard deviation (s_p).

The normal distribution is:

$$F(\rho, \mu, \sigma) = \int_{-\infty}^{\rho} \frac{1}{\sigma \cdot \sqrt{\pi}} \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{z-\mu}{\sigma}\right)^2\right] dz \tag{1}$$

where $\mu = \mu_p$ and $\sigma = s_p$.

The logarithmic normal distribution is:

$$F(\rho, \lambda, \xi) = \int_{-\infty}^{\rho} \frac{1}{z \cdot \xi \cdot \sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{\ln(z)-\lambda}{\xi}\right)^2\right] dz \tag{2}$$

where $\lambda = \ln(\mu_\rho) - 0.5 \cdot \ln\left(1 + \left(\frac{s_\rho}{\mu_\rho}\right)^2\right)$

and $\xi = \sqrt{\ln\left(1 + \left(\frac{s_\rho}{\mu_\rho}\right)^2\right)}$

The 2P Weibull distribution is:

$$F(\rho, \eta, m) = 1 - \exp\left[-\left(\frac{\rho}{\eta}\right)^m\right] \tag{3}$$

where $\eta = \dfrac{\mu_\rho}{\Gamma\left[\dfrac{m+1}{m}\right]}$ and $\eta = \dfrac{s_\rho}{\sqrt{\Gamma\left[\dfrac{m+2}{m}\right] - \Gamma\left[\dfrac{m+1}{m}\right]^2}}$

Here, $\Gamma[\varpi] = \int_0^{\infty} t^{\omega-1} \cdot \exp(-t) dt$.

The 3P Weibull distribution is:

$$F(\rho, \eta, m, \gamma) = 1 - \exp\left[-\left(\frac{\rho-\gamma}{\eta}\right)^m\right] \tag{4}$$

Table 1. Reference distribution databases for Monte Carlo simulation

No.	Database	Number of data sets	Density (kg/m^3)			Distribution of density (kg/m^3)	MOE in bending (GPa)			Coefficient		
			Minimum	Mean	Maximum	Type[a]; coefficients[b]; lower limit (*Ll*)	Minimum	Mean	Maximum	a^c	b^c	SD_i^d
1	Todomatsu	605	298.3	394.7	545.7	LN; $\gamma = 5.975$, $\xi = 0.084$; $Ll = 341.3$	4.71	9.33	13.41	0.0191	1.80	1.287
2	Japanese cedar	7875	231.7	410.6	605.7	LN; $\gamma = 6.012$, $\xi = 0.110$; $Ll = 339.7$	2.38	7.24	14.28	0.0116	2.48	1.686
3	Jezo spruce	860	272.1	419.2	631.9	NO; $\mu = 419.508$, $\sigma = 39.172$; $Ll = 353.6$	5.09	10.09	18.14	0.0274	−1.41	1.500
4	ALL[e]	12957	231.7	439.8	885.9	LN; $\gamma = 6.078$, $\xi = 0.152$; $Ll = 338.7$	2.38	8.37	19.38	0.0203	−0.55	1.949
5	FFPRI[f]	59	354.0	467.5	607.0	3PW[g]; $\eta = 165.897$, $m = 2.768$, $\gamma = 319.979$; $Ll = 370.2$	5.15	10.26	15.10	0.0317	−4.56	2.226
6	Hiba	315	360.0	472.1	624.5	3PW; $\eta = 139.563$, $m = 2.618$, $\gamma = 348.222$; $Ll = 390.9$	5.08	9.75	13.94	0.0031	8.29	1.425
7	Hemlock	259	349.0	491.1	629.0	3PW; $\eta = 234.853$, $m = 4.192$, $\gamma = 277.719$; $Ll = 389.4$	4.30	10.96	19.17	0.0226	−0.15	1.970
8	Larch	1215	320.8	500.9	714.2	LN; $\gamma = 6.210$, $\xi = 0.114$; $Ll = 411.5$	3.37	8.85	14.97	0.0178	−0.06	1.535
9	Douglas fir	673	392.1	518.7	763.9	LN; $\gamma = 6.247$, $\xi = 0.098$; $Ll = 437.9$	6.01	11.58	19.38	0.0243	−1.02	2.027
10	Japanese cypress	378	399.9	521.1	634.9	3PW; $\eta = 223.455$, $m = 6.020$, $\gamma = 313.329$; $Ll = 447.1$	6.56	10.74	14.10	0.0118	4.59	1.237
11	Japanese red pine	507	378.1	527.9	750.4	NO; $\mu = 527.938$, $\sigma = 48.244$; $Ll = 446.3$	3.70	10.26	17.18	0.0257	−3.31	1.698
12	Siberian larch	270	470.7	619.9	885.9	LN; $\gamma = 6.424$, $\xi = 0.107$; $Ll = 512.9$	5.13	12.56	19.12	0.0204	−0.10	1.955

MOE, Young's modulus; NO, normal distribution; LN, logarithmic normal distribution; 3PW, 3P Weibull distribution

[a] The best fitted distribution type

[b] Coefficients of cumulative distribution functions refer to Eqs. 1–4 in the text

[c] *a*, *b*: Regression coefficients of relationship between MOE in bending and density, i.e., MOE in bending = *a* × Density + *b*

[d] SD_i = (standard deviation of MOE in bending) × $(1 - R^2)^{1/2}$, where R^2 is the coefficient of determination for the relationship between density and MOE in bending

[e] ALL: Database including data of all species used in this report, i.e., databases 1–3 and 6–12

[f] FFPRI: Database of strength performance of commercial lumber (mechanical classification data); No. 7 from the research group of the Forestry and Forest Products Research Institute

[g] The validity of this fitting is not accepted by the K-S test

where $\eta = \dfrac{s_\rho}{\sqrt{\Gamma\left[\dfrac{2+m}{m}\right] - \Gamma\left[\dfrac{1+m}{m}\right]^2}}$ and $\gamma = \mu_\rho - \eta \cdot \Gamma\left[\dfrac{m+1}{m}\right]$

The Kolmogorov-Smirnov test (K-S test) was used as the goodness-of-fit test, and the most suited distribution type was selected among these four distributions. The coefficients in the cumulative distribution function of the most suited distribution for each database are also shown in Table 1. Figure 2 shows an example of cumulative frequency curve for the density data of reference distribution database 5 with the most suited distribution type (3P Weibull distribution).

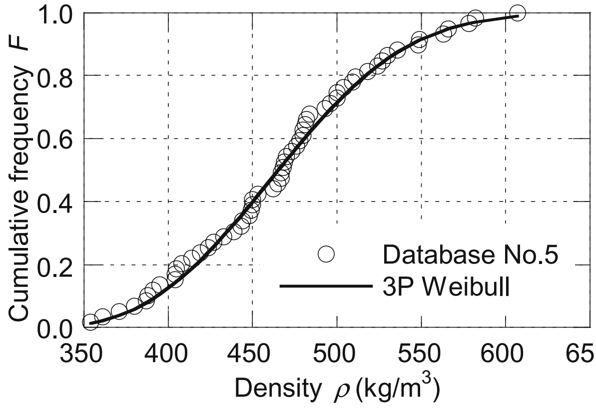


Fig. 2. Example of reference distribution database (database 5). *Circles*, points from the database; *line*, the best fit distribution according to the Kolmogorov-Smirnov test

Results and discussion

In this study, we considered two effects of the reference distribution database on the estimated Young's modulus: (1) the effect of the reference distribution database on the distribution of accepted E_i data and (2) the effect of the reference distribution database on the accuracy of the estimated Young's modulus. As examples of the former, the two cases $v = 4000$ m/s and $v = 5000$ m/s were examined; however, the effect of the latter was considered by comparing the estimated Young's modulus (MOE-e) with the measured values, which were MOE-d calculated from ρv^2 and MOE-b obtained using a bending test.

Effect of reference distribution database on the distribution of Young's moduli estimated from the stress wave propagation velocity for v = 4000 m/s and v = 5000 m/s

First, to determine the stress wave propagation velocity used in the analyses, the range of measured stress wave propagation velocities was examined. Figure 3 shows a histogram of the stress wave propagation velocities, which were collected by our previous inspection of new (220 specimens) and used (796 specimens) timbers. From Fig. 3, the minimum, average ± standard deviation, and maximum of the data are 2209.8, 4721.1 ± 504.2, and 6896.6 m/s, respectively. Therefore, 4000 m/s (the cumulative probability among all data is 0.079) and 5000 m/s (cumulative probability 0.715) were chosen as representative stress wave propagation velocities for the analysis in this section. Then, using these two stress wave propagation velocities, the distribution of accepted E_i data, which was a group of estimated Young's moduli, was analyzed for each reference distribution database. The error range in the estimation process was set to be within 5%.

Figure 4 shows histograms of accepted E_i data for a stress wave propagation velocity of 4000 m/s for which the reference distribution databases for Japanese cedar (database 2) and Siberian larch (database 12) were employed (because the results from these two databases have particularly large differences). As shown in Fig. 4, when database 2 was employed, the minimum, average ± standard deviation, and maximum accepted E_i data were 4.74, 6.71 ± 0.72, and 8.94 GPa, respectively, and the distribution is closest to a normal distribution. On the other hand, when database 12 was employed, the corresponding values were 7.44, 9.95 ± 1.09, and 13.74 GPa, respectively. This average is about 50% higher than the average when database 2 is employed. The distribution obtained using database 12 is closer to a log normal distribution, which differs from that in the case of using database 2.

Figure 5 shows box-and-whisker plots of accepted E_i data for each reference distribution database for stress wave propagation velocities of 4000 m/s (Fig. 5a) and 5000 m/s (Fig. 5b). Figure 5 shows that for both stress wave propagation velocities, accepted E_i data generally increase with the database number, which corresponds to an increase in the

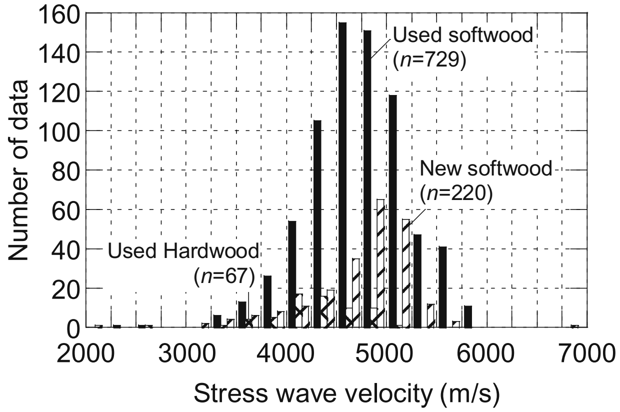


Fig. 3. Measurement of distribution of stress wave velocity in timber in existing facilities (50–250 years old): temples and their peripheral facilities ($n = 12$), shrine ($n = 1$), and private houses ($n = 2$)

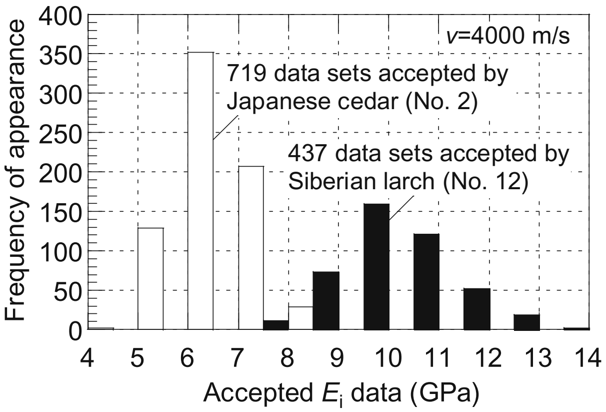
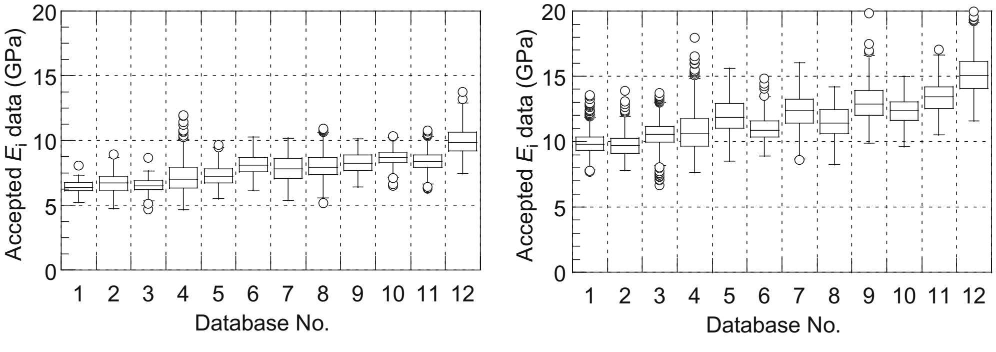


Fig. 4. Distribution of accepted Young's modulus (E_i) data within an error range of 5% when the stress wave velocity was 4000 m/s

Fig. 5. Effect of reference distribution database on accepted E_i data when stress wave velocities were 4000 m/s and 5000 m/s. *Circles* denote outliers



(a) v=4000 m/s

(b) v=5000 m/s

Table 2. Test results of sawn timber

Species	Number of specimens	Moisture content (%)		Density (ρ) (kg/m^3)		Stress wave velocity (μ) (m/s)		MOE-d (GPa)		MOE-b (GPa)	
		Mean	*SD*	Mean	*SD*	Mean	*SD*	Mean	*SD*	Mean	*SD*
Japanese cedar	31	25.2	4.7	568.4	70.5	3814.4	353.3	8.2	1.1	7.7	1.1
Japanese cypress	30	19.9	3.4	558.2	28.6	4628.9	203.6	12.0	1.3	10.3	1.3
Douglas fir	10	14.3	0.9	507.9	48.1	5309.0	320.4	14.5	2.8	12.6	2.1

MOE-d, Young's modulus obtained from the measured density and stress wave propagation velocity; MOE-b, Young's modulus obtained using a bending test

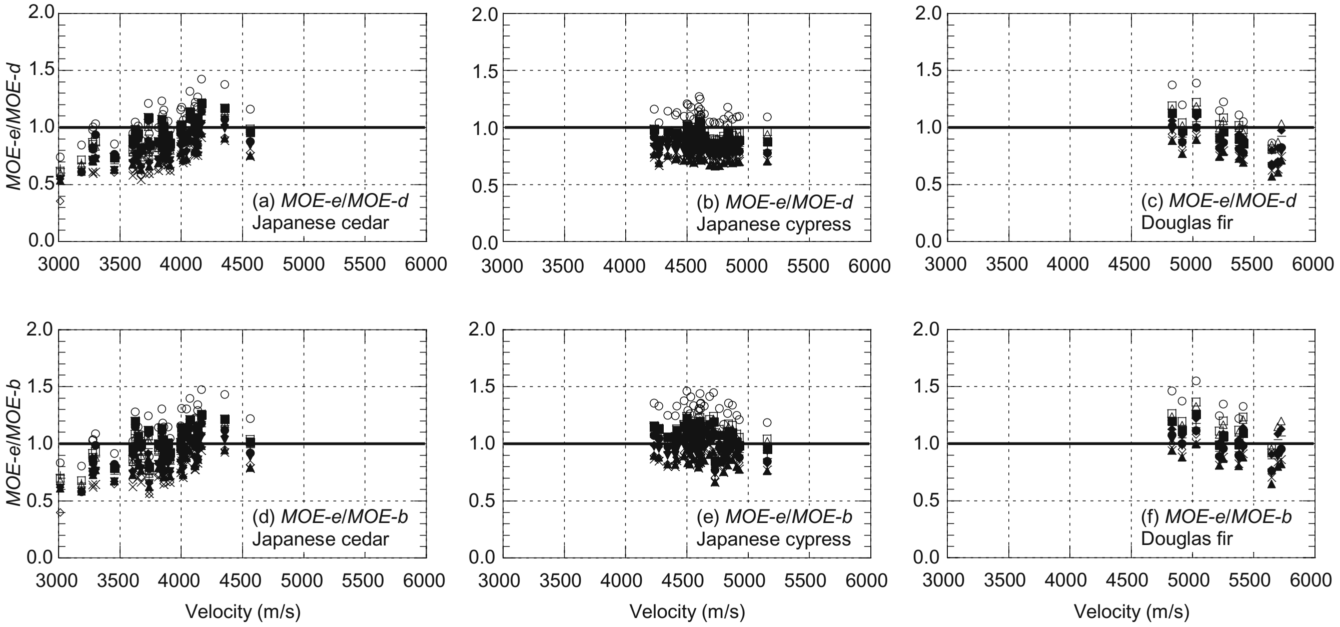


Fig. 6. Relationship between stress wave velocity and estimation accuracy expressed by the ratio of estimated Young's modulus to measured Young's modulus using the timber density (*MOE-e/MOE-d*) and by the ratio of estimated Young's modulus to measured Young's modulus using a bending test (*MOE-e/MOE-b*). ×, obtained using database 1; ▲, database 2; ◇, database 3; ▼, database 4; ◆, database 5; ●, database 6; –, database 7; +, database 8; △, database 9; ■, database 10; □, database 11; ○, database 12

average density of the database. In addition, the range of accepted E_i data is wider for database 4 (containing all species) than for the other databases. The reason is considered to be that database 4 has the widest distribution ranges of densities and Young's moduli, as shown in Table 1. As discussed above, these simulation results reveal that the Young's modulus estimated for the same stress wave propagation velocity differs significantly and depends on the reference distribution database employed.

Effect of reference distribution database on accuracy of estimated Young's modulus

As discussed in the preceding section, the accepted E_i data produced by simulation in this method depend on the reference distribution database employed. Therefore, in order to improve the accuracy of the estimated Young's modulus, an appropriate database must be selected. In this section, we discuss the estimation accuracy for each reference distribution database by comparing the estimated values obtained by simulation with the experimental values. Here, since the accepted E_i data are distributed as shown in Figs. 4 and 5, the average accepted E_i data is used as the representative estimated Young's modulus (MOE-e).[1]

Our specimens were sawn timbers of Japanese cedar (n = 31), Japanese cypress (n = 30), and Douglas fir (n = 10). The timber specimens measured 120 mm (width) × 200 mm (depth) × 4000 mm (length). In the experiments, the density (ρ) determined from the size and weight of each specimen and the measured stress wave propagation velocity (v) were used to obtain Young's modulus: MOE-d = ρv^2. The stress wave propagation velocity of each specimen was the average of three measurements. In addition, the Young's modulus of bending (MOE-b) was measured by a four-point bending test. Table 2 shows the results.

The following parameters were used to indicate estimation accuracy: the ratio of MOE-e to MOE-d and the ratio of MOE-e to MOE-b. Figure 6 shows the relationships between these accuracy indicators of the estimation and

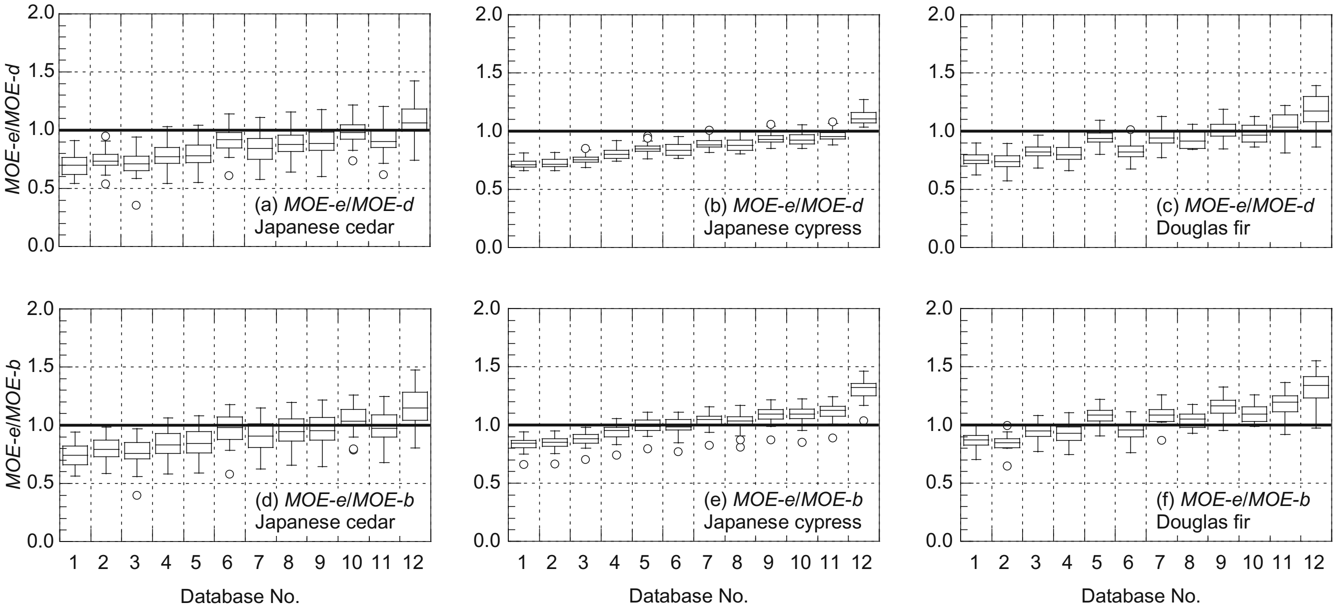


Fig. 7. Effect of reference distribution database on estimation accuracy. *Open circles* mark denote outliers

stress wave propagation velocities. In Fig. 6, 12 plots are shown for the stress wave velocity data measured for each timber specimen because there are 12 different MOE-e data sets obtained using 12 databases. In the case of MOE-e/MOE-d (Fig. 6a: Japanese cedar, Fig. 6b: Japanese cypress, and Fig. 6c: Douglas fir), the values are generally less than unity, i.e., the estimated Young's modulus (MOE-e) is smaller than the measured MOE-d. However, note that some databases yield MOE-e/MOE-d values greater than unity, particularly when database 12 for Siberian larch was employed, as indicated by open circles. On the other hand, for MOE-e/MOE-b (Fig. 6d: Japanese cedar, Fig. 6e: Japanese cypress, and Fig. 6f: Douglas fir), the values are generally higher across the three species than the MOE-e/MOE-d values plotted in Fig. 6a–c because the MOE-b values are smaller than the MOE-d values for all the species, as shown in Table 2. As was also the case for MOE-e/MOE-d, the accuracy indicators are affected by the reference distribution database, and some databases yield an overestimated MOE-b.

Figure 7 summarizes the accuracy indicators (MOE-e/MOE-d and MOE-e/MOE-b) using box-and-whisker plots. One box-and-whisker plot expresses the statistical data of the same species (n = 31, 30, and 10 for Japanese cedar, Japanese cypress, and Douglas fir, respectively), which were analyzed using the same database. As seen from the plots, the database significantly affects the degree of accuracy. Figures 7 and 5 show that the accuracy indicators increase as the database number increases. This increase corresponds to an increase in average density in each database. In particular, the use of database 12 leads to an overestimation compared with that of the measured values.

When databases of the same species as specimens in the test were used for the simulation, the averages and the standard deviations of MOE-e/MOE-d for Japanese cedar, Japanese cypress, and Douglas fir were 0.75 ± 0.09 (Fig. 7a, database 2), 0.93 ± 0.06 (Fig. 7b, database 10), and 1.01 ± 0.11 (Fig. 7c, database 9), and those of MOE-e/MOE-b were 0.79 ± 0.10 (Fig. 7d, database 2), 1.09 ± 0.08 (Fig. 7e, database 10), and 1.16 ± 0.10 (Fig. 7f, database 9), respectively. For Japanese cedar, both accuracy indicators obtained using databases of the same species show poor estimation results, but the results obtained using the databases of different species yield good estimation results. The databases employed in this study were all for air-dried timber or data that were compensated for air-dried timber (moisture content, 15%). The Japanese cedar specimens tested, however, had high moisture contents at the time of the experiment, and consequently, had higher densities than commonly available air-dried Japanese cedar timber. All the databases that provide good estimation accuracies for Japanese cedar in Fig. 7 have higher densities than the Japanese cedar database (database 2) (refer to Table 1). Therefore, it is considered that databases with density distributions closer to that of Japanese cedar specimens yield better estimation accuracies than the Japanese cedar database (database 2). On the other hand, the average densities of the Japanese cypress and Douglas fir specimens were 558.2 kg/m^3 and 507.9 kg/m^3, which are similar to those in the Japanese cypress and the Douglas fir databases, databases 10 and 9, respectively (refer to Tables 1 and 2). From these results, it is considered that reasonably good estimation accuracy is obtained when databases of the same species are used for analyzing Japanese cypress and Douglas fir.

The above discussion clearly indicates that it is more important in our estimation method that the density of the specimen (which might have a high moisture content) at the time of the stress wave propagation velocity measurement be included in the density distribution in the database than to use a database of the same species as the specimen in the test. Because our estimation method is more effective in cases where it is difficult to measure density, a database with a wide density distribution is advantageous. Databases 4 and 5 contain data of numerous species and have a wide density distribution. Using these databases, the estimation accuracy indicators for Japanese cedar, Japanese cypress, and Douglas fir were 0.79–0.85, 0.81–0.99, and 0.82–1.08, respectively. Hence, these databases are versatile and effective for use in our estimation method. Moreover, they are also useful when it is difficult to identify the species of the timber.

Conclusions

In this study, the effect of the reference distribution database on the accuracy of the estimated Young's modulus by our developed method, which is based on the use of stress wave propagation velocity and an existing strength database, was investigated. The results obtained are as follows:

(1) The distribution of Young's moduli estimated using an arbitrary stress wave propagation velocity depends on the reference distribution database employed.
(2) Databases providing higher estimation accuracy by this method usually have wide density distributions that include the density of the target timber, rather than data of the same species as the target timber.
(3) The estimation accuracy is about 80% or better when a database containing many species or a database of all commercial lumber species with a wide density range is employed.

This estimation method was developed in order to measure the Young's modulus of timber whose density cannot be measured. Considering that the quality of lumber has a large variation, such estimation accuracy is acceptable for practical use.

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