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ORIGINAL ARTICLE

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Simulation method to generate the strength of glulam using correlated random variables

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Abstract Many reports have been published about designing the strength of glulam using simulation methods. In simulation methods, one of the most important problems is how to deal with correlations among strength factors, i.e., modulus of elasticity (MOE), modulus of rupture (MOR), tensile strength (σ_T), and compression strength (σ_C). For example, in the case that the MOR criteria of glulam is $\sigma_{ni}/f_{ni} + \sigma_{bi}/f_{bi} \geq 1$ (where σ_{ni} and σ_{bi} are the axial stress and the bending stress of the i-th lamina respectively, and f_{ni} and f_{bi} are the axial strength and the bending strength of the i-th lamina respectively), a correlation between f_{ni} and f_{bi} exists. How can we account for this correlation when calculating the strength of glulam, bearing in mind that it is very difficult to measure the correlation coefficients among MOR, σ_T, and σ_C? We developed a method by which these problems could be solved, and, using random variables generated by this method, the strengths of glulam were simulated. The simulated values were almost the same as the experimental values. The results indicated the usefulness of the method.

Key words Simulation method · Glulam strength · Correlation coefficients · Lamina strength

Introduction

Several simulation methods[1–7] for designing the strength of glulam have been proposed. In the simulation method, an important problem is how to deal with correlations among strength factors such as modulus of elasticity (MOE), modulus of rupture (MOR), tensile strength (σ_T), and compression strength (σ_C), because some of them are involved in the strength criteria of glulam. For instance, in the case of two factors, MOE and σ_T, the simulation method to generate them using correlated random variables was shown in Taylor and Bender,[8] but no method has been put forward to incorporate additional factors. The reason might be that it is very difficult to measure the correlation coefficients among MOR, σ_T, and σ_C. Even if the matched specimen method is used, we cannot measure the strength factors of the exact same specimens.

Therefore, we developed a method by which the correlation coefficients among the strength factors could be calculated. Using this method, the strengths of glulam were simulated. To validate the method, experiments were carried out to determine MOE, MOR, σ_T, and σ_C of glulam; the results are given in this report.

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Materials and methods

Specimens and experiments for laminae

MOE, MOR, σ_T, and σ_C were measured for Sugi (Japanese cedar) and Douglas-fir laminae. Before tension and compression testing, MOE values were measured by bending tests, as shown in Fig. 1. When σ_T and σ_C were measured, Young's modulus in tension (E_T) and compression (E_C) were also measured. E_C and E_T were measured by displacement transducers fixed on four sides of the specimens. The grades of Sugi laminae were L30–L90 and those of Douglas fir laminae were L90–L160. The grades for laminae were determined according to the minimum MOE measured using a continuous MOE measuring machine. Compatibility conditions for each lamina grade in the Japanese Agricultural Standard (JAS) for glulam are shown in Table 1. In this table, 5% values are calculated under the assumption that the strength is normally distributed.

The experimental setup is shown in Fig. 1. The width and thickness of laminae were 105 mm and 30 mm, respectively, and each lamina had a single finger joint. MOE values of each specimen for tensile and compression strengths were measured before strength testing and the grade was decided. The number of specimens for each grade was about 30.

Correlation coefficients between strength factors of laminae

Each lamina broke at the finger joint. The results of the strength experiments are summarized in Table 2; in this table, the best-fit probability distributions are also shown. Kolmogorov-Smirnov examination was used to fit the strength data to the probability distribution.

The relations between MOE and strengths in Douglas fir and Sugi are shown in Figs. 2 and 3, respectively. Using these regression analyses, correlation coefficients between MOE and the strengths were calculated. The results are shown in Tables 3 and 4.

Calculation of correlation coefficients among strength factors other than those measured by experiment

In Tables 3 and 4, correlation coefficients could not be measured rigorously, even though matched specimens were used. Therefore, we calculated the values by the following method. First, a combination of uniform random variables from 0 to 1 with no correlation was generated by the Mersenne twister method.[9] The number of generated random variables for each strength factor was 1000. Next, these variables were transformed into a combination of standard normal variables with no correlation. Third, they were transformed into a combination of standard normal variables with correlation. Fourth, they were further transformed into non-normal variables with correlation by Rosenblatt transformation. Using this method, correlation coefficients among the strength factors such as MOR, σ_T, and σ_C were calculated. The results are shown in Tables 5 and 6. We can see that the values measured by experiments are almost same as those generated by this method.

Validation of the method

In order to validate the method, using all correlation coefficients in Tables 5 and 6, 1000 strength values for each factor were generated and correlation coefficients were calculated by the above-mentioned method. The results are shown in Tables 7 and 8. Again, we can see that the values in Table 7 and 8 are almost the same as those in Table 5 and 6. These results indicate the usefulness of the method

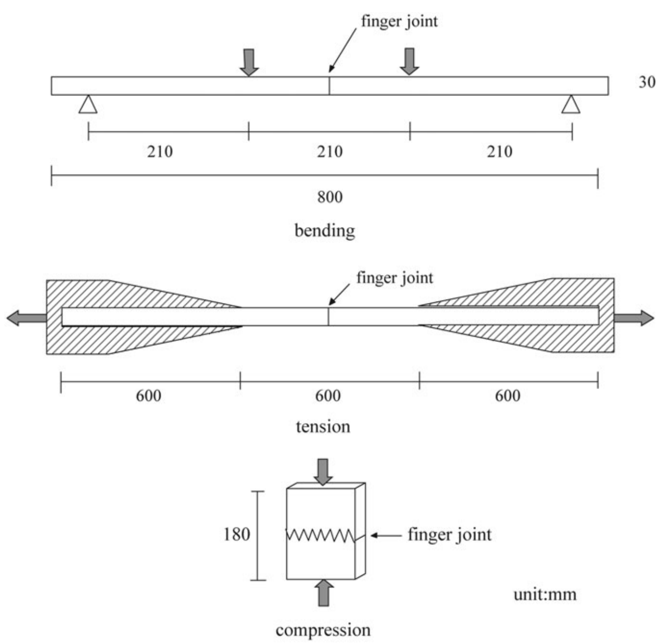

Fig. 1. Setups for various experiments on lamina

Table 1. Compatibility conditions for modulus of elasticity (MOE) in Japanese Agricultural Standard

Grade	Minimum MOE (kN/mm^2)	Bending strength (N/mm^2)		Tensile strength (N/mm^2)	
		Mean	5% value	Mean	5% value
L200	20.0	81.0	61.0	48.0	36.0
L180	18.0	72.0	54.0	42.5	32.0
L160	16.0	63.0	47.5	37.5	28.0
L140	14.0	54.0	40.5	32.0	24.0
L125	12.5	48.5	36.5	28.5	21.5
L110	11.0	45.0	34.0	26.5	20.0
L100	10.0	42.0	31.5	24.5	18.5
L90	9.0	39.0	29.5	23.5	17.5
L80	8.0	36.0	27.0	21.5	16.0
L70	7.0	33.0	25.0	20.0	15.0
L60	6.0	30.0	22.5	18.0	13.5
L50	5.0	27.0	20.5	16.5	12.0
L40	4.0	24.0	18.0	14.5	10.5
L30	3.0	21.0	16.0	12.5	9.5

Table 2. Strength properties of laminae

Species	Strength factor	MOE (kN/mm^2)	MOR (N/mm^2)	σ_T (N/mm^2)	σ_C (N/mm^2)
Douglas fir	No. of specimens	420	150	145	125
	Mean	13.68	58.61	40.35	47.90
	Coefficient of variation	0.17	0.21	0.21	0.13
	Probability distribution function	3P-Weibull	Normal	Normal	Log-normal
Sugi	No. of specimens	498	170	158	170
	Mean	5.49	34.57	23.76	31.73
	Coefficient of variation	0.34	0.25	0.28	0.17
	Probability distribution function	Normal	Log-normal	Log-normal	Log-normal

MOR, modulus of rupture; σ_T, tensile strength; σ_C, compression strength; 3P, three-parameter

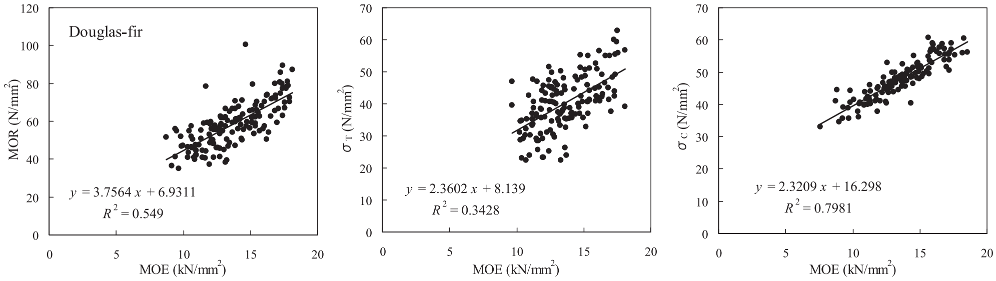

Fig. 2. Relations between modulus of elasticity (*MOE*) and modulus of rupture (*MOR*), tensile strength (σ_T), and compression strength (σ_C) in Douglas fir

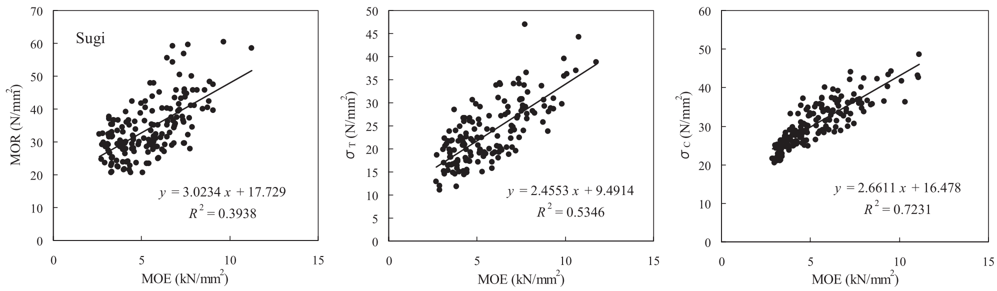

Fig. 3. Relations between MOE and various strength indicators in sugi

Table 3. Calculated correlation coefficients for Douglas-fir laminae

	MOE	MOR	E_T	σ_T	E_C	σ_C
MOE	1	0.741	0.881	0.585	0.810	0.893
MOR		1	?	?	?	?
E_T			1	0.503	?	?
σ_T				1	?	?
E_C		Sym.			1	0.745
σ_C						1

E_T, Young's modulus in tension; E_C, Young's modulus in compression; ?, unknown

Table 4. Calculated correlation coefficients for sugi laminae

	MOE	MOR	E_T	σ_T	E_C	σ_C
MOE	1	0.628	0.977	0.731	0.943	0.850
MOR		1	?	?	?	?
E_T			1	0.718	?	?
σ_T				1	?	?
E_C		Sym.			1	0.805
σ_C						1

Table 5. Correlation coefficients for Douglas-fir laminae by the proposed method

	MOE	MOR	E_T	σ_T	E_C	σ_C
MOE	1	0.749	0.882	0.583	0.809	0.892
MOR		1	0.669	0.464	0.600	0.674
E_T			1	0.508	0.731	0.797
σ_T				1	0.459	0.517
E_C		Sym.			1	0.736
σ_C						1

Table 6. Correlation coefficients for sugi laminae by the proposed method

	MOE	MOR	E_T	σ_T	E_C	σ_C
MOE	1	0.638	0.979	0.754	0.951	0.855
MOR		1	0.619	0.476	0.596	0.525
E_T			1	0.732	0.929	0.838
σ_T				1	0.718	0.652
E_C		Sym.			1	0.810
σ_C						1

Table 7. Correlation coefficients for Douglas-fir laminae using correlated random variables

	MOE	MOR	E_T	σ_T	E_C	σ_C
MOE	1	0.744	0.878	0.566	0.807	0.880
MOR		1	0.673	0.434	0.617	0.662
E_T			1	0.488	0.714	0.786
σ_T				1	0.454	0.502
E_C		Sym.			1	0.742
σ_C						1

Table 8. Correlation coefficients for sugi laminae using correlated random variables

	MOE	MOR	E_T	σ_T	E_C	σ_C
MOE	1	0.647	0.976	0.760	0.946	0.853
MOR		1	0.638	0.480	0.605	0.524
E_T			1	0.742	0.932	0.829
σ_T				1	0.735	0.661
E_C		Sym.			1	0.805
σ_C						1

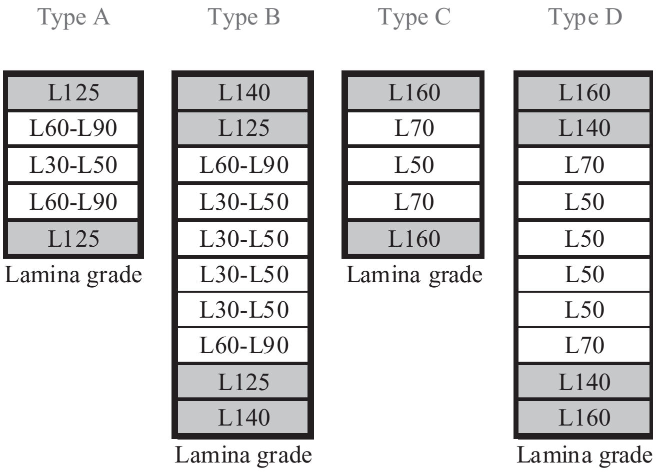

Fig. 4. Specimens for glulam experiments. The width of all glulam specimens was 105 mm. The thickness of type A and C specimens was 150 mm and the thickness of type B and D specimens was 300 mm. *Shaded laminae,* Douglas-fir; *white laminae*, sugi

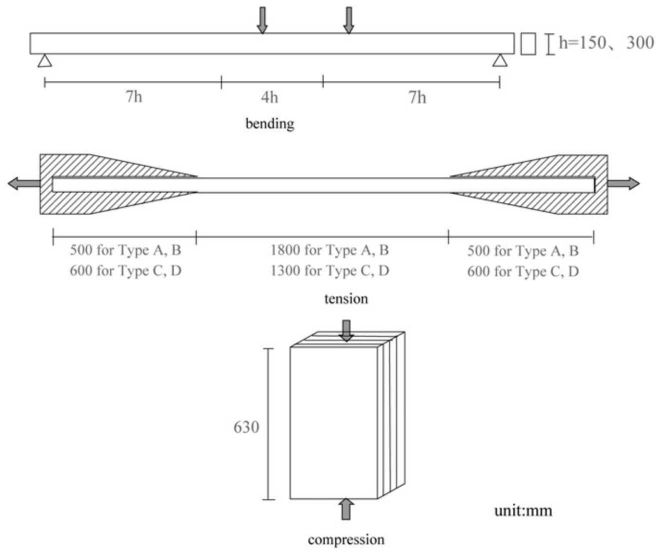

Fig. 5. Setups for the glulam experiments

proposed here. In this method, all correlation coefficients were transformed according to the best-fit probability distributions.[10]

Simulation of glulam strengths using correlated random variables

Using random variables generated by the above-mentioned method, glulam strengths were generated and compared to those of experiments.

Specimens and experiments of glulam

Glulam specimens are shown in Fig. 4, in which the combinations of laminae are also shown. Bending, tensile, and compression strengths were measured in about six specimens for each experiment. Bending experiments were conducted according to JAS, but compression experiments were conducted according to Testing and Evaluation Method for Full-size Structural Timber Strength,[11] as shown in Fig. 5. In contrast, tension experiments were not conducted to the specified standard, because the length of specimens was not as long as shown in Fig. 5.

Results and discussion

An example of the relationships between strength and cumulative probability for type A and B are shown in Fig. 6, and also type C and D in Fig. 7. Results of experiments are also shown. Each glulam specimen broke at the finger joint and the results for other glulam grades were similar. The simulation method is that described above. Five hundred virtual glulam specimens were generated. The fracture criteria for bending and axial strength are $\sigma_{ni}/f_{ni} + \sigma_{bi}/f_{bi} \geq 1$ and

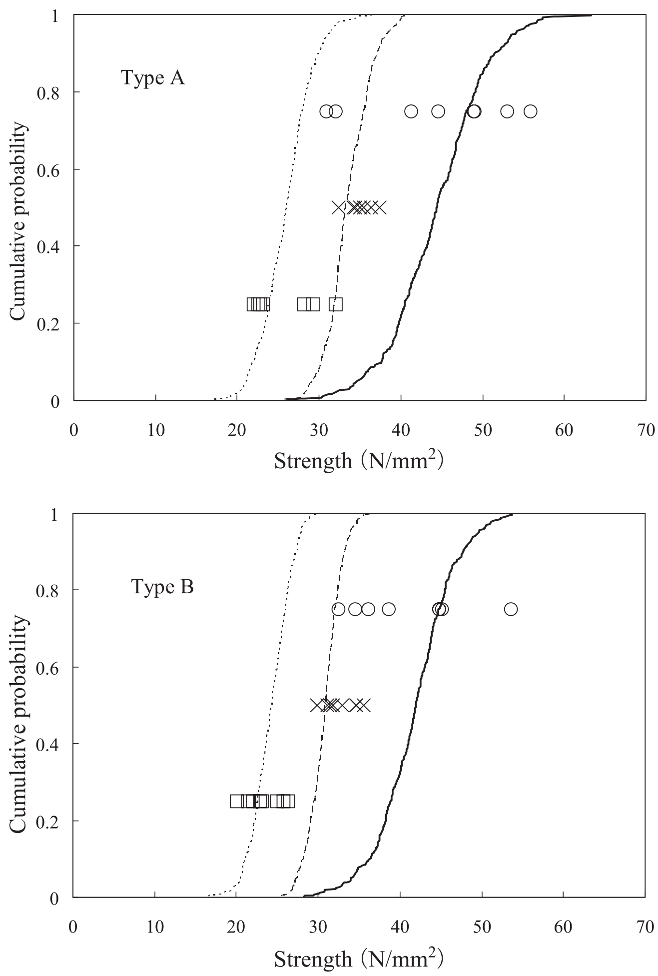

Fig. 6. Results of simulation and experiments (type A and B specimens). *Solid line*, MOR (simulation); *circles*, MOR (experiment); *dotted line*, σ_T (simulation); *squares*, σ_T (experiment); *dashed line*, σ_C (simulation); *crosses*, σ_C (experiment)

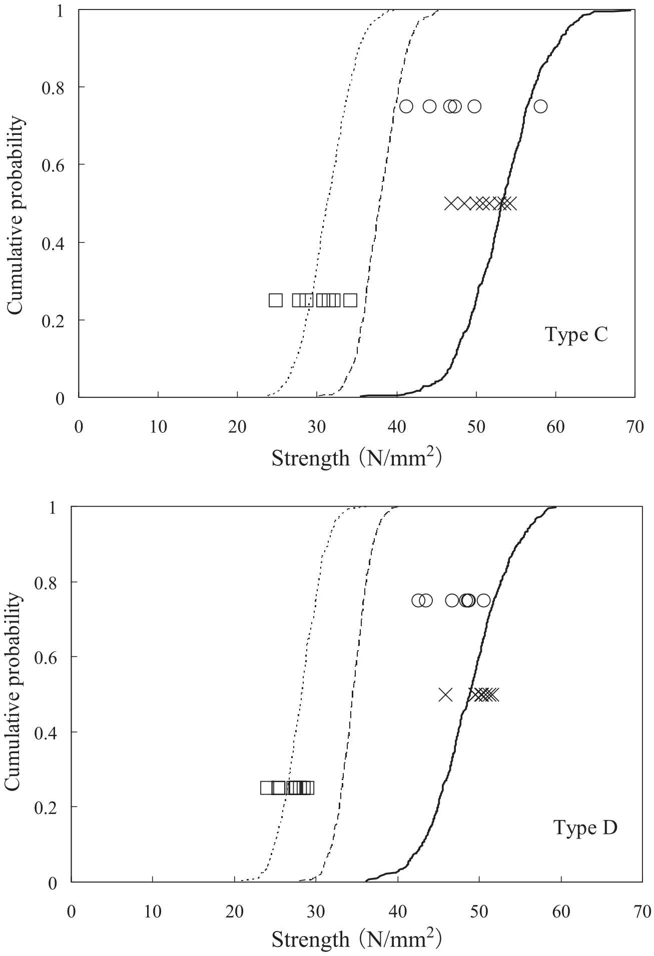

Fig. 7. Results of simulation and experiments (type C and D specimens)

$\sigma_{ni}/f_{ni} \geq 1$, respectively. Here, σ_{ni} and σ_{bi} are the axial stress and the bending stress of the i-th lamina, respectively, and f_{ni} and f_{bi} are the axial strength and the bending strength of the i-th lamina, respectively. Note that the cumulative probability of the experiments is plotted for convenience, but is not particularly meaningful. It can be seen that the experimental measurements are roughly in the same range as the simulation, except for compression strengths of type C and D specimens. The reason that the simulated values are smaller than the experimental values for compression strength is not clear, but the fracture criteria for compression strength might not be appropriate.

Conclusions

A simulation method for generating the strength of glulam using correlated random variables is described here; the correlated random variables are MOE, MOR, σ_T, and σ_C, among others. Experiments to validate the method were conducted. By comparing the simulation and experiment results, this method was shown to be useful for generating the strength of glulam.

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