

Probabilistic evaluation of the final moisture content of kiln-dried lumber using the bootstrap method

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Abstract This paper presents a probabilistic method of evaluating the final moisture content (MC) of lumber obtained at the end of the kiln-drying process. The final MC data of three different drying tests conducted in past studies were analyzed using the bootstrap method. Target MC was tentatively set below 20 % in the analysis. Two characteristic parameters representing the final MC were estimated with bootstrap confidence intervals. These parameters were the standard deviation (SD) and the percentage of the population that met the MC requirement of less than 20 % (P_{20}). The histograms of the final MC and the subsequent goodness-of-fit tests revealed that the final MC data of two drying tests did not follow any classical probability distributions, including Normal, Log-Normal, Weibull, and Gamma distributions, thus indicating the need for nonparametric statistics. The uncertainty of the final MC could be evaluated with the estimated SD and P_{20} . After deriving the relationships between P_{20} and the corresponding probability that P_{20} is not achieved, we demonstrated how such relationships could provide a kiln operator with information to facilitate better decision-making in optimizing a drying schedule.

Keywords Probabilistic evaluation · Final moisture content · Kiln-drying · Probability distribution · Bootstrap method

Introduction

Lumber drying is one of the most time- and energy-consuming processes in producing lumber products. As the main purpose of lumber drying is to reduce moisture content (MC) to a specific target value, the final MC that is measured at the end of drying is a primary consideration. In general, a large number of lumber are dried simultaneously in a batch kiln, and in such a condition, considerable variability in the final MC occurs [1]. Consequently, after kiln-drying, a batch may contain an unacceptable number of lumber pieces that do not meet the requirement of the appropriate moisture specification [2]. This outcome can lead to increased costs and the lower rates of lumber recovery.

To check the variability in the MC, most kiln operators select a limited number of test samples from the charge, and measure their MC routinely. The MC information is helpful, for example, at the end of drying process, in determining whether or not he or she should stop the kiln containing thousands of pieces dried simultaneously. However, this judgement relies largely on the operator's experience, since the MC information sampled from the whole charge involves uncertainty which is difficult to quantify. If the final MC data can be systematically evaluated in a probabilistic way, such as data analysis for structural lumber [3–6], the operator could make an objective and accurate decision in optimizing the drying schedule.

Understanding the final MC distribution is very important in the probabilistic analysis, because classical

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parametric statistics is based on the assumption of a specific probabilistic distribution. Rice and Shepard [1] extensively measured the MC of about 3500 eastern white pine (*Pinus strobus* L.) lumbers from 14 kiln charges that were dried in 7 sawmills. Substantial MC variation existed within a given charge and between kilns at each sawmill, and most of the MC distributions were found to be different from Normal distribution. Milota et al. [7] calculated the final MC distribution of mixed hem-fir species and mountain hemlock (*Tsuga mertensiana*) using a computer simulation, and showed that a number of slowly drying boards tended to skew the MC distribution toward the right. Tenori and Moya [8] and Moya et al. [9] statistically evaluated a variability in the final MC of kiln-dried lumber from plantations in Costa Rica. In the statistical analysis, the normality assumption was assessed, and positive skewness was found in the distribution [9]. The factors affecting the variability in the final MC of kiln-dried lumber was also examined using Pearson's correlation matrix [10] and general linear model [11], assuming that the final MC follows a Normal distribution. Numerical stochastic models have been intensively developed to simulate the MC dispersion during and after conventional drying [12, 13] and radio-frequency vacuum drying [14–16]. The developed stochastic models could be used to reproduce the final MC distributions, and some of the simulated distributions had a long right tail [12, 14]. The results of the above studies indicate that the final MC does not necessarily follow a normal distribution. However, as far as we know, a comprehensive evaluation of the final MC in relation to probabilistic distribution has not been carried out. Moreover, the final MC data have been assessed using only descriptive statistics, such as mean, standard deviation, coefficient of variation, and the percentage of the population whose MC is within or without the target MC range. The inference of these parameters has not been attempted at all, which means that no uncertainty assessment of the final MC has been carried out. Therefore, establishing a probabilistic evaluation method of the final MC is an important research endeavor.

The purpose of this study was to develop a probabilistic method by which to evaluate the final MC measured at the end of kiln-drying. The final MC data of three different drying tests conducted in the past works were analyzed using the bootstrap method.

Materials and methods

First, the bootstrap method, a modern computer-intensive statistical method, was introduced and its methodology was briefly described. Second, the probability distributions of the final MC were examined to determine whether the

parametric approach can be used for the analysis. Third, the final MC was evaluated with two characteristic parameters, namely, standard deviation (SD) and the percentage of the population that met the MC requirement (P_{req}). SD is the population parameter representing the variability of the final MC, whereas P_{req} is one of the most important indicators that have a significant impact on lumber recovery and productivity. The uncertainties of SD and P_{req} were estimated using the bootstrap method, after which we demonstrated how to integrate the bootstrap estimate into the decision-making related to lumber drying.

Principle of the bootstrap method

The bootstrap was first introduced by Efron [17] as a computer-based simulation method for estimating the standard error of a parameter estimate. Over the next decades, the theory and applications of the bootstrap have been developed [18], and the bootstrap became a very practical approach to making statistical inferences without strong parametric assumptions. The bootstrap is a type of Monte Carlo simulation based on resampling from observed data; its algorithm is briefly described as follows [19].

Suppose that a random sample $x = (x_1, x_2, \dots, x_n)$ from an unknown probability distribution F has been observed and we wish to estimate a parameter of interest $\theta = s(F)$ on the basis of x . For this purpose, an estimate $\hat{\theta} = s(x)$ is typically calculated from x . Here, $s(\cdot)$ is a function by which to measure a parameter estimate, such as mean, median, standard deviation, bias, quantiles, and so on.

Let \hat{F} be the empirical distribution, putting probability $1/n$ on each of the observed values x_i , $i = 1, 2, \dots, n$. A bootstrap sample is defined to be a random sample of size n , which is drawn with replacement from \hat{F} , say $x^* = (x_1^*, x_2^*, \dots, x_n^*)$. The star notation indicates that x^* is not the actual data set x , but rather, a randomized or resampled version of x . Thus, the bootstrap sample $x^* = (x_1^*, x_2^*, \dots, x_n^*)$ consists of members of the original data set (x_1, x_2, \dots, x_n) , with some appearing zero times, some appearing once, some appearing twice, etc.

Corresponding to a bootstrap sample x^* is the bootstrap replication of $\hat{\theta}$ given as follows:

$$\hat{\theta}^* = s(x^*). \quad (1)$$

The quantity $s(x^*)$ is the result of applying the same function $s(\cdot)$ to x^* as applied to x .

By generating independent bootstrap samples x^{*b} ($b = 1, 2, \dots, B$) repeatedly, the bootstrap replications $\hat{\theta}^*(b)$ corresponding to each bootstrap sample can be obtained as follows:

$$\hat{\theta}^*(b) = s(x^{*b}), \quad b = 1, 2, \dots, B. \tag{2}$$

From the distribution of $\hat{\theta}^*(b)$, the uncertainty of the parameter estimate $\hat{\theta}$ can be estimated, for example, by measuring the standard deviation or confidence intervals. A schematic diagram of the bootstrap algorithm is shown in Fig. 1.

There are several approaches to construct confidence intervals based on attempts to approximate the percentiles of the distribution of $\hat{\theta}^*(b)$. Efron’s percentile confidence intervals [17] are the two values that cut-off fixed percentages in the tails of the bootstrap distribution of an estimate. For example, the bootstrap 95 % intervals are the two values that include 95 % of the bootstrap distribution of an estimate between them. This notion is justified on the basis of the assumption that a transformation exists which can convert the bootstrap distribution of an estimate into a normal distribution [20]. Thus, in small to moderate samples for asymmetric or heavy-tailed distributions, the percentile method is vulnerable [18], and an improved version of the percentile method called “bias-corrected and accelerated percentile method” (BCa) [21] is required. The BCa adjusts for the median of the distribution of an estimate

that is not equal to the mean and for the standard deviation of the distribution varying with the mean of the distribution. A more detailed description of BCa is provided by Efron [22].

Data preparation

The current study incorporated the final MC data obtained from three different drying tests reported in the literature [23, 24]. The drying methods and specimens for each test are briefly described as follows. All the drying tests were conventional kiln-drying tests with schedules listed in Table 1. In drying test 3, the second step with a dry-bulb temperature of 120 °C for 18 h represents a high-temperature and low-humidity pretreatment, which is effective in preventing surface checks [25]. Next, 222 boards [23], 357 square lumbers [24], and 115 square lumbers [24] were used for drying tests 1, 2 and, 3, respectively. The summary statistics of the final MC data are listed in Table 2. In drying test 2, additional drying runs were conducted by Matsumoto and Ishida [24], so the sample size was larger than that in the report. The specific wood species used for the drying tests was sugi (*Cryptomeria Japonica*).

Fig. 1 Bootstrap algorithm for estimating the uncertainty of parameter estimate $\hat{\theta}$

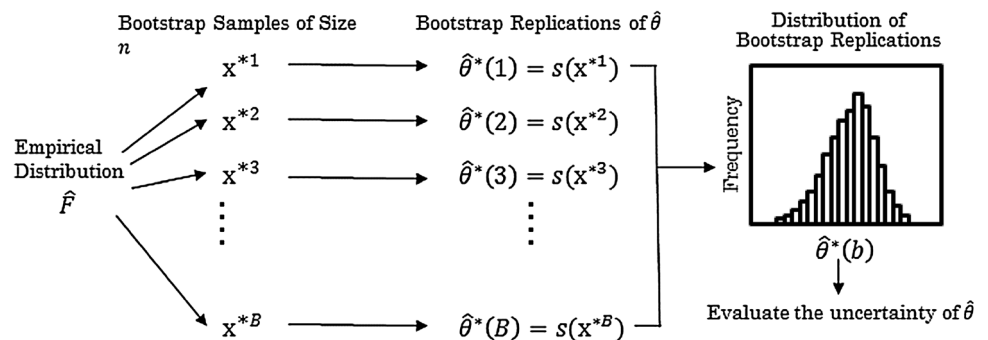


Table 1 Drying schedules of the three drying tests

Drying test 1 [23]			Drying test 2 [24]			Drying test 3 [24]		
Time (h)	DBT (°C)	WBT (°C)	Time (h)	DBT (°C)	WBT (°C)	Time (h)	DBT (°C)	WBT (°C)
8	85	85	12	70	70	8	95	95
8	70	67	168	70	65	18	120	90
16	71	67	168	70	60	504	90	60
16	73	67	168	70	55			
16	75	67	168	70	50			
16	77	68						
24	79	68						
20	80	68						
8	80	75						

DBT dry-bulb temperature, WBT wet bulb temperature

Table 2 Summary statistics of the final moisture content for all the three drying tests

Drying test	<i>n</i>	Mean (%)	SD (%)	CV	<i>P</i> ₂₀ (%)
1 [23]	222	16.8	3.7	0.22	92.3
2 [24]	357	16.4	6.4	0.39	80.7
3 [24]	115	13.3	6.1	0.50	89.6

SD standard deviation, CV coefficient of variation, *P*₂₀ the percentage of the population that met the moisture content requirement of less than 20 %

Fitting classical probability distributions

Two goodness-of-fit tests, the Kolmogorov–Smirnov test (KS test) and the Anderson–Darling test (AD test), were performed to evaluate whether the final MC data of each drying test followed classical probability distributions. The parameters of each probability distribution were estimated by maximum likelihood, and then subjected to the goodness-of-fit tests. The classical probability distributions employed were Normal, Log-Normal, Weibull, and Gamma distributions. The former three distributions are often used for strength data of structural lumber [3–6], whereas Gamma distribution is a standard probability distribution fitted to continuous data. Moreover, Gamma distribution is used for a random variable *y* that has positive values, and its probability density function is expressed as follows:

$$f(y; \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-y\beta}, \quad (3)$$

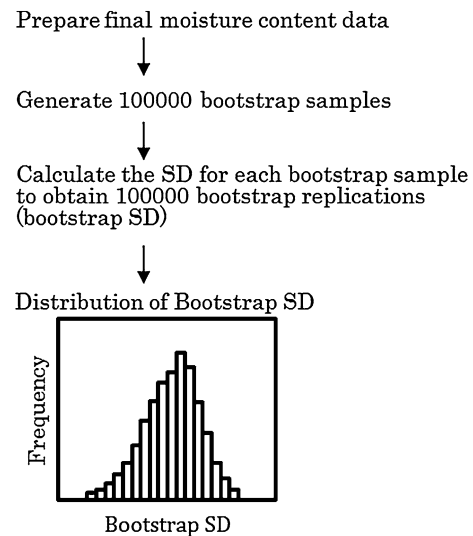
where β is a scale parameter, which is the parameter of interest; α is a known shape parameter; and $\Gamma(\alpha)$ is the complete gamma function defined by the following:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \quad \alpha > 0. \quad (4)$$

Horie et al. [3] evaluated the strength data of structural lumber, and used Chi-square test and KS test as goodness-of-fit measures. In the current study, however, the Chi-square test was not used, because the sample data had to be binned before generating the Chi-square test. As we know, the value of the Chi-square test statistic is dependent on how the data are binned. In addition, the Weibull distribution employed in this study was not 3P Weibull but 2P Weibull, because the theoretical and physical meanings of the location parameter in 3P Weibull distribution were not clear [3].

Bootstrap estimation of the characteristic parameters for the final MC

The final MC data were evaluated with the two characteristic parameters, SD and *P*_{req}. In this study, the MC

**Fig. 2** Procedure to calculate bootstrap standard deviation (bootstrap SD)

requirement was tentatively set below 20 %. The SD and *P*₂₀ of the final MC were estimated with 100000 bootstrap replications, and their 95 % confidence intervals were calculated using the BCa method. The procedure to calculate the bootstrap SD was shown in Fig. 2. Statistical analysis was performed using the MASS package and the boot package in R, version 3.2.3 [26].

Results and discussion

Distribution of the final MC

The histograms of the final MC and the fitted probability distributions are shown in Fig. 3. In all the drying tests, the distributions of the final MC had a long right tail. A certain percentage of the population fell above the MC of 30 %. Moreover, these samples remained wet, thereby demonstrating the difficulty in uniformly drying sugi lumber.

To examine whether the final MC data followed the fitted probability distributions (Fig. 3), the goodness-of-fit tests were performed and their results were listed in Table 3. In the table, high *p* values indicate that data probably follow a probability distribution. As can be seen, in the case of drying test 1, the final MC data did not follow any classical probability distribution, which may be attributed to the heavy-tailed nature of the distribution (Fig. 3).

In the case of drying test 2, the results of the AD test suggested that all the probability distribution gave a poor fit to the final MC data. Although the *p* value of the KS test against the Log-Normal null hypothesis was slightly higher than 0.05, this was not enough to ensure that the data

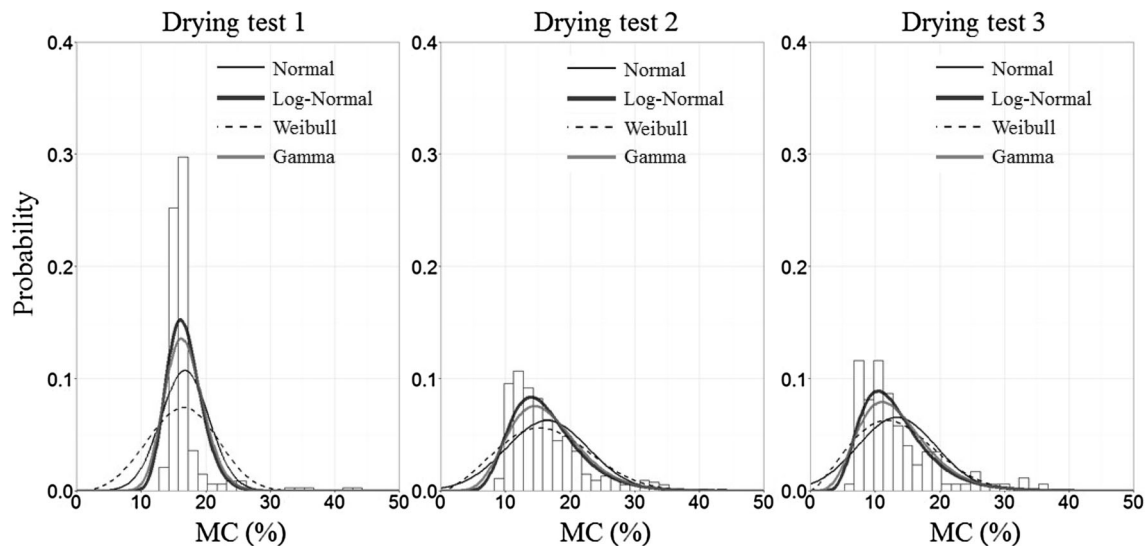


Fig. 3 Histograms of the final moisture content (MC) and fitted probability distributions

Table 3 Results of the goodness-of-fit tests for each drying test

Drying test	Goodness-of-fit test	Fitted distribution			
		Normal	Log-Normal	Weibull	Gamma
1	KS test	**	**	**	**
	AD test	**	**	**	**
2	KS test	**	0.064	**	**
	AD test	**	**	**	**
3	KS test	**	0.462	*	0.139
	AD test	**	0.226	**	0.061

Numbers represent *p* values. The *p* values exceeding 0.05 indicate that null hypothesis was not rejected with 95 % significance level and that the data may probably follow a probability distribution

KS test Kolmogorov–Smirnov test, AD test Anderson–Darling test

* *p* values less than 0.05

** *p* values less than 0.01

followed the Log-Normal distribution. The *p* values in the AD test were consistently lower than those in the KS test. Thus, in evaluating whether the data followed the probability distributions, the AD test tended to judge more conservatively than the KS test.

Meanwhile, in the case of drying test 3, the *p* values against the Log-Normal null hypothesis were much higher than 0.05 in both the KS test and the AD test. This result implies that the Log-Normal distribution is a good candidate for the distribution of the final MC of drying test 3.

The histograms of the final MC (Fig. 3) and the subsequent goodness-of-fit tests (Table 3) revealed that the final MC data did not necessarily follow a classical probability distribution. Therefore, the conventional parametric statistics are of limited use in evaluating the uncertainty of the final MC. Furthermore, the bootstrap method is thought to be a preferable alternative to parametric statistics.

Bootstrap estimation of characteristic parameters for the final MC

The SD of the final MC was estimated by the bootstrap method (“bootstrap SD”), and the histograms of bootstrap SD were depicted for each drying test in Fig. 4. The uncertainty in the estimated SD can be measured by the histograms. For example, the bootstrap SD for drying test 1 ranged from 1.2 to 6.3 %, whereas the SD of the sample was 3.7 % (Table 2). Given that the same drying test was attempted repeatedly, the SD should be within this range. This range can be assessed with confidence intervals in a more probabilistic way. Table 4 lists the 95 % BCa confidence intervals for the bootstrap SD. The coverage property of this interval implies that 95 % of the time, a random interval constructed in this way will contain the true value [19].

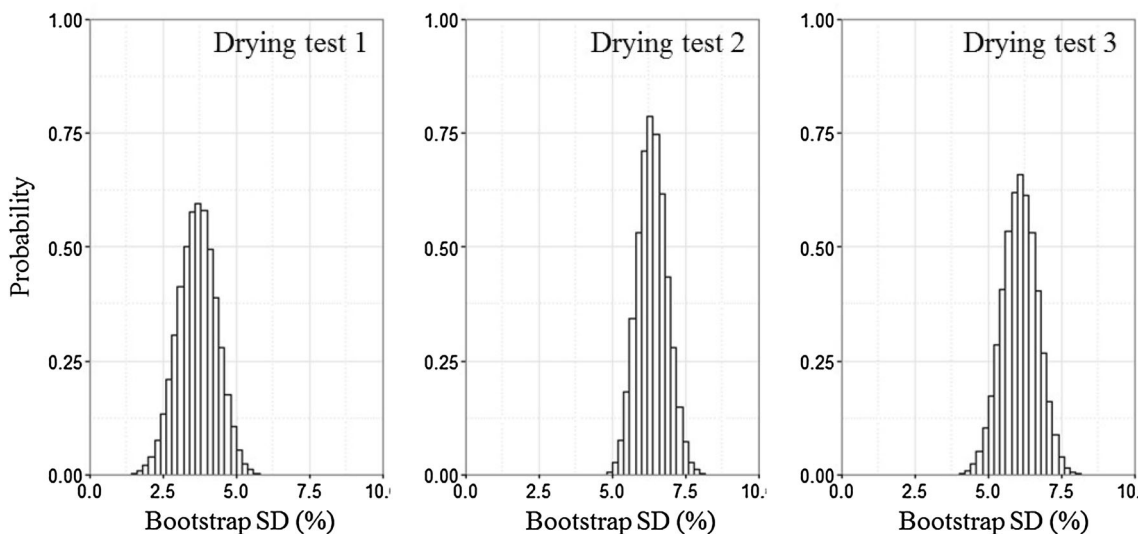


Fig. 4 Histograms of bootstrap standard deviation (bootstrap SD) for each drying test

Table 4 95 % BCa confidence intervals for bootstrap estimates

Drying test	SD (%)	P_{20} (%)
1	2.6–5.3	87.8–95.1
2	5.6–7.7	75.9–84.3
3	5.1–7.5	81.7–93.9

BCa bias-corrected and accelerated percentile method, SD standard deviation, P_{20} the percentage of the population that met the moisture content requirement of less than 20 %

P_{req} , the percentage of the population that met the MC requirement, is one of the most important indicators that have a significant impact on lumber recovery and productivity. In this study, the MC requirement was tentatively set

below 20 %, and the P_{20} was estimated by the bootstrap (“bootstrap P_{20} ”). Similar to the estimated SD, the uncertainty in the estimated P_{20} can also be measured with the 95 % confidence intervals (Table 4).

Suppose that a kiln operator wishes to dry at least $\alpha\%$ of the total population within a batch to less than 20 % MC. In other words, α is the acceptable percentage, and the target P_{20} is $\alpha\%$. The probability that the target P_{20} of $\alpha\%$ will not be achieved can be estimated from the histograms of the bootstrap P_{20} in Figs. 5 and 6. We call this probability “operator’s risk” (OR), which is expressed as follows:

$$OR = \text{Prob}\{P_{20} < \alpha\}. \tag{5}$$

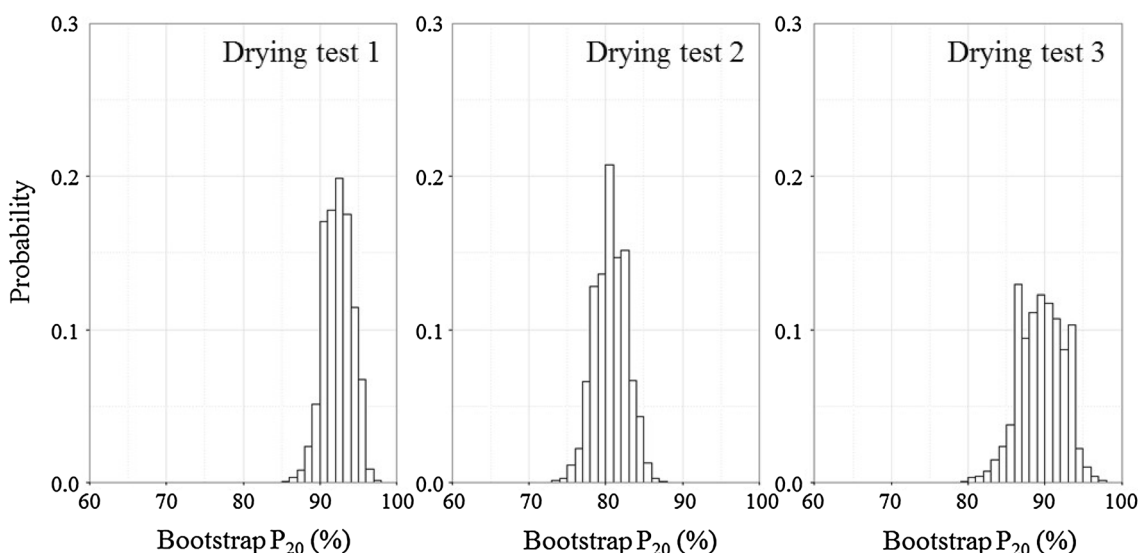
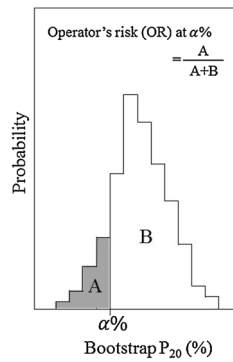


Fig. 5 Histograms of bootstrap P_{20} for each drying test. P_{20} is the percentage of the population that met the moisture content requirement of less than 20 %

Fig. 6 Schematic diagram of Fig. 5. OR operator’s risk, α acceptable percentage, A area below $\alpha\%$, B area above $\alpha\%$



As can be seen in Fig. 6, the OR at $\alpha\%$ is expressed as the ratio of the area below $\alpha\%$ (A) to the total area (A + B). From Fig. 5, the OR was calculated for the whole α to obtain the OR as a function of target P_{20} (Fig. 7). In the case of drying test 1, for example, the OR at 95 and 90 % are 0.92 and 0.09, respectively. This means that 92 % will fail trying to dry 95 % of the total population to less than 20 % MC; in comparison, the risk of failure can be drastically reduced to 9 % by changing the P_{20} from 95 to 90 %. If a kiln operator aims to achieve a target P_{20} of 95 %, then the current drying operation should be continued to reduce the MC, because the risk of failure is considered to be very high. Meanwhile, if a target P_{20} of 90 % and the risk of failure of 9 % can be accepted, the kiln operator makes a decision that the current drying operation is working well and should thus be stopped. These results demonstrate that the uncertainty of the final MC could be evaluated using the bootstrap, and that the relationships between the target P_{20} and the OR could provide a kiln operator with information that can facilitate a better decision-making in optimizing a drying schedule.

The bootstrap method may not be reliable for very small sample sizes, regardless of how many bootstrap samples are generated. Thus, a certain sample size should be acquired. In this study, the sample size of each drying test was more than 100, so a sufficient sample size was prepared for the bootstrap analysis.

Conclusions

This study examined the process of evaluating the final MC of lumber obtained at the end of kiln-drying. The goodness-of-fit tests revealed that the final MC data do not necessarily follow a classical probability distribution, and that the conventional parametric statistics are of limited use in the probabilistic analysis of the final MC. This is the reason why we utilized the bootstrap method without any assumptions about the underlying probability distribution. The bootstrap method may be suitable for analyzing not only data obtained in the process of lumber drying, but also the strength data of structural lumber, because the simplicity of the bootstrap method allows its application in a wide variety of fields.

Our results demonstrated that the bootstrap method is a powerful approach to evaluate the uncertainty and variability of the final MC data. The confidence intervals of SD and P_{20} were computed from the bootstrap estimates, so that the uncertainty of these parameters could be assessed. Based on the relationships between P_{20} and the corresponding probability that P_{20} is not achieved, probabilistic risk assessment of the final MC can be implemented in a kiln-drying operation. For example, at the end of drying process, a kiln operator measures the MC of a limited

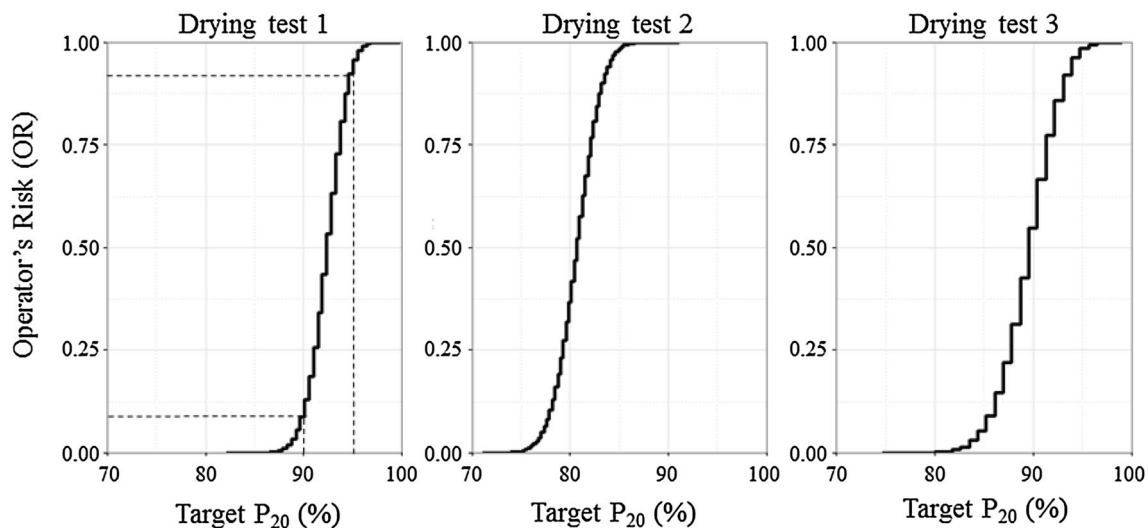


Fig. 7 Operator’s risk (OR) as a function of target P_{20} for each drying test. P_{20} is the percentage of the population that met the moisture content requirement of less than 20 %

number of test samples to know if he or she should stop the kiln containing thousands of pieces dried simultaneously. These findings may lead to a higher quality control of kiln-dried lumber.

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