

# Applicability of effective medium theory to wood density measurements using terahertz time-domain spectroscopy

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**Abstract** The use of an effective medium theory is important when accurately measuring wood density using millimeter and terahertz wave techniques. To confirm the applicability of this theory to the evaluation of wood density, the relative permittivity and dielectric loss of oven-dry flat-sawn specimens of 11 different wood species were measured in a frequency range of 0.15–1.2 THz using a transmission measurement system for terahertz time-domain spectroscopy. A mixture model based on the effective medium theory well explained the density dependence of relative permittivity over the entire frequency range, while it did not fully explain that of dielectric loss, especially for higher frequencies. This indicates that wood scatters the terahertz wave with a wavelength close to the transverse sectional dimensions of the pores in wood in the same way as Mie scattering. It was found from the dielectric loss spectrum of wood substance that the frequency around 0.23 THz was preferable for the nondestructive evaluation of wood.

**Keywords** Oven-dry wood · Effective medium theory · Dielectric mixture · THz-TDS · Density evaluation

## Introduction

Wood is a natural material and there are fluctuations in its physical properties, such as its moisture content, grain direction, and density, and thereby it should be evaluated nondestructively to promote its wide usage in industry [1]. The fluctuation of density in wood is especially important in determining the strength of wood [2].

Terahertz and millimeter wave techniques, which use electromagnetic waves with frequencies of 0.1–10 and 0.03–0.3 THz, respectively, have been developed so that the waves that penetrate through and reflect from a wood specimen can be detected with low noise [3–14]. These techniques have attracted notice as new tools for the non-destructive evaluation (NDE) of wood density because they are contact-free, noninvasive and safe, and can provide higher resolution images [6–8] than the microwave techniques [15–21].

The method to evaluate density while using these techniques is generally based on effective medium theory [22–24], in which wood is regarded as a mixture of pores (air in the cell lumina) and wood substance (cell wall) [3, 11] and responds to electromagnetic excitation as if it were homogeneous [23]. This theory, however, has a lower wavelength limit and the following rule of thumb is often employed [23]: “the size of an inclusion in the mixture must not exceed a tenth of the wavelength in the effective medium”. In general, wood has many pores of transverse sectional sizes ranging from about 10 to 200  $\mu\text{m}$ , and thereby the lower wavelength limit for wood is to be about 100–2000  $\mu\text{m}$  or 0.15–3 THz in frequency. Therefore, to correctly evaluate

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wood density using the terahertz and millimeter wave techniques, it is necessary to examine the validity of the effective medium theory in this frequency range.

In our other paper, which was submitted to the *Journal of Wood Science* and is currently under review, the effect of pores on wood dielectric anisotropy was examined with the assumption that the effective medium theory is applicable to the relative permittivity of oven-dry wood obtained by terahertz time-domain spectroscopy (THz-TDS) at 0.15 THz. The purpose of this paper is to examine the applicability of the effective medium theory to the density evaluation of wood over a frequency range of 0.15–1.2 THz. The complex permittivity along the longitudinal axis of flat-sawn oven-dry specimens of 11 different wood species was measured using a THz-TDS transmission measurement system. The fitness of a mixture model based on the effective medium theory to the measured relative permittivity and dielectric loss was examined, and the relationships of the averages and standard deviations of the dielectric parameters of wood substance to the frequency were also examined.

## Theory

On the basis of effective medium theory, wood is regarded as a mixture of pores (air) and wood substance [11, 25–27], and the relative permittivity and dielectric loss along the longitudinal axis,  $\varepsilon'$  and  $\varepsilon''$ , of a piece of oven-dry wood are generally related to its density,  $\rho_0$ , by using the following equations [11]:

$$\varepsilon' = 1 + \frac{(\varepsilon'_{\text{WS}} - 1)\rho_0}{\rho_{\text{WS}}} \quad (1)$$

$$\varepsilon'' = \frac{\varepsilon''_{\text{WS}}\rho_0}{\rho_{\text{WS}}} \quad (2)$$

where  $\varepsilon'_{\text{WS}}$ ,  $\varepsilon''_{\text{WS}}$ , and  $\rho_{\text{WS}}$  are the relative permittivity, dielectric loss, and density ( $=1.5 \text{ g/cm}^3$ ), respectively, of wood substance. The effective medium theory, however, has a lower wavelength limit, which is at least ten times larger than the size of the mixture element [23]. In the wavelength range close to this limit, the elements of the mixture scatter the electromagnetic waves in the same way as Mie scattering [28].

## Experimental

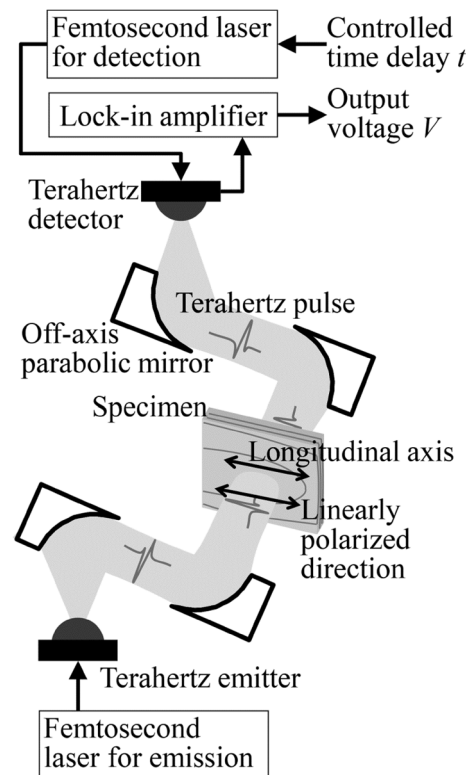
### Sample preparation

Flat-sawn specimens, 25 mm square and 3 mm thick, were prepared from softwoods: hinoki (*Chamaecyparis*

*obtusa*), sugi (*Cryptomeria japonica*), and akamatsu (*Pinus densiflora*); ring-porous hardwoods: kiri (*Paulownia tomentosa*), kuri (*Castanea crenata*), and keyaki (*Zelkova serrata*); diffuse-porous hardwoods: kusunoki (*Cinnamomum camphora*), tochinoki (*Aesculus turbinata*), buna (*Fagus crenata*), and isunoki (*Distylium racemosum*); and radial-porous hardwood: shirakashi (*Quercus myrsinaefolia*). Five specimens were prepared from each species. All specimens were dried to a constant weight at 105 °C.

### THz-TDS transmission measurement

Figure 1 shows an experimental setup of the THz-TDS transmission measurement system (TAS7500SP, Advantest Co.). A terahertz emitter was irradiated by a femtosecond laser pulse to obtain a terahertz pulse. The terahertz pulse was radiated perpendicular to a wood specimen via a set of off-axis parabolic mirrors. The transmitted pulse was focused by another set of off-axis parabolic mirrors on a terahertz detector, in which the pulse component was synchronized with another femtosecond laser pulse, of which the time delay,  $t$  (ps), was changed stepwise at constant intervals (2 fs), detected, and processed into a



**Fig. 1** Experimental setup for terahertz transmission time-domain spectroscopy. A specimen is placed so that its longitudinal axis is parallel to the polarized direction of the terahertz pulse

voltage,  $V$  (mV), in a lock-in amplifier. The voltage  $V$  for each time delay  $t$  was measured 4096 times and averaged to obtain the waveform  $V(t)$ .

The phase and amplitude spectrums,  $\theta(f)$  (rad) and  $A(f)$  (mV), at frequencies  $f$  of 0.15–1.2 THz with a frequency resolution of 7.6 GHz were obtained through Fourier transformation of the wave signal  $V(t)$ . The relative permittivity and dielectric loss spectrums of the specimen,  $\epsilon'(f)$  and  $\epsilon''(f)$ , were obtained using the following equations:

$$\epsilon'(f) = \{n(f)\}^2 - \{\kappa(f)\}^2 \tag{3}$$

$$\epsilon''(f) = 2n(f)\kappa(f) \tag{4}$$

where  $n$  and  $\kappa$  are the refractive index and extinction coefficient, respectively, and formulated as follows:

$$n(f) = 1 + \frac{c\{\theta(f) - \theta_0(f)\}}{2\pi fd} \tag{5}$$

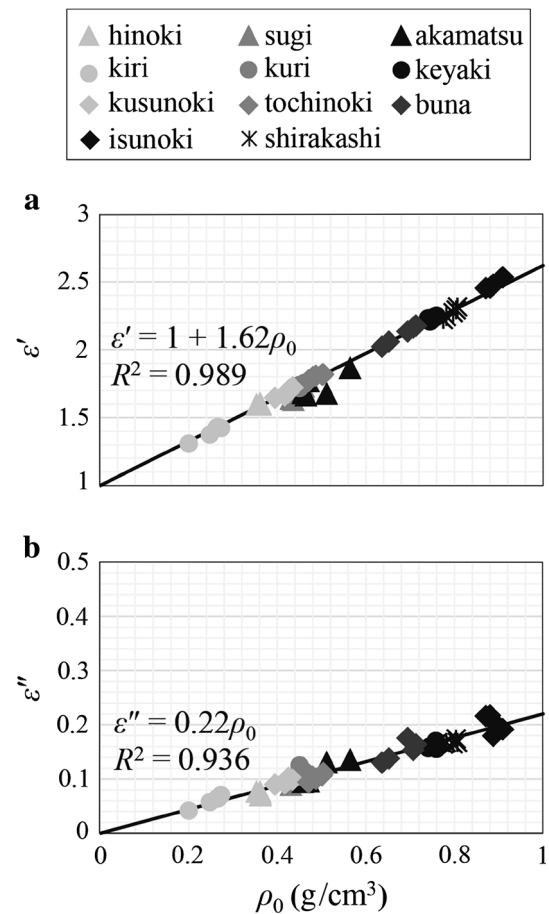
$$\kappa(f) = -\frac{c}{2\pi fd} \ln \left[ \frac{\{1 + n(f)\}^2 A(f)}{4n(f) A_0(f)} \right] \tag{6}$$

where  $\theta_0(f)$  and  $A_0(f)$  represent the phase and amplitude without a specimen, respectively,  $d$  is the specimen thickness ( $=3$  mm), and  $c$  is speed of light in a vacuum ( $=3.0 \times 10^8$  m/s). A specimen was set on the measurement system so that the linearly polarized direction of the terahertz pulse, the beam of which was approximately collimated and found to have a width of about 10–20 mm, was parallel to the longitudinal axis to obtain the dielectric parameters along this axis (Fig. 1).

### Results and discussion

Relation between dielectric parameters and density of wood at a low frequency

Figure 2 shows the relations of the relative permittivity  $\epsilon'$  and dielectric loss  $\epsilon''$  to the density  $\rho_0$  of oven-dry wood at a frequency of 0.15 THz. In the figure, the regression lines derived from Eqs. (1) and (2) were fitted to the plots using the least-squares method. These regression lines fitted well to the plots for  $\epsilon'$  and  $\epsilon''$  and had determination coefficients of  $R^2 = 0.989$  and  $0.936$ , respectively. From the slopes and determination coefficients of the regression lines, the relative permittivity and dielectric loss of wood substance were estimated to be  $\epsilon'_{WS} = 3.43$  and  $\epsilon''_{WS} = 0.33$  with standard deviations of  $1.08 \times 10^{-3}$  and  $0.35 \times 10^{-3}$ , respectively. This indicates that the effective medium theory is valid at 0.15 THz, because the transverse sectional dimension of the wood cells, about 10–200  $\mu\text{m}$ , is much smaller than the wavelength, 2 mm at 0.15 THz. The estimated dielectric parameters,  $\epsilon'_{WS}$  and  $\epsilon''_{WS}$ , were close

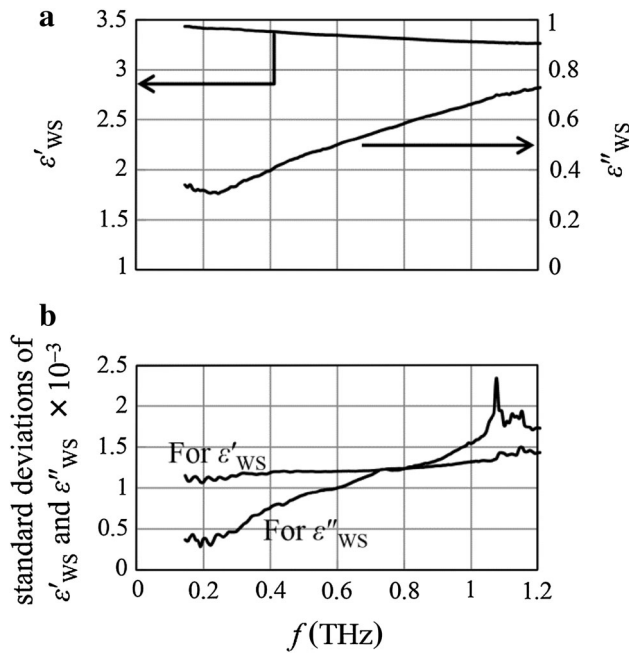


**Fig. 2** Relations of **a** relative permittivity,  $\epsilon'$ , and **b** dielectric loss,  $\epsilon''$ , along the longitudinal axis to the density,  $\rho_0$ , of oven-dry wood at a frequency of 0.15 THz

to and apart from those obtained using a millimeter wave technique, 3.5 and 0.14, at a frequency of 0.1 THz, respectively [11]. This indicates that the relative permittivities obtained using terahertz and millimeter wave techniques can be used in common with each other. The dielectric loss may be more precise in this study than that in the previous study [11], because it was shown in the previous study that the dielectric loss of wood substance was largely affected by the relative permittivity along longitudinal and transverse axes of wood [11]. The estimated values for the frequencies of 0.15–1.2 THz are discussed later.

#### Dielectric properties of wood substance

Figure 3 shows relations of the relative permittivity ( $\epsilon'_{WS}$ ), dielectric loss ( $\epsilon''_{WS}$ ), and their standard deviations to the frequency ( $f$ ).  $\epsilon'_{WS}$  decreased constantly as  $f$  increased, while  $\epsilon''_{WS}$  had a minimum value of 0.31 around 0.23 THz (Fig. 3a), at which the relations of  $\epsilon'$  and  $\epsilon''$  to  $\rho_0$  were



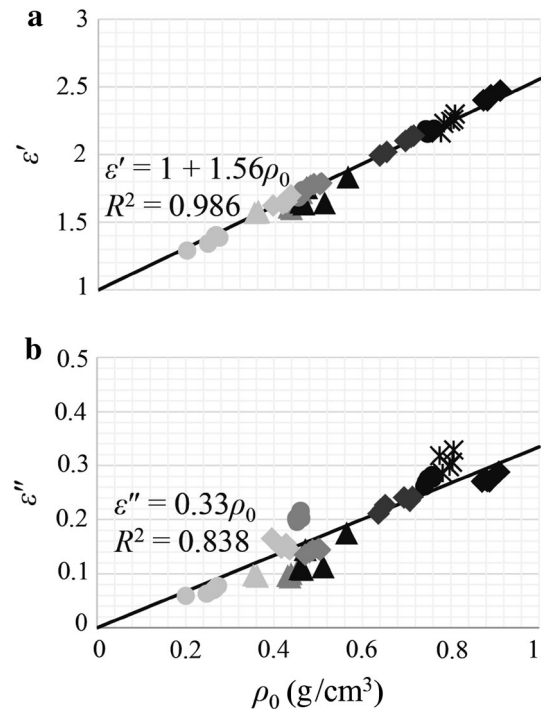
**Fig. 3** Relation of the **a** averages and **b** standard deviations of the dielectric parameters ( $\epsilon'_{ws}$ ,  $\epsilon''_{ws}$ ) of wood substance to the frequency ( $f$ )

similar to the linear relations at 0.15 THz (Fig. 2). Torgovnikov [29] reported that dielectric loss decreases with increases in frequency above 10 MHz to about 0.1 THz. These findings indicate that wood substance has a maximum transmittance around 0.23 THz for the millimeter and terahertz waves, and that this frequency is more preferable for the NDE of oven-dry wood when using the terahertz and millimeter wave techniques, since the highest gain of the detected signal is obtained.

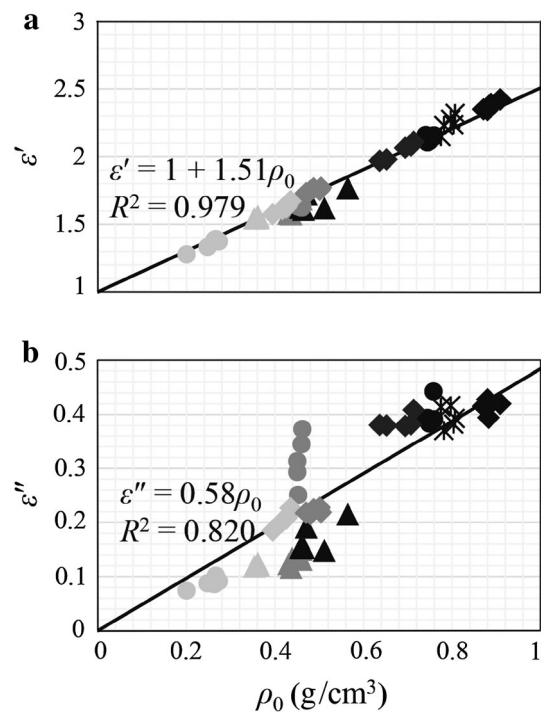
The standard deviation of  $\epsilon'_{ws}$  slightly increased with  $f$ , while that of  $\epsilon''_{ws}$  significantly increased with  $f$  (Fig. 3b). This indicates that the applicability of effective medium theory is inferior at a higher frequency or at a shorter wavelength, especially for dielectric loss.

Effect of frequency on relation between dielectric parameters and wood density

The relations of relative permittivity  $\epsilon'$  and dielectric loss  $\epsilon''$  to density  $\rho_0$  are shown in Figs. 4 and 5 for frequencies of 0.6 and 1.2 THz, respectively. In Figs. 4a and 5a,  $\epsilon'$  and  $\rho_0$  show a good linear relation similar to that for 0.15 THz (Fig. 2a). On the other hand, some plots that relate  $\epsilon''$  to  $\rho_0$  deviate considerably from the best-fit line (Figs. 4b, 5b). This indicates that the effective medium theory is not well applicable for dielectric loss at higher frequencies. This probably results from the scattering of the terahertz wave in wood. At a lower frequency of 0.15 THz, the



**Fig. 4** Relations of **a** relative permittivity,  $\epsilon'$ , and **b** dielectric loss,  $\epsilon''$ , along the longitudinal axis to the density,  $\rho_0$ , of oven-dry wood at a frequency of 0.6 THz. The plot symbols are the same as in Fig. 2



**Fig. 5** Relations of **a** relative permittivity,  $\epsilon'$ , and **b** dielectric loss,  $\epsilon''$ , along the longitudinal axis to the density,  $\rho_0$ , of oven-dry wood at a frequency of 1.2 THz. The plot symbols are the same as in Fig. 2

corresponding wavelength of 2 mm is much larger than the transverse sectional dimension of the wood cells (approximately 10–200  $\mu\text{m}$ ) and wood responds to the excitation of the terahertz wave as if it is homogeneous, where the waves are not scattered. At higher frequencies of 0.6 and 1.2 THz, the corresponding wavelengths of 500 and 250  $\mu\text{m}$  are of the same order as the dimensions of the wood cells, and thereby the terahertz wave is scattered by the pores of the cells in the same way as Mie scattering [28]. The influence of the scattering on the relative permittivity is smaller than that of the dielectric loss. This is because the scattering significantly affects the terahertz wave attenuation, which primarily determines the amount of dielectric loss, due to the constructive and destructive wave interference, and because it does not affect the terahertz wave phase shift, which primarily determines the amount of relative permittivity [12]. Further studies are needed to quantitatively evaluate the effect of the scattering on the dielectric loss.

## Conclusion

To examine the applicability of an effective medium theory to the evaluation of wood density in the terahertz frequency range, the complex permittivity along the longitudinal axis of flat-sawn oven-dry specimens of 11 wood species was measured in a frequency range of 0.15–1.2 THz using a transmission measurement system for THz-TDS. The effective medium theory explained the density dependence of relative permittivity for the entire frequency range, but did not explain that of the dielectric loss for a higher frequency range. This indicates that the terahertz waves are more scattered at higher frequencies. It was concluded from the dielectric loss spectrum of wood substance that a frequency of approximately 0.23 THz is preferable for the NDE, since a high gain of the detected signals was obtained. A quantitative evaluation of the effect of the scattering on the dielectric loss should be examined in the future.

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