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ORIGINAL ARTICLE

The influence of shear strain on critical load bearing capacity. Determination of instability factor k_c^G

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Abstract The paper is concerned with timber constructions. The paper presents a solution of a rod deflection differential equation. The rod deflection equation describes a compressed rod to which a transverse force is applied. The equation accounts for the influence of shearing forces on the magnitude of displacement. Based on the equation, the critical force and the rod deflection function at the rod's initial curvature defined by a sinusoid were determined. Having found the rod deflection function and the critical force, a formula for the instability factor k_c^G was derived to account for the influence of shear strain. The paper demonstrates in which particular cases it is necessary to take into account the influence of deflection caused by shearing on the critical load bearing.

Keywords Wood constructions · Shear strain · Instability factor

Notations

$\sigma_{c,0,d}$	Maximum compressive stress
P	Compressive force
A	Cross-sectional area
$y_{\max}$	Maximum deflection caused by compressive force P
$f_{c,0,k}$, $f^d_{c,0,k}$	Specific compressive strength for wood along fibres
$z_{\max}$	Distance from the neutral axis to external fibre
i	Radius of inertia
$y(x)$	Deflection function
EI	Stiffness of a rod
E	Modulus of elasticity
I	Moment of inertia
$M(x)$	Bending moment
a	Maximum initial curvature of a rod
l_c	Rod's buckling length
k_c	Instability factor
$\sigma_{c,\mathrm{crit}}$	Critical stress
μ	Energetic shear coefficient
G, $G(A)$	Modulus of shear strain
$T(x)$	Shear force
$q(x)$	Transverse load
l	Rod's length
P_e, P_e^G	Critical forces
$f_{c,0,d}$	Calculated compressive strength for wood along fibres
k_c^G	Instability factor
$E_{0.05}$, $E^d_{0.05}$	5 % quantile of elasticity modulus for wood
I_d	Moment of inertia for wood
I_{md}	Moment of inertia for wood-based material
E_d	Modulus of longitudinal elasticity for wood
E_{md}	Modulus of elasticity for wood-based material
A, A_d	Cross-sectional area of wood
A_{md}	Cross-sectional area of wood-based material
λ	Rod's slenderness
n	Safety factor
α_d, α_{md}	Energetic shear deformability
$E^{md}_{0.05}$	5 % quantile of elasticity modulus for wood-based material
$f^{md}_{c,0,k}$	Specific compressive strength for wood-based material along fibres

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$S(z)$	The static moment of the cut-off part of a cross-section
$b(z)$	The width of a cross-section in the distance z from the neutral axis
$G^{d}_{0.05}$	5 % quantile of shear modulus for wood
$G^{md}_{0.05}$	5 % Quantile of shear modulus for wood-based material
$\tau_{s,md}(A_{s,md})$	Shear stress at the web and walls made of wood-based material
$\tau_{p,d}(A_{p,d})$	Shear stress at wood flanges, shear stress at the wood part of the flange
$\tau_{p,md}(A_{p,md})$	Shear stress at the wood-based part of the flange
$A_{s,md}$	The cross-sectional area of the web, and the walls
$A_{p,d}$	The cross-sectional area of the wood flange, the cross-sectional area of the wood part of the flange
$A_{p,md}$	The cross-sectional area of the wood-based part of the flange
ε	Linear deformation
E_E	Experimental modulus of elasticity

Introduction

Civil engineering computations according to second-order theories can be carried out in two ways. In the first method, we assume that normal stresses are the sum of all stresses resulting from compressive force and stresses caused by bending moment. Bending moment is conceived as depending on the kind of transverse load and the magnitude of longitudinal force applied at the eccentricity of loads that results from the shape of a deformed rod's axis.

In the second method which is analysed in the present paper, normal stress is determined using a formula in which the first term defines stress that is the result of compressive stresses caused by longitudinal force divided by instability factor k_c and the second term defines stress caused by bending moments which, in this case, solely depend on transverse loads. Second-order effects are expressed in the formula used to determine stress with the second method as values of instability factor k_c. The factor can be basically determined with two methods: first, using second-order rod analysis described with a sinusoid and second, using longitudinal force applied at the eccentricity of loads. The paper focuses on the first method. The calculation of the instability factor k_c with the second method was presented in the author's different paper.

Compared to steel structures, wood structures have a relatively high ratio of longitudinal modulus of elasticity (Young's modulus) to shear modulus, which for C24 timber is $\frac{E}{G} = 16$. Therefore, any analysis of timber structures and, particularly, any analysis of structures made of wood and wood-based composite materials calculated using first- and second-order theories should also consider the influence of shear strain on critical load. For steel structures, the ratio of Young's modulus to shear modulus is very low, $\frac{E}{G} = 2.5$, which means that shear strain effects on critical load bearing capacity and displacement in static analysis of steel elements are negligible.

The study presents how to derive a formula for the instability factor k_c^G which considers the influence of shear strain. The formula is derived using a differential equation of a deformed axis of a rod which accounts for the influence of shear strain on displacement values.

A comparative analysis was conducted on the magnitude of instability factors k_c^G and k_c with and without shear strain. The analysis seems to indicate the need to consider the influence of shear strain while determining critical load bearing capacity. It is of particular importance in any analysis of wood and wood-based constructions. The study is based on Timoshenko's Theory of Elastic Stability, Timoshenko and Gere [1].

Determination of instability factor k_c; the state of the art

The inequality which is the starting point for determining the instability factor k_c is one which describes the maximum compressive stress in external fibres excited by the longitudinal force P acting at the eccentricity of loads. It is assumed that in timber work such stress forces should be smaller or equal to wood specific compressive strength $f_{c,0,k}$:

$$\sigma_{c,0,d} = \frac{P}{A}\left(1 + \frac{1}{c} y_{\max}\right) \leq f_{c,0,k}, \tag{1}$$

where

$$c = \frac{I}{A \cdot z_{\max}} = \frac{i^2}{z_{\max}} \tag{2}$$

The function $y_{\max}$ is determined using the differential equation of the beam deflection which has the following form:

$$\frac{d^2 y(x)}{dx^2} + k^2 y(x) = -\frac{1}{EI} M(x), \tag{3}$$

where $k^2 = \frac{P}{EI}$. The function $y_{\max}$ is determined for a rod's initial curvature defined by a sinusoid which can be given by $y_{\max} = \dfrac{a}{1 - \dfrac{P}{P_e}}$ where $P_e = \dfrac{\pi^2 EI}{l_c^2}$. Substituting $y_{\max} = \dfrac{a}{1 - \dfrac{P}{P_e}}$ into Eq. (1), after transformations we receive

$$k_c = 0.5\left\{\left[1 + \left(1 + \frac{a}{c}\right)k_E\right] - \sqrt{\left[1 + \left(1 + \frac{a}{c}\right)k_E\right]^2 - 4k_E}\right\}, \tag{4}$$

where

$$k_E = \frac{\sigma_{c,\mathrm{crit}}}{f_{c,0,k}} \tag{5}$$

Formula (4) can be transformed to a form that is to be found in the Eurocode 5 standard [2]. In order to do that, it must be multiplied and divided by the complex conjugate expression.

The method of determining the rod's initial curvature was given in section "Determination of initial maximum curvature of the rod a".

Derivation of a differential equation of rod deflection with shear strain

The differential equation was derived based on Timoshenko's theory, Timoshenko and Gere [1]. The relation between the deflection function and bending moments can be written as

$$\frac{d^2 y_M(x)}{dx^2} = -\frac{1}{EI} M_c(x), \tag{6}$$

where $M_c(x)$ is a bending moment excited by the transverse force and the longitudinal force P causing instability:

$$M_c(x) = M(x) + P \cdot y(x) \tag{7}$$

The relation between the deflection function and the shear force in the rod is based on the equation of the potential energy of the system induced by shear in $V_p = \mu \int_0^l \frac{[T_c(x)]^2}{2GA} dx$ and can be given by

$$\frac{d^2 y_T(x)}{dx^2} = \frac{\mu}{GA} \frac{d}{dx} T_c(x) \tag{8}$$

In Eq. (8) $T_c(x)$ is the total shearing force which accompanies bending. The formula for shear force can be obtained by differentiating Eq. (7) and can be written as

$$T_c(x) = T(x) + P\frac{dy(x)}{dx} \tag{9}$$

Then the total curvature of the rod is the sum of all curvatures induced by bending moments and shear force:

$$\frac{d^2 y(x)}{dx^2} = -\frac{1}{EI}[M(x) + Py(x)] + \frac{\mu}{GA}\frac{d}{dx}\left[T(x) + P\frac{dy(x)}{dx}\right] \tag{10}$$

Assuming that $\dfrac{dT(x)}{dx} = -q(x)$ after transformations, we obtain

$$\frac{d^2 y(x)}{dx^2} + k^2 y(x) = -\frac{1}{EI\left(1 - \frac{\mu P}{GA}\right)} M(x) - \frac{\mu}{GA\left(1 - \frac{\mu P}{GA}\right)} q(x), \tag{11}$$

in which

$$k^2 = \frac{P}{EI\left(1 - \frac{\mu P}{GA}\right)} \tag{12}$$

The equation was solved twice for two cases of static work of the beam. The first solution was found for a beam compressed with a longitudinal force P. The second solution was found for the beam compressed with a longitudinal force P at its initial curvature defined by a sinusoid. While in the former case a homogeneous equation was analysed, the latter was an inhomogeneous equation. The homogeneous equation was used to determine the displacement function and the critical force. The displacement function was derived to be later used to solve the inhomogeneous equation. The critical force was derived to compare it with the critical force found in the inhomogeneous equation. The beam deflection function at its initial curvature defined by a sinusoid derived from the inhomogeneous equation was later used to determine the instability factor k_c^G, which can be used to calculate stress in complex structures according to the second-order theories and taking into account shear strain.

Determination of deflection function and critical force for a rod loaded with longitudinal force *P*

A solution of the homogenous equation. Determination of the displacement function *y*(*x*)

Since, in this case, there is no transverse load $M(x) = 0$ and $q(x) = 0$, Eq. (11) takes the following form:

$$\frac{d^2 y(x)}{dx^2} + k^2 y(x) = 0 \tag{13}$$

The equation was solved using the operational calculus based on the Laplace transform Osiowski [3]. The following formula was obtained:

$$y(x) = w_0 \cos kx + w_1 \frac{1}{k} \sin kx \tag{14}$$

Function (14), i.e. the solution of the homogeneous equation, is used below to solve the inhomogeneous equation.

Determination of critical force P_e^G

In order to satisfy the boundary conditions $y(x=0)$ and $y(x=l)=0$ we have in Eq. (14) $w_0 = 0$ and w_1 equals zero or that the argument value of the sine function at the point $x = l\ k{\cdot}l$ is $k{\cdot}l = n{\cdot}\pi$. Substituting k to the equation, where $k^2 = \dfrac{P}{EI\left(1-\frac{\mu P}{GA}\right)}$ and $n = 1$, we obtain a formula for the critical force $P = P_e^G$:

$$P_e^G = \frac{P_e}{1 + P_e \frac{\mu}{GA}}, \tag{15}$$

where

$$P_e = \frac{\pi^2 EI}{l_c^2} \tag{16}$$

Equation (15) can also be found in Timoshenko and Gere [1].

Determination of the rod deflection function compressed with the force *P* at its initial curvature defined by a sinusoid. Solution of the inhomogeneous equation

If in Eq. (11) $M(x) = P \cdot a \cdot \sin\frac{\pi}{l}x$, $q(x) = -\frac{d^2M(x)}{dx^2} = \frac{\pi^2 \cdot a \cdot P}{l^2} \cdot \sin\frac{\pi}{l}x$ after transformations we obtain

$$\frac{d^2 y_\square(x)}{dx^2} + k^2 y_\square(x) = -\left(1 + P_e\frac{\mu}{GA}\right)k^2 a \sin\frac{\pi}{l}x \tag{17}$$

The equation was solved using the operational calculus based on the Laplace transform Osiowski [3]. A solution of the equation, using Eq. (14), is the following function:

$$y_\square(x) = w_0 \cos kx + w_1 \frac{1}{k}\sin kx + \frac{k^2 l^2\left(1 + P_e\frac{\mu}{GA}\right)}{\pi^2 - k^2 l^2} a \sin\frac{\pi}{l}x - \frac{k^2 l^2\left(1 + P_e\frac{\mu}{GA}\right)}{\pi^2 - k^2 l^2} a \sin kx \tag{18}$$

Boundary conditions have the following form: $y_\square(0) = 0$, $y_\square(l) = 0$. The deflection function after taking into consideration the boundary conditions and after adding the initial curvature defined by the sinusoid $y = a\sin\frac{\pi}{l}x$ can be given by

$$y(x) = \frac{a}{1 - \frac{P}{P_e^G}} \sin\frac{\pi}{l}x, \tag{19}$$

where the critical force P_e^G is $P_e^G = \dfrac{P_e}{1 + P_e\frac{\mu}{GA}}$, $P_e = \dfrac{\pi^2 EI}{l_c^2}$.

The critical force P_e^G, which was derived from the equation, is the same as that given by Eq. (15), which was derived from the homogenous equation.

Determination of the factor k_c^G

The instability factor k_c^G was determined based on the author's original derivation, based on Timoshenko's theory presented in Timoshenko and Gere [1]. To derive the formula, we use the beam deflection equation at the beam's initial curvature defined by a sinusoid, Eq. (19). The maximum deflection $y_{\max}$ is

$$y_{\max} = \frac{a}{1 - \alpha\left(1 + P_e\frac{\mu}{GA}\right)}, \tag{20}$$

where $\alpha = \frac{P}{P_e}$. Equation (20) is substituted into the formula for maximum compressive stress $\sigma_{c,0,d}$ given by (1), and then we obtain the following inequality:

$$\sigma_{c,0,d} = \frac{P}{A}\left[1 + \frac{a}{c}\frac{1}{1 - \alpha\left(1 + P_e\frac{\mu}{GA}\right)}\right] \le f_{c,0,k} \tag{21}$$

Dividing the numerator and denominator of the expression $\alpha = \frac{P}{P_e}$ where $P_e = \frac{\pi^2 EI}{l_c^2}$ by A and assuming that $\sigma_{\mathrm{MID}} = \frac{P}{A}$, after transformations the coefficient α is $\alpha = \frac{\sigma_{\mathrm{MID}}}{\sigma_{c,\mathrm{crit}}}$ where $\sigma_{c,\mathrm{crit}} = \frac{\pi^2 E_{0.05}}{\lambda^2}$ and $\lambda = \frac{l_c}{i}$. Substituting the value of α and $\sigma_{\mathrm{MID}} = \frac{P}{A}$ into Eq. (21), we obtain

$$\sigma_{c,0,d} = \sigma_{\mathrm{MID}}\left[1 + \frac{a}{c}\frac{1}{1 - \frac{\sigma_{\mathrm{MID}}}{\sigma_{c,\mathrm{crit}}}\left(1 + P_e\frac{\mu}{GA}\right)}\right] \le f_{c,0,k} \tag{22}$$

We are seeking the mean value of compressive stress σ_{MID} causing in external wood fibres maximum stress equal to the elastic limit. In wood constructions, maximum stress $\sigma_{c,0,d}$ values are equal or smaller than the compressive strength for wood along fibres $f_{c,0,k}$. This inequality can be solved by finding the value of σ_{MID}. After transformations, we obtain the following quadratic inequality:

$$\sigma_{\mathrm{MID}}^2 - \left[f_{c,0,k} + \left(1 + \frac{a}{c}\right)\frac{\sigma_{c,\mathrm{crit}}}{1 + P_e\frac{\mu}{GA}}\right]\sigma_{\mathrm{MID}} + f_{c,0,k}\frac{\sigma_{c,\mathrm{crit}}}{1 + P_e\frac{\mu}{GA}} \ge 0.$$

By comparing the expressions to zero, we obtain a quadratic equation relative to σ_{MID}. Because the discriminant of the trinomial square Δ is always greater than zero $\Delta > 0$, the equation has two real roots $\sigma^G_{\mathrm{MID}(1)}$ and $\sigma^G_{\mathrm{MID}(2)}$:

$$\sigma^G_{\mathrm{MID}(1)(2)} = 0.5\left\{\left[f_{c,0,k} + \left(1+\frac{a}{c}\right)\cdot\sigma^G_{c,\mathrm{crit}}\right] \mp\sqrt{\left[f_{c,0,k} + \left(1+\frac{a}{c}\right)\cdot\sigma^G_{c,\mathrm{crit}}\right]^2 - 4f_{c,0,k}\cdot\sigma^G_{c,\mathrm{crit}}}\right\}, \quad (23)$$

where

$$\sigma^G_{c,\mathrm{crit}} = \frac{\sigma_{c,\mathrm{crit}}}{1 + P_e\cdot\frac{\mu}{GA}} \quad (24)$$

The inequality is satisfied if σ_{MID} belongs to the interval $\sigma_{\mathrm{MID}} \in \left(-\infty, \sigma^G_{\mathrm{MID}(1)}\right\rangle \cup \left\langle\sigma^G_{\mathrm{MID}(2)}, +\infty\right)$. Taking into further analysis the smaller root $\sigma^G_{\mathrm{MID}(1)}$ given by Eq. (23), the following inequality is obtained: $\sigma_{c,0,d} = \frac{P}{A} \le \sigma^G_{\mathrm{MID}(1)}$. By transforming the inequality and introducing the safety factor which is defined by a norm, we obtain a maximum compressive stress equation at a longitudinal force P causing buckling: $\sigma_{c,0,d} = \frac{P}{A\cdot k^G_c} \le f_{c,0,d}$. The instability factor k^G_c in the equation is determined in the following manner. Let us assume that the stress $\sigma^G_{\mathrm{MID}(1)}$ is a part of characteristic wood strength $f_{c,0,k}$ $\sigma^G_{\mathrm{MID}(1)} = k^G_c\cdot f_{c,0,k}$ and hence we have

$$k^G_c = \frac{\sigma^G_{\mathrm{MID}(1)}}{f_{c,0,k}} \quad (25)$$

By substituting (23) into (25) we obtain a relation defining the instability factor k^G_c

$$k^G_c = 0.5\left\{\left[1+\left(1+\frac{a}{c}\right)k^G_E\right] - \sqrt{\left[1+\left(1+\frac{a}{c}\right)k^G_E\right]^2 - 4k^G_E}\right\}, \quad (26)$$

where

$$k^G_E = \frac{\sigma^G_{c,\mathrm{crit}}}{f_{c,0,k}} \quad (27)$$

The stress $\sigma^G_{c,\mathrm{crit}}$ based on formula (24) is

$$\sigma^G_{c,\mathrm{crit}} = \frac{\sigma_{c,\mathrm{crit}}}{1 + \sigma_{c,\mathrm{crit}}A\cdot\frac{\mu}{GA}} \quad (28)$$

The influence of shear strain is expressed in Eq. (28) by the coefficient $1 + \sigma_{c,\mathrm{crit}}A\cdot\frac{\mu}{GA}$. After substituting $\sigma^G_{c,\mathrm{crit}} = \frac{\sigma_{c,\mathrm{crit}}}{1 + \sigma_{c,\mathrm{crit}}A\cdot\frac{\mu}{GA}}$ and then $\sigma_{c,\mathrm{crit}} = \frac{\pi^2 E_{0.05}}{\lambda^2}$ into Eq. (27), it can be written as:

$$k^G_E = \pi^2\frac{E_{0.05}}{f_{c,0,k}}\left[\lambda^2 + \frac{\mu}{GA}\pi^2 E_{0.05}A\right]^{-1} \quad (29)$$

Equation (29) applies to constructions made of wood. While analysing composite elements made of wood and wood-based materials, into functions λ, A in Eq. (29), $I = I_d + I_{md}\frac{E_{md}}{E_d}$ and $A = A_d + A_{md}\frac{E_{md}}{E_d}$ should be substituted to determine stress in wood and $I = I_{md} + I_d\frac{E_d}{E_{md}}$ and $A = A_{md} + A_d\frac{E_d}{E_{md}}$ should be substituted to determine stress in wood-based materials. An additional assumption was also made that when analysing wood elements, α_d should be used instead of $\frac{\mu}{GA}$ in Eqs. (28) and (29) and α_{md} should be used for wood-based materials. Formulas that define the functions α_d and α_{md} are given below in the paper. Given the above, the factor k^G_c with shear strain effect can be determined using (26). The function k^G_E in (26) is presented in Table 1.

Table 1 Set of factors k^G_E

1.	k^G_E	$\pi^2\frac{E_{0.05}}{f_{c,0,k}}\left[\lambda^2 + n\cdot\alpha_d\pi^2 E_{0.05}A\right]^{-1}$	(30)
2.		$\pi^2\frac{E^d_{0.05}}{f^d_{c,0,k}}\left[\lambda^2 + n\cdot\alpha_{md}\pi^2\frac{E^d_{0.05}}{E_d}\left(A_dE_d + A_{md}E_{md}\right)\right]^{-1}$	(31)
3.		$\pi^2\frac{E^{md}_{0.05}}{f^{md}_{c,0,k}}\left[\lambda^2 + n\cdot\alpha_{md}\pi^2\frac{E^{md}_{0.05}}{E_{md}}\left(A_dE_d + A_{md}E_{md}\right)\right]^{-1}$	(32)

$$i = \sqrt{\frac{E_d I_d + E_{md} I_{md}}{E_d A_d + E_{md} A_{md}}} \quad (33)$$

Formula (30) in Table 1 is used to determine normal stress for wood in a wood element.

Formula (31) in Table 1 is used to determine normal stress for wood in an element with a composite cross-section, made of wood and wood-based materials.

Formula (32) in Table 1 is used to determine normal stress for a wood-based material in an element with a composite cross-section, made of wood and wood-based materials.

Formula (26), just like Eq. (4), can be transformed to a form that is to be found in the Eurocode 5 standard [2]. However, formula (26), contrary to the formula in the standard, will contain the coefficient $1 + \sigma_{c,\mathrm{crit}} A \cdot \frac{\mu}{GA}$.

Determination of initial maximum curvature of the rod *a*

Based on the literature (Eurocode 5 standard [2]), while determining the instability factor k_c without shear strain, the expression $\frac{a}{c}$ was substituted with $\beta\left(\sqrt{\frac{f_{c,0,k}}{\sigma_{c,\mathrm{crit}}}} - 0.3\right)$ where the critical stress $\sigma_{c,\mathrm{crit}} = \frac{\pi^2 E_{0.05} I}{\lambda^2}$ depends on the critical force $P_e = \frac{\pi^2 E_{0.05} I}{l_c^2}$. The coefficient β is $\beta = 0.2$ for solid wood and $\beta = 0.1$ for glued laminated wood. Therefore, for determination of the instability factor k_c^G with shear strain, the author suggests the expression $\frac{a}{c}$ to be substituted with $\beta\left(\sqrt{\frac{f_{c,0,k}}{\sigma^G_{c,\mathrm{crit}}}} - 0.3\right)$ where the critical stress $\sigma^G_{c,\mathrm{crit}} = \frac{\sigma_{c,\mathrm{crit}}}{1 + \alpha_d \sigma_{c,\mathrm{crit}} \cdot A}$ depends on the critical force $P_e^G = \frac{P_e}{1 + \alpha_d \cdot P_e}$. Hence, we have that:

$$a = \begin{cases} \frac{i^2}{z_{\max}}\left[\frac{1}{5\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}\left(\lambda^2 + n\cdot\alpha_d \pi^2 E_{0.05} A\right)} - 0.06\right] & \text{for solid wood} \quad (34) \\ \frac{i^2}{z_{\max}}\left[\frac{1}{10\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}\left(\lambda^2 + n\cdot\alpha_d \pi^2 E_{0.05} A\right)} - 0.03\right] & \text{for glued laminated wood} \quad (35) \end{cases}$$

In case of constructions made of wood and wood-based materials, in Eqs. (34) and (35) A should be substituted with either $A = A_d + A_{md}\frac{E_{md}}{E_d}$ or $A = A_{md} + A_d\frac{E_d}{E_{md}}$ depending on whether stress is determined for wood or wood-based materials, respectively.

When determining stress in wood-based materials, in Eqs. (34) and (35), the wood strength $f_{c,0,k}$ and 5 % quantile of modulus of elasticity for wood $E_{0.05}$ should be substituted for the wood-based material strength $f^{md}_{c,0,k}$ and 5 % quantile of modulus of elasticity for the wood-based material $E^{md}_{0.05}$. Also, in the same formulae, the coefficient α_d should be substituted with α_{md}. The expression c is defined with Eq. (2).

Calculation of coefficients α_d and α_{md}

The method of calculating coefficients α_d and α_{md} is based on the author's own analyses. As the starting equation to determine α_d and α_{md} coefficients, the following potential energy caused by shear was assumed Piechnik [5]:

$$E_p = \int_0^l \left[\iint_A \frac{1}{2G(A)} \tau^2 dydz\right] dx \quad (36)$$

For elements made of the same material, the shear strain modulus *G(A)* can be placed before the sign of the double integral in Eq. (36), which means that in this case we may apply the energetic shear coefficient Jakubowicz and Orłoś [4], Piechnik [5] and Bielajew [6]:

$$\mu = \frac{A}{I^2} \iint_A \frac{S^2(z)}{b^2(z)} dA \quad (37)$$

For elements made of materials with different modulus of shear strain, e.g. timber and plywood, the $G(A)$ values cannot be placed before the sign of the double integral, which is why the energetic shear coefficient cannot be determined. Therefore, in this case the author suggests to use the term, introduced by the author himself, of energetic shear deformability α_{md} and calculate it from Eq. (38) based on Eq. (36).

$$\alpha_{md} = \frac{1}{T^2(x)} \iint_A \frac{1}{G(A)} \tau^2 dA, \quad (38)$$

where

$$\tau = \frac{T(x) \cdot S(z)}{I \cdot b(z)} \quad (39)$$

Equation (38) was derived based on Piechnik [5].

For elements made of the same material, energetic shear deformability α_d can be obtained from equation $\alpha_d = \frac{\mu}{GA}$, whereas for elements made of different materials, energetic shear deformability α_{md} should be used instead of $\frac{\mu}{GA}$.

The coefficients α_d and α_{md} were determined for three most common cross-sections; a rectangular cross-section made of timber, a composite I-section with a wood-based web and for a box section with wood-based walls (Fig. 1).

Case 1: Fig. 1a

$$\alpha_d = \frac{1.2}{G_{0.05}^{d} \cdot b \cdot h} \quad (40)$$

Case 2: Fig. 1b

$$\alpha_{md} = \alpha_{s,md} + \alpha_{p,d} \quad (41)$$

$$\alpha_{s,md} = \frac{1}{T^2(x)} \int_{-\left(\frac{h}{2}-t\right)}^{+\left(\frac{h}{2}-t\right)} \int_{-\frac{g}{2}}^{+\frac{g}{2}} \frac{\tau_{s,md}^2\left(A_{s,md}\right)}{G_{0.05}^{md}} dydz, \quad (42)$$

where

$$\tau_{s,md}\left(A_{s,md}\right) = \frac{T(x)\left\{\frac{g}{2}\left[\left(\frac{h}{2}-t\right)^2 - z^2\right] + \frac{bt}{2}(h-t)\frac{E_d}{E_{md}}\right\}}{\left[\frac{g(h-2t)^3}{12} + bt\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right)\frac{E_d}{E_{md}}\right]g} \quad (43)$$

$$\alpha_{s,md} = \frac{(h-2t)\left\{\left[\frac{bt}{2}(h-t)\frac{E_d}{E_{md}} + \frac{g}{2}\left(\frac{h}{2}-t\right)^2\right]^2 - \frac{g}{3}\left[\frac{bt}{2}(h-t)\frac{E_d}{E_{md}} + \frac{g}{2}\left(\frac{h}{2}-t\right)^2\right]\left(\frac{h}{2}-t\right)^2 + \frac{g^2}{20}\left(\frac{h}{2}-t\right)^4\right\}}{G_{0.05}^{md}\left[\frac{g(h-2t)^3}{12} + b \cdot t\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right)\frac{E_d}{E_{md}}\right]^2 g} \quad (44)$$

$$\alpha_{p,d} = \frac{2}{T^2(x)} \int_{\left(\frac{h}{2}-t\right)}^{\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{\tau_{p,d}^2\left(A_{p,d}\right)}{G_{0.05}^{d}} dydz \quad (45)$$

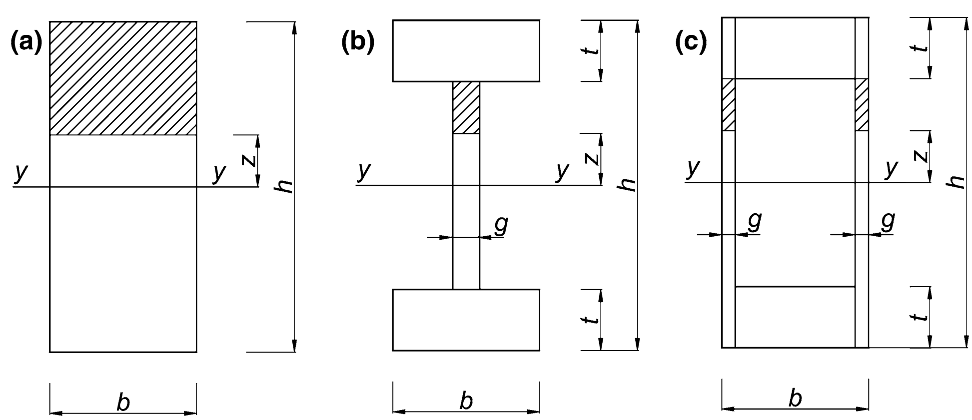

Fig. 1 Three cross-sections used for determining the coefficients α_d and α_{md}. **a** Rectangular cross-section made of timber. **b** Composite I-section with a wood-based web. **c** Composite box section with wood-based walls

where

$$\tau_{p,d}\left(A_{p,d}\right) = \frac{T(x)\frac{b}{2}\left(\frac{h^2}{4} - z^2\right)}{\left[bt\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right) + \frac{g(h-2t)^3}{12}\frac{E_{md}}{E_d}\right]b} \quad (46)$$

$$\alpha_{p,d} = \frac{\frac{bh^4 t}{32} - \frac{bh^2}{96}\left[h^3 - (h-2t)^3\right] + \frac{b}{320}\left[h^5 - (h-2t)^5\right]}{G^d_{0.05}\left[bt\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right) + \frac{g(h-2t)^3}{12}\frac{E_{md}}{E_d}\right]^2} \quad (47)$$

$$\alpha_{p,d} = \frac{2}{T^2(x)} \int_{\left(\frac{h}{2}-t\right)}^{\frac{h}{2}} \int_{-\left(\frac{b}{2}-g\right)}^{+\left(\frac{b}{2}-g\right)} \frac{\tau^2_{p,d}\left(A_{p,d}\right)}{G^d_{0.05}} dydz, \quad (52)$$

where

$$\tau_{p,d}\left(A_{p,d}\right) = \frac{T(x)\left[\frac{1}{2}(b-2g) + g\frac{E_{md}}{E_d}\right]\left(\frac{h^2}{4} - z^2\right)}{\left[(b-2g)t\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right) + \frac{gh^3}{6}\frac{E_{md}}{E_d}\right]b} \quad (53)$$

$$\alpha_{p,d} = \frac{2(b-2g)\left[\frac{1}{2}(b-2g) + g\frac{E_{md}}{E_d}\right]^2\left\{\frac{1}{16}h^4 t - \frac{1}{48}h^2\left[h^3 - 8\left(\frac{h}{2} - t\right)^3\right] + \frac{1}{160}\left[h^5 - 32\left(\frac{h}{2} - t\right)^5\right]\right\}}{G^d_{0.05}\left[(b-2g)t\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right) + \frac{gh^3}{6}\frac{E_{md}}{E_d}\right]^2 b^2} \quad (54)$$

Case 3: Fig. 1c

$$\alpha_{md} = \alpha_{s,md} + \alpha_{p,d} + \alpha_{p,md} \quad (48)$$

$$\alpha_{s,md} = \frac{2}{T^2(x)} \int_{-\left(\frac{h}{2}-t\right)}^{+\left(\frac{h}{2}-t\right)} \int_{\left(\frac{b}{2}-g\right)}^{\frac{b}{2}} \frac{\tau^2_{s,md}\left(A_{s,md}\right)}{G^{md}_{0.05}} dydz, \quad (49)$$

where

$$\tau_{s,md}\left(A_{s,md}\right) = \frac{T(x)\left\{g\left[\left(\frac{h}{2} - t\right)^2 - z^2\right] + (h-t)\left[gt + \frac{1}{2}(b-2g)t\frac{E_d}{E_{md}}\right]\right\}}{\left[\frac{gh^3}{6} + (b-2g)t\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right)\frac{E_d}{E_{md}}\right]2g} \quad (50)$$

$$\alpha_{s,md} = \frac{(h-2t)\left\{\left\{(h-t)\left[gt + \frac{1}{2}(b-2g)t\frac{E_d}{E_{md}}\right] + g\left(\frac{h}{2} - t\right)^2\right\}^2 - \frac{2}{3}g\left\{(h-t)\left[gt + \frac{1}{2}(b-2g)t\frac{E_d}{E_{md}}\right] + g\left(\frac{h}{2} - t\right)^2\right\}\left(\frac{h}{2} - t\right)^2 + \frac{1}{5}g^2\left(\frac{h}{2} - t\right)^4\right\}}{2G^{md}_{0.05}\left[\frac{gh^3}{6} + (b-2g)t\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right)\frac{E_d}{E_{md}}\right]^2 g} \quad (51)$$

$$\alpha_{p,md} = \frac{4}{T^2(x)} \int_{\left(\frac{h}{2}-t\right)}^{\frac{h}{2}} \int_{\left(\frac{b}{2}-g\right)}^{\frac{b}{2}} \frac{\tau^2_{p,md}\left(A_{p,md}\right)}{G^{md}_{0.05}} dydz, \quad (55)$$

where

$$\tau_{p,md}\left(A_{p,md}\right) = \frac{T(x)\left[g + \frac{1}{2}(b-2g)\frac{E_d}{E_{md}}\right]\left(\frac{h^2}{4} - z^2\right)}{\left[\frac{gh^3}{6} + (b-2g)t\left(\frac{1}{2}h^2 - ht + \frac{2}{3}t^2\right)\frac{E_d}{E_{md}}\right]b} \quad (56)$$

$$\alpha_{p,md} = \frac{4g\left[g+\frac{1}{2}(b-2g)\frac{E_d}{E_{md}}\right]^2\left\{\frac{1}{16}h^4t-\frac{1}{48}h^2\left[h^3-8\left(\frac{h}{2}-t\right)^3\right]+\frac{1}{160}\left[h^5-32\left(\frac{h}{2}-t\right)^5\right]\right\}}{G_{0.05}^{md}\left[\frac{gh^3}{6}+(b-2g)t\left(\frac{1}{2}h^2-ht+\frac{2}{3}t^2\right)\frac{E_d}{E_{md}}\right]^2 b^2} \quad (57)$$

In any analysis of I-sections and box sections made of wood, the following values of $\frac{E_{md}}{E_d}=1$, $\frac{E_d}{E_{md}}=1$ and $G_{0.05}^{md}=G_{0.05}^{d}$ should be used in the above formulae.

A comparative analysis of instability factors k_c^G and k_c determined with and without shear strain depending on the kind of a section

A comparative analysis was conducted for two instability factors k_c^G and k_c determined with and without shear strain. The analysis was performed for a grid structure which is the subject of the author's monograph on steel-wood grid structures. The structure is presented in Fig. 2.

First, beams of the structure were analysed and then columns of the grid structure. Sections made of wood and composite sections made of wood and wood-based materials were analysed according to Figs. 3a–j.

10 types of beam cross-sections were analysed: wood sections according to Figs. 3a–f and composite wood and wood-based material sections according to Figs. 3g–j. The sections presented in Figs. 3a–f show beams of the structure whereas sections according to Figs. 3g–j refer to calculations of both beams and columns, with dimensions of the webs and walls given in brackets apply to the analysis of the columns. All dimensions shown in Figs. 3a–j are given in millimetres.

The predefined and analysed I- and box-sections according to Figs. 3c, d, g, h have the same width b as the basic rectangular section analysed in this study according to Fig. 3a. The I- and box-sections according to Figs. 3e, f, i, j have the side ratio of $\frac{h}{b}=1$.

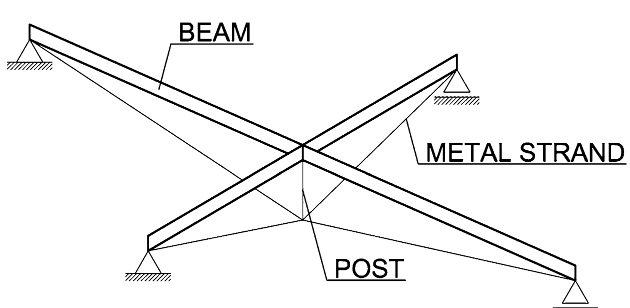

Fig. 2 Timber-steel grid structure for the analysis of instability factors k_c^G and k_c

It was assumed that the webs of I-sections and the walls of box sections (Figs. 3g–j) were made of plywood, particle board and fibre board. The thickness of webs in the I-sections and walls in the box sections was selected so that they would be able to carry transverse forces applied on the beams of the grid structure. Given no transverse load, the magnitude of the shear force is only dependent on the compressive force. Therefore, webs and section walls were assumed to have smaller thickness dimensions: 10 mm for the webs and 5 mm for the walls, which should satisfy strength requirements. The analysis of beams and columns to determine the course and maximum values defined below the function $F(k)$ $F(n)$ was conducted for 11 values of rod's slenderness. The following values of rod's slenderness were assumed: $\lambda = [17.68, 20, 30, 40,\ldots,100, 150]$, where $\lambda = 17.68$ and $\lambda = 150$ are the minimum and maximum slenderness available in the literature. It was assumed that the elements were made from C24 wood. The sizes of the cross-sections were selected so that the flexural rigidity EI was approximately the same for all the analysed cross-sections. The calculations were made using the following values of the shear strain modulus $G_{m,0,\mathrm{mean}}$ contained in Table 2.

The values of the instability factor k_c^G with the shear strain of the factors k_c without shear strain and the function $F(k)=\frac{k_c-k_c^G}{k_c}\cdot 100\,\%$ $F(n)=\left(\frac{k_c}{k_c^G}-1\right)\cdot 100\,\%$ were determined using the algorithm in Table 3.

The function $F(k)$ determines the percentage differences between k_c^G and k_c, depending on rod's slenderness λ.

The function $F(n)$ determines the degree to which the load bearing capacity has been exceeded if in the formula $n=\frac{P}{A\cdot k_c\cdot f_{c,0,d}}$ the coefficient k_c^G will be used instead of k_c. While deriving formula (65) in Table 3 it was assumed that if the formula of the load bearing capacity contains the coefficient k_c; then it is used in 100 % ($n = 1$).

A list of determined functions $F(k)$ and $F(n)$ are presented in Tables 4 and 5.

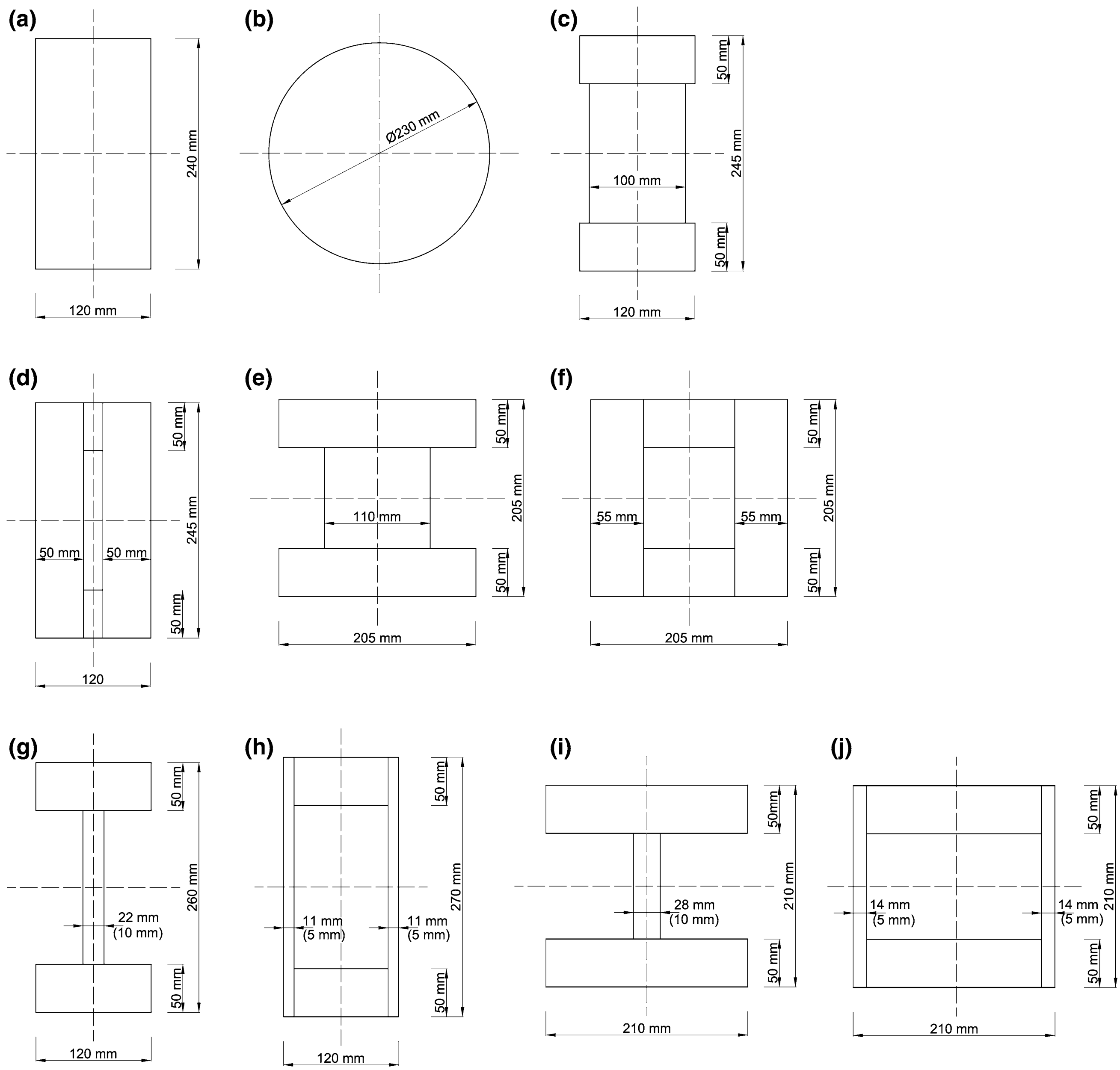

Fig. 3 Cross-section of the beams and columns used for determining the instability factors k_c^G and k_c. **a** Wooden rectangular cross-section. **b** Wooden circular cross-section. **c** Wooden narrow I-section. **d** Wooden narrow box cross-section. **e** Wooden wide I-section. **f** Wooden wide box cross-section. **g** Narrow I-section with a web made from wood-based material. **h** Narrow box cross-section with walls made from wood-based material. **i** Wide I-section with a web made from wood-based material. **j** Wide box section with walls made from wood-based material

Experimental determination of factors k_c^G and k_c

To determine the factors k_c^G and k_c experimentally for their comparison with the factors obtained from a theoretical analysis, one needs to design the research model first. Then, the factors can be determined using, for example, the two methods described below.

Southwell method

The method assumes that the designed rod is compressed with a load smaller than critical and that the rod has a certain initial curvature a. It is based on the Eq. (66) of Thimoshenko and Gere [1], in which the deflection $y_{\max}$ depends on the given compressive force P:

Table 2 The values of shear strain modulus $G_{m,0,\mathrm{mean}}$ [7–9]

	Type of material	$G_{m,0,\mathrm{mean}}$ (MPa)
1.	Wood	690
2.	Plywood	550
3.	Particle board	860
4.	Fibre board	2000

$$y_{\max} = \frac{a}{\frac{P_e}{P} - 1} \tag{66}$$

When calculating deflection with shear strain, we can use an equation analogous to the Eq. (66) in which, instead of the force P_e there is a force P_e^G. This equation can be derived from the Function (18) which satisfies given boundary conditions. Given the above, the critical force P_e^G can be determined from Eq. (67):

$$P_e^G = P\left(1 + \frac{a}{y_{\max}}\right) \tag{67}$$

depending on the compressive force P and deflection $y_{\max}$, caused by the force and occurring at half the length of a pivotally supported rod. To determine forces P_e^G, values of deflection $y_{\max}$ must be measured at given values of force P. To find an experimental value of the instability factor k_c^G, the force P_e^G should be divided by the cross-sectional area A, thus obtaining $\sigma_{c,\mathrm{crit}}^G$. Then, the factor k_E^G should be determined from Eq. (27) and the sought factor k_c^G from Eq. (26).

Table 4 Analysis of beams

	$F(k)$ (%)		$F(n)$ (%)	
	$F(k)_{\mathrm{MIN}}$	$F(k)_{\mathrm{MAX}}$	$F(n)_{\mathrm{MIN}}$	$F(n)_{\mathrm{MAX}}$
1.	0.80	3.64	0.81	3.77
2.	1.80	10.09	1.83	11.22
3.	1.00	6.44	1.01	6.88
4.	0.51	3.10	0.51	3.20

1. section made of wood
2. composite section made of wood and plywood
3. composite section made of wood and particle board
4. composite section made of wood and fibre board

Table 5 Analysis of columns

	$F(k)$ (%)		$F(n)$ (%)	
	$F(k)_{\mathrm{MIN}}$	$F(k)_{\mathrm{MAX}}$	$F(n)_{\mathrm{MIN}}$	$F(n)_{\mathrm{MAX}}$
2.	3.62	23.52	3.75	30.75
3.	2.17	15.84	2.22	18.82
4.	1.03	7.50	1.04	8.11

2. composite section made of wood and plywood
3. composite section made of wood and particle board
4. composite section made of wood and fibre board

Deformation method

This method is based on the equation determining compressing stress at a given force P causing buckling:

$$\sigma_T = \frac{P}{A \cdot k_c^G} \le f_{c,0,d} \tag{68}$$

Table 3 Functions $F(k)$ and $F(n)$ algorithm

$\sigma_{c,\mathrm{crit}} = \frac{\pi^2 \cdot 7400}{\lambda^2}$ (58)	$\sigma_{c,\mathrm{crit}} = \frac{\pi^2 \cdot 7400}{\lambda^2}$ (58)
$\sigma_{c,\mathrm{crit}}^G = \frac{\sigma_{c,\mathrm{crit}}}{1 + \sigma_{c,\mathrm{crit}} \cdot A \cdot \alpha_{d(md)}}$ (59)	
$k_E^G = \frac{\sigma_{c,\mathrm{crit}}^G}{21}$ (60)	$k_E = \frac{\sigma_{c,\mathrm{crit}}}{21}$ (61)
$a = \frac{i^2}{z_{\max}}\left[\frac{1}{5\pi}\sqrt{\frac{21}{7400}\left(\lambda^2 + \alpha_{d(md)}\pi^2 E_{0.05} A\right)} - 0.06\right]$ (62)	$a = \frac{i^2}{z_{\max}}\left[\frac{\lambda}{5\pi}\sqrt{\frac{21}{7400}} - 0.06\right]$ (63)
$k_c^G = 0.5\left\{\left[1 + \left(1 + \frac{a}{c}\right)k_E^G\right] - \sqrt{\left[1 + \left(1 + \frac{a}{c}\right)k_E^G\right]^2 - 4k_E^G}\right\}$ (26)	$k_c = 0.5\left\{\left[1 + \left(1 + \frac{a}{c}\right)k_E\right] - \sqrt{\left[1 + \left(1 + \frac{a}{c}\right)k_E\right]^2 - 4k_E}\right\}$ (4)
$F(k) = \frac{k_c - k_c^G}{k_c} \cdot 100\%$ (64)	
$F(n) = \left(\frac{k_c}{k_c^G} - 1\right) \cdot 100\%$ (65)	

In this method the values of compressive force, P should be adjusted so that they would cause appropriate values of theoretical normal stress σ_T in external wood fibres at half the length of a pivotally supported rod which should be smaller than the assumed maximum stress. When the model is made, with a certain imperfection a and the measuring equipment is installed in place (e.g. electrofusion tensometers) where the maximum normal stress is thought to occur at half the length of the rod and after the introduction of values of the force P (according to the project design), measurements must be made to obtain appropriate linear deformations ε. By multiplying the deformation by the elasticity modulus E_E determined in material tests, we receive the appropriate stress $\sigma_E = E_E \cdot \varepsilon$. The experimental factor can be determined with Eq. (69), based on Eq. (68) replacing the stress values σ_T with stress values σ_E.

$$k_c^G = \frac{1}{\varepsilon} \frac{P}{E_E A} \tag{69}$$

Both presented methods allow to determine instability factors k_c^G and k_c for a given model of compressed column and specific load rate.

It would be recommended to calculate the two factors k_c^G and k_c using both methods to determine which one is more precise.

Summary and conclusions

1. The following conclusions can be drawn from the conducted comparative analysis:
   - 1.1 The functions $F(k)$ and $F(n)$ when analysing the columns assume values larger compared to those obtained in the analysis of beams. This is due to the fact that the webs of I-sections and the walls of box sections may have in case of unloaded columns significantly smaller thickness than the webs and walls of beams to which transverse load was applied. Decreased thickness values directly affect the magnitude of energetic shear deformability α_d and α_{md}, notions defined in the paper, and consequently increase the differences between the compared factors k_c and k_c^G and affect the growth of the functions $F(k)$ and $F(n)$.
   - 1.2 In the analysis of beams and columns of the structure, the functions $F(k)$ and $F(n)$ have minimum values for composite sections made of wood and fibreboard and maximum values for composite sections made of wood and plywood.
   - 1.3 Owing to the shape of the cross-section, in case of wood and wood-based materials the functions $F(k)$ and $F(n)$ have the minimum values for the narrow I-section and the narrow box and the maximum values for the wide I-section and the wide box. The values of the function $F(k)$ for the narrow I-section and the narrow box differ insignificantly. The values of the function $F(k)$ for the wide I-section and the wide box also differ only insignificantly. The same applies to the function $F(n)$. In case of wood elements the functions $F(k)$ and $F(n)$ have the minimum values for rectangular and circular cross-sections, and the maximum values for the wide I-section and the wide box.
   - 1.4 Owing to the slenderness of the analysed elements, the functions $F(k)$ and $F(n)$ have the maximum values for thick columns and the extremum for the slenderness of $\lambda = 60$. The minimum values of the functions $F(k)$ and $F(n)$ occur at the slenderness of $\lambda = 150$.
   - 1.5 The conducted comparative analysis shows the need to use the formulae presented in the paper instead of those found in the literature. As was shown, this is particularly important in case of columns with small slenderness and cross-section, composite wide I-sections and wide box sections made of wood and plywood. In this case, the percentage differences between the compared factors k_c and k_c^G are $F(k)_{\max} = 23.52$ % and the load bearing capacity is exceeded by $F(n)_{\max} = 30.75$ %.
2. For the formulae (30)–(32), the author substituted the constants $E_{0,\text{mean}}$ and $G_{0,\text{mean}}$ with $E_{0.05}$ and $G_{0.05}$ assuming such a factor of safety k that $E_{0.05} \approx \frac{E_{0,\text{mean}}}{k}$ and $G_{0.05} \approx \frac{G_{0,\text{mean}}}{k}$, where $k = 1.5$, $E_{0,\text{mean}}$ is the mean modulus of elasticity along the fibres, $G_{0,\text{mean}}$ is the mean modulus of shear strain, $E_{0.05}$ is 5 % quantile of modulus of elasticity along the fibres and $G_{0.05}$ 5 % quantile of modulus of shear strain.
3. As stated in point 2, while calculating the instability factor k_c, $E_{0.05}$ 5 % quantile of modulus of elasticity should be used instead of $E_{0,\text{mean}}$ the mean modulus of elasticity. The introduction of 5 % quantile of modulus of elasticity $E_{0.05}$ into Eq. (28) results in lowering the values of the critical stress $\sigma_{c,\text{crit}}$ in the numerator, and thus in lowering the magnitude of the critical stress $\sigma_{c,\text{crit}}^G$ and the instability factor k_c^G which positively affects safety. The substitution of $E_{0.05}$ into $E_{0,\text{mean}}$ to the critical stress $\sigma_{c,\text{crit}}$ in the denominator of Eq. (28) increases the critical stress $\sigma_{c,\text{crit}}^G$ and the instability factor k_c^G which has adverse effect on safety.

Therefore, the author is of the opinion that a discussion should be held whether an additional safety factor n with the value of $n \geq 1$ should be introduced into Eqs. (30)–(32) or not. Percentage differences between the factors k_c^G and k_c were determined for $n = 1$. For $n > 1$ these differences are bound to be greater.

4. Although the cross-sections of axially compressed columns presented in Figs. 3g–j are theoretically possible, they are rarely used. Such cross-sections can be used for rods in which the longitudinal force has a decisive effect on the magnitude of stress and the transverse load is insignificant.

References

1. Timoshenko S, Gere J (1961) Theory of elastic stability. McGraw-Hill International, London, pp 52, 131–132, 180–181
2. Standard EN 1995 Eurocode 5 (2004) Design of timber structure Part 1-1: General-Common rules and Rules for buildings. European Committee for Standardization, Brussels, pp 43–44
3. Osiowski J (1981) Introduction to the operational calculus. WNT, Warsaw, pp 335–343
4. Jakubowicz A, Orłoś Z (1984) Strength of materials. WNT, Warsaw, p 248
5. Piechnik S (1980) Strength of materials for civil engineering faculties. PWN, Warsaw, p 283
6. Bielajew MM (1956) Strength of materials. MON, Warsaw, p 369
7. Standard EN 338 (2009) Structural timber - Strength classes. European Committee for Standardization, Brussels, p 7
8. Standard EN 12369-2 (2011) Wood-based panels - Characteristic values for structural design - Part 2: Plywood. European Committee for Standardization, Brussels, p 11
9. Standard EN 12369-1 (2001) Wood-based panels - Characteristic values for structural design - Part 1: OSB, particleboards and fibreboards. European Committee for Standardization, Brussels, pp 9, 13