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ORIGINAL ARTICLE

Static analysis of lattice columns

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Abstract

The paper focuses on timber structures. Static analysis was performed in the paper of timber lattice columns with N and V lattice configuration made in variants of timber, plywood, fibreboard and particleboard. As a part of the study, formulae determining critical force in the column and the column slenderness ratio were derived basing on the theory by Timoshenko and Gere. In addition, the paper includes formulae applicable to shearing forces occurring in the column as well as maximum shearing forces that a column can carry, also based on the theory by Timoshenko and Gere. Basing on the formulae described above and the formulae given in the literature (EN-1995 Eurocode 5 Standard), a comparative analysis was carried out of the load-bearing capacity of columns and calculations for the truss. The calculations demonstrate that there are discrepancies between the static values being compared and both calculation methods lead to partially divergent results.

Keywords Lattice columns · Truss · Shear strain · Critical load-bearing capacity · Shearing in columns

Symbols

Latin letters

a Initial maximum column curvature
A Cross-sectional area
A_f Cross-sectional area of a flange
A_s Cross-sectional area of a vertical
A_k Cross-sectional area of a diagonal
e Eccentric of the joints, eccentric of the force application P
E Modulus of elasticity
E_{mean} Mean value of modulus of elasticity
E_k Modulus of elasticity of a diagonal
E_s Modulus of elasticity of a vertical
$E_{0.05}$ Fifth percentile of the modulus of elasticity of column shafts
$E^k_{0.05}$ Fifth percentile of the modulus of elasticity of diagonals,
$E^s_{0.05}$ Fifth percentile of the modulus of elasticity of verticals
$f_{c,0,k}$ Characteristic compressive strength of timber along the grain
$f_{c,0,d}$ Design compressive strength of timber along the grain
$f_{r,k}$ Characteristic planar (rolling) shear strength
$f_{v,k}$ Characteristic shear strength
g Truss elements thickness
G Shear modulus
h Distance of the flanges
i Radius of gyration of a column considered as solid
I Second moment of area of a section
I_f Second moment of area of a flange
k_c Instability factor
l Span (height of a latticed column)
l_c Buckling span of a column
l_1 Distance between adjacent nodes
$M(x)$ Bending moment
n Load-bearing capacity of a column
P Compressive force
$q(x)$ Transverse load
V_p Shear force
V_p^a Maximum shear force, caused by compressive force P, for a column with its initial curvature described with a sinusoid
V_p^e Maximum shear force, caused by compressive force P, for a column, where force P acts on eccentric e
$V^a_{p,\max}$ Maximum shear force a column can carry, for a column with its initial curvature described with a sinusoid
$V^e_{p,\max}$ Maximum shear force a column can carry, for a column, where force P acts on eccentric e

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$y(x)$ Deflection line function
z_{max} Distance from the neutral axis to the extreme grain

Greek letters

α Angle between a diagonal and a vertical
λ_{ef} Slenderness ratio for a column
μ Energetic shear coefficient

Introduction

Comments on the used wood

Wood together with wood-based materials is the basic building material, just like steel and concrete constructions. The wood used for making the columns in question can be divided into two groups: wood of coniferous trees and wood of deciduous trees. The first group comprises spruce, fir, pine, larch, western hemlock and Douglas-fir. The second group is made of oak, beech, teak, Angelique/Basralocus, bloodwood, greenheart, Azobe/Bongossi. Out of those aforementioned trees, it is such tree as pine is most often used in Europe. Information about density of timber used to make columns in question can be found in Sect. 1.3 entitled "timber density" and in Table 1.2 in Neuhaus [1]. The essential property of timber is its moisture which influences its strength. Comments on the moisture of timber can be found in Sect. 1.5 entitled "timber moisture" and in the Table 1.3 in Neuhaus [1]. Strength and elasticity properties as well as density of used wood from coniferous and deciduous trees were assumed basing on EN 338 [2] Standard. However, strength and elasticity properties of plywood as well as fibreboard and particleboard were assumed basing on EN 12369-2 [3] and EN 12369-1 [4] standards respectively.

The subject of the paper

The subject of the paper is lattice columns with N and V lattice configurations. Column flanges are commonly made of solid wood or glued laminated timber, and the truss can be made of wood, plywood, fibreboard or particleboard.

The theory given in Timoshenko and Gere [5] shows that in the case of lattice columns their critical load-bearing capacities are lower than the critical load-bearing capacities of solid columns of the same slenderness ratio and cross-sectional area. This is mainly due to the fact that the influence of shearing on the displacement in the lattice columns is significantly higher than in the solid columns. At the same time, the critical load-bearing capacity depends on the elastic and strength characteristics of the truss as well as its arrangement in the column. Due to the above, the author carried out a comparative static analysis basing on the formulae derived from the theory given in Timoshenko and Gere [5] that includes the influence of shearing on displacements as well as the formulae given in EN-1995 Eurocode 5 [6] without taking into account these influences.

Calculations for lattice columns with N and V lattice configurations based on the literature (EN-1995 Eurocode 5 [6]) state-of-the-art

The slenderness ratio of a lattice columns with N and V lattice configurations fixed rigidly to the column flanges is derived from the formula:

$$\lambda_{ef} = \max\begin{cases} \lambda_{tot}\sqrt{1+\mu_{ef}} \\ 1.05\cdot\lambda_{tot} \end{cases} \tag{1}$$

where:

λ_{tot}—is the slenderness ratio for a solid column with the same length, the same area and the same second moment of area:

$$\lambda_{tot} = \frac{2\cdot l}{h} \tag{2}$$

For a glued N-truss:

$$\mu_{ef} = \frac{e^2\cdot A_f}{I_f}\left(\frac{h}{l}\right)^2 \tag{3}$$

For a glued V-truss:

$$\mu_{ef} = 4\cdot\frac{e^2\cdot A_f}{I_f}\left(\frac{h}{l}\right)^2 \tag{4}$$

Based on the literature, the shear forces in columns V_p and the maximum shear forces $V_{p,\max}$ that a column can carry are derived from the formulae:

$$V_p = \begin{cases} \dfrac{P}{120\cdot k_c} & \text{for} \quad \lambda_{ef} < 30 \\ \dfrac{P\cdot\lambda_{ef}}{3600\cdot k_c} & \text{for } 30 \le \lambda_{ef} < 60 \\ \dfrac{P}{60\cdot k_c} & \text{for} \quad 60 \le \lambda_{ef} \end{cases} \tag{5}$$

In order to determine the maximum shear force $V_{p,\max}$ that the column can carry, the value of the maximum force P, at which the load-bearing capacity of the column is not exceeded needs to be substituted in the formulae (5).

Derivation of formulae for calculations of the column load-bearing capacity based on the theory given in Timoshenko and Gere [5] taking into account the influence of shear strains

Determination of critical force

In order to determine critical force, taking shear into account, a rod deflection differential equation, as derived by the author, was used. That equation accounts for the influence of shear forces on deflections, and it has the following form:

$$\frac{d^2y(x)}{dx^2} + k^2y(x) = -\frac{1}{EI\left(1-\frac{\mu P}{GA}\right)}M(x) - \frac{\mu}{GA\left(1-\frac{\mu P}{GA}\right)}q(x) \tag{6}$$

where k^2 equals:

$$k^2 = \frac{P}{EI\left(1-\frac{\mu P}{GA}\right)} \tag{7}$$

Derivation of Eq. (6) is presented in the author's article Śliwka [7].

Formula (7) can also be found in Łubiński et al. [8].

In that paper, a case of a rod loaded with longitudinal compressive force P was analysed, which results in the fact that the bending moment $M(x)$, present in the equation, and transverse load $q(x)$ take up the 0 value $M(x) = 0$ $q(x) = 0$. That yielded the following equation:

$$\frac{d^2y(x)}{dx^2} + k^2y(x) = 0 \tag{8}$$

The Eq. (8) can also be found in Piechnik [9] as well as in Jakubowicz and Orłoś [10].

The equation was solved, using the method of operational calculus, based on Laplace transformation Osiowski [11]. The solution of the equation is the following function:

$$y(x) = w_0 \cos kx + w_1 \frac{1}{k} \sin kx \tag{9}$$

The Eq. (9) can also be found in Piechnik [9] as well as in Jakubowicz and Orłoś [10].

Critical force was derived from a boundary condition, saying that the value of the argument of function sin at point $x = l$ $k{\cdot}l$ equals:

$$k \cdot l = n\pi \tag{10}$$

Replacing k in the equation, where $k^2 = \frac{P}{EI\left(1-\frac{\mu P}{GA}\right)}$ and $n = 1$, the following formula for critical force $P = P_{c,\mathrm{crit}}$, was obtained:

$$P_{c,\mathrm{crit}} = \frac{P_e}{1 + P_e\frac{\mu}{GA}} \tag{11}$$

where:

$$P_e = \frac{\pi^2 EI}{l_c^2} \tag{12}$$

Formulae (11) and (12) are presented in the author's articles Śliwka [7] Śliwka [12].

Formula (11) can also be found in Timoshenko and Gere [5] and Łubiński et al. [8] and the formula (12) in Piechnik [9] as well as in Jakubowicz and Orłoś [10].

Determination of slenderness ratio λ_{ef} for a lattice column basing on the theory given in Timoshenko and Gere [5]

In the formula (11) the coefficient $\frac{\mu}{GA}$ is substituted with the coefficient η. The coefficient η is the value, by which the shear force V_p needs to be multiplied in order to obtain an additional angle of the deflection line caused by shearing. Hence, we get:

$$\gamma = \eta \cdot V_p \tag{13}$$

In order to determine the η value, horizontal displacements caused by shear force need to be examined in every particular case. For this purpose, the column presented below was used for analyses:

Horizontal displacement is caused by elongations and shortenings of the truss columns in every area (Fig. 1b, c). Elongation of a diagonal caused by shear force V_p amounts to:

$$\frac{V_p \cdot l_1}{2 \cdot E_k \cdot A_k \cdot \sin\alpha \cdot \cos\alpha} \tag{14}$$

Based on the expression (14) the horizontal displacement u_1 equals:

$$u_1 = \frac{V_p \cdot l_1}{2 \cdot E_k A_k \cdot \sin\alpha \cdot \cos^2\alpha} \tag{15}$$

Horizontal displacement resulting from the shortening of the truss horizontal members equals:

$$u_2 = \frac{V_p \cdot h}{2 \cdot E_s A_s} \tag{16}$$

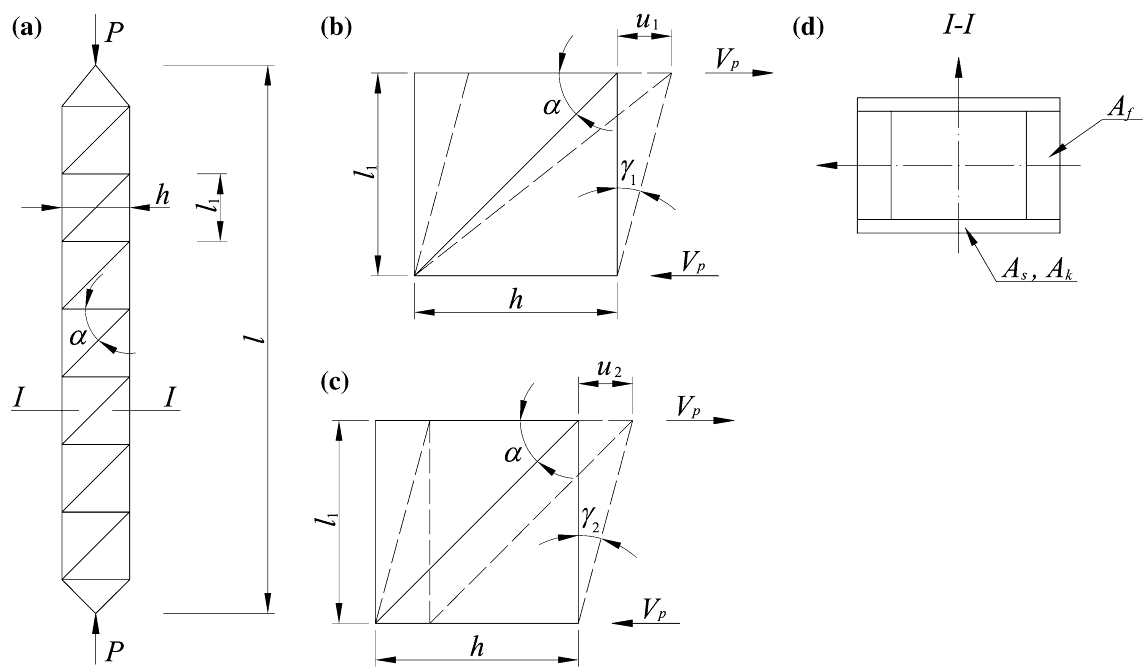


Fig. 1 Analysed lattice column with N-truss

The total angular displacement γ caused by the shear force V_p is determined on the basis of the formulae (15) and (16):

$$\gamma = \frac{u_1 + u_2}{l_1} = \frac{V_p}{2 \cdot E_k A_k \sin\alpha \cos^2\alpha} + \frac{V_p \cdot h}{2 \cdot E_s A_s \cdot l_1} \tag{17}$$

Using the formula (13) we get:

$$\eta = \frac{1}{2 \cdot E_k \cdot A_k \cdot \sin\alpha \cdot \cos^2\alpha} + \frac{h}{2 \cdot E_s A_s \cdot l_1} \tag{18}$$

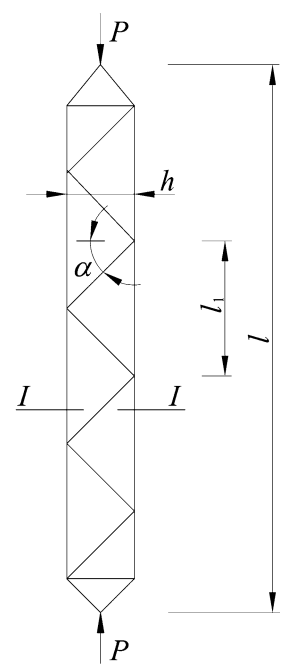
By substituting the expression (18) in the formula (11) for $\frac{\mu}{GA}$ and substituting the mean moduli of elasticity E_k, E_s with fifth percentiles $E^k_{0.05}$, $E^s_{0.05}$ the following formula for the critical force for the column with N-truss was derived:

$$P_{c,crit} = P_e \cdot \left[1 + P_e\left(\frac{1}{2 \cdot E^k_{0.05} \cdot A_k \cdot \sin\alpha \cdot cos^2\alpha} + \frac{h}{2 \cdot E^s_{0.05} \cdot A_s \cdot l_1}\right)\right]^{-1} \tag{19}$$

The formula for the critical force for the column with V-truss according to the Fig. 2 was obtained from the formula (19) by neglecting in it the expression including the area of the vertical A_s. Thus, the following formula was derived:

$$P_{c,crit} = P_e\left[1 + P_e\left(\frac{1}{2 \cdot E^k_{0.05} \cdot A_k \cdot \sin\alpha \cdot \cos^2\alpha}\right)\right]^{-1} \tag{20}$$

Fig. 2 Analysed lattice column with V-truss



where $P_e = \frac{\pi^2 E_{0.05} I}{l_c^2}$ is the critical force for the column considered as solid. On the basis of the formulae (19) and (20) the buckling spans of the columns with built-up cross-sections are the following:

For columns with N-truss:

$$l_{ef} = \sqrt{1 + P_e\left(\frac{1}{2 \cdot E_{0.05}^k \cdot A_k \cdot \sin\alpha \cdot \cos^2\alpha} + \frac{h}{2 \cdot E_{0.05}^s A_s l_1}\right)} \cdot l_c \tag{21}$$

For columns with V-truss:

$$l_{ef} = \sqrt{1 + P_e\left(\frac{1}{2 \cdot E_{0.05}^k \cdot A_k \cdot \sin\alpha \cdot \cos^2\alpha}\right)} \cdot l_c \tag{22}$$

Thus, using the formulae (21) and (22), the author proposes the following formulae for the column slenderness ratio λ_{ef}.

For columns with N-truss:

$$\lambda_{ef} = \sqrt{\lambda^2 + \pi^2 E_{0.05} \cdot A(\eta_1 + \eta_2)} \tag{23}$$

For columns with V-truss:

$$\lambda_{ef} = \sqrt{\lambda^2 + \pi^2 E_{0.05} \cdot A \cdot \eta_1} \tag{24}$$

where:

$$\eta_1 = \frac{1}{2 \cdot E_{0.05}^k \cdot A_k \cdot \sin\alpha \cdot \cos^2\alpha} \tag{25}$$

$$\eta_2 = \frac{h}{2 \cdot E_{0.05}^s A_s l_1} \tag{26}$$

$$\lambda = \frac{l_c}{i} \tag{27}$$

Calculations of truss based on the theory given in Timoshenko and Gere [5]: determination of shear forces

The formulae used to determine maximum shear forces occurring in the columns V_p are of the following form:

$$V_p = \max\begin{cases} P\dfrac{a}{i}\dfrac{\pi^3 E_{0.05} A}{\pi^2 E_{0.05} A \cdot \lambda_{ef} - P\lambda_{ef}^3} & (28) \\[2ex] P \cdot \dfrac{e}{i}\dfrac{\left(\dfrac{P}{E_{0.05} A}\right)^{0.5}}{\sin\left[\dfrac{\lambda_{ef}}{2}\left(\dfrac{P}{E_{0.05} A}\right)^{0.5}\right]} & (29) \end{cases}$$

The formulae, used to determine maximum shear forces $V_{p,\max}$ that the column can carry, are of the following form:

$$V_{p,\max} = \min\begin{cases} \dfrac{1}{2}\dfrac{c}{i}\dfrac{\pi A}{\lambda_{ef}}\left[f_{c,0,k} - \left(1 + \dfrac{a}{c}\right)\dfrac{\pi^2 E_{0.05}}{\lambda_{ef}^2} + \sqrt{\left[f_{c,0,k} + \left(1 + \dfrac{a}{c}\right)\dfrac{\pi^2 E_{0.05}}{\lambda_{ef}^2}\right]^2 - 4 f_{c,0,k}\dfrac{\pi^2 E_{0.05}}{\lambda_{ef}^2}}\right] & (30) \\[2ex] \sigma_{\text{mid}} \cdot \dfrac{eA}{i}\dfrac{\left(\dfrac{\sigma_{\text{mid}}}{E_{0.05}}\right)^{0.5}}{\sin\left[\dfrac{\lambda_{ef}}{2}\left(\dfrac{\sigma_{\text{mid}}}{E_{0.05}}\right)^{0.5}\right]} & (31) \end{cases}$$

the mean stress σ_{mid} in the formula (31) is derived from the equation:

$$\sigma_{\text{mid}}\left\{1 + \frac{e}{c}\text{cosec}\left[\frac{\lambda_{ef}}{2}\left(\frac{\sigma_{\text{mid}}}{E_{0.05}}\right)^{0.5}\right]\right\} - f_{c,0,k} = 0 \tag{32}$$

Derivations of the formulae (28)–(32) are given in the author's article Śliwka [12].

Determination of initial maximum curvature of column *a*, and of force application eccentric *e*

In accordance with the literature (EN-1995 Eurocode 5 [6]), upon determining instability factor k_c, based on column deflection line at its initial curvature described with a sinusoid, expression $\frac{a}{c}$ was substituted with $\beta\left(\frac{\lambda_{ef}}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}}-0.3\right)$, where coefficient β was $\beta=0.2$ for solid timber and $\beta=0.1$ for glued laminated timber. Therefore, the author suggests using the following formulae, concerning a and e. In expression $\frac{a}{c}$ $c=\frac{i^2}{z_{\max}}$.

Table 1 compiles the strength and the modulus of elasticity of materials used in column analyses.

Fifth percentiles of the moduli of elasticity were derived from the formula $E_{0.05}=\frac{E_{\text{mean}}}{1.5}$. The load-bearing capacity was derived from the formula $n=\frac{P}{A\cdot k_c\cdot f_{c,0,d}}$. In the analyses the columns with the slenderness ratios of $\lambda_{ef}=[20, 30, 40, 50, 100, 150]$ were taken into consideration.

The paper presents the calculation results for the columns with the slenderness ratios of $\lambda_{ef}=50$, the maximum slenderness ratio of $\lambda_{ef}=150$, the intermediate slenderness ratio of $\lambda_{ef}=100$ and the slenderness ratio of $\lambda_{ef}=30$, for which the differences between the compared static values of λ_{ef}, k_c, n, $(n-1)\cdot 100\%$ are the greatest. The slenderness ratios values $\lambda_{ef}=[20, 30, 40, 50, 100, 150]$ refer to calculation based on the formulae given in EN-1995 Eurocode 5 [6]. It was assumed that the column flanges were made of solid wood of class C24, and the truss elements were made in variants of timber, plywood, fibreboard and particleboard. It was assumed that the truss elements were glued to the column flanges. Aminoplastic resin, phenolic resin and polycondensation adhesive, described in the standard EN 301 [13], are among the possible materials that can be used for gluing. Columns with flanges with the dimensions of 100×100 mm were analysed. It was assumed that the distance of the flanges of the column was $h=400$ mm. The thickness of the truss elements was assumed as equal to $g=20$ mm. As the widths of the truss elements made of particleboard 20 mm wide obtained from calculations turned out to be significantly larger than the widths of the

$$a=e=\begin{cases}\dfrac{i^2}{z_{\max}}\left[\dfrac{\lambda_{ef}}{5\pi}\left(\dfrac{f_{c,0,k}}{E_{0.05}}\right)^{0.5}-0.06\right] & \text{for solid timber} \quad (33)\\[2ex] \dfrac{i^2}{z_{\max}}\left[\dfrac{\lambda_{ef}}{10\pi}\left(\dfrac{f_{c,0,k}}{E_{0.05}}\right)^{0.5}-0.03\right] & \text{for glued laminated timber} \quad (34)\end{cases}$$

Comparative static analysis of the load-bearing capacity of columns

A comparative static analysis of columns was conducted with regards to their critical load-bearing capacity, using the formulae given in EN-1995 Eurocode 5 [6] and those based on the theory presented in Timoshenko and Gere [5]. The comparative analysis was based on a value λ_{ef} described with the formulae (1), (23) and (24). The formula (1) was used to determine critical load-bearing capacity of a column without taking the influence of shearing into consideration. However, the formulae (23) and (24) were used to determine critical load-bearing capacity taking the influence of shearing into consideration.

Table 1 Static values used in the analysis of columns. Standards EN 338 [2], EN 12369-2 [3], EN 12369-1 [4]

	$f_{c,k}$	$f_{r,k}$	$f_{v,k}$	E_{mean}
	[MPa]			
Timber	21	-	4.0	11000
Plywood	17	1.2	-	4500
Fibreboard	24	2.5	-	4600
Particleboard g=20 mm	11.1	1.6	-	1700
Particleboard g=24 mm	9.6	1.4	-	1600

$f_{c,0,k}$ characteristic compressive strength of timber along the grain, $f_{r,k}$ characteristic planar (rolling) shear strength, $f_{v,k}$ characteristic shear strength, E_{mean} mean value of modulus of elasticity, g truss elements thickness truss elements made of wood, plywood and fibreboard, an additional analysis of the column with particleboard truss of the thickness of $g = 24$ mm was carried out. The analysis shows that the change in thickness significantly affects the static quantities compared. The width of the truss elements was obtained from the load-bearing capacity condition of the compressed column under buckling and the load-bearing capacity of the glued joint connecting the truss elements to the column flanges.

The values of the compressive forces in the columns were selected so that the load-bearing capacities of the columns calculated on the basis of the formulae given in EN-1995 Eurocode 5 [6] with the assumed dimensions of column elements were $n = 1$. The load-bearing capacity with the value of $n = 1$ means that the stresses in the element are equal to the stress values acceptable for the assumed wood class. Values of $n > 1$ indicate how much the load-bearing capacity is exceeded. Values of $n < 1$ indicate the value of the load-bearing capacity reserve. In order to determine how much the load-bearing capacity is exceeded and the value of the load-bearing capacity reserve, the value of the expression $(n - 1)\cdot 100\%$ needs to be calculated.

Tables 2, 3, 4, 5, 6, 7, 8 and 9 summarise the static values based on the comparative analyses. These values are the slenderness ratio λ_{ef}, instability factors k_c load-bearing capacity n, and percentage differences concerning load-bearing capacity $(n - 1)\cdot 100\%$. These values were determined on the basis of the formulae given in EN-1995 Eurocode 5 [6]

Table 2 Analysis of load-bearing capacity of columns N-truss $\lambda_{ef} = 30$

N-truss					
λ_{ef}=30	1)	2)	3)	4)	5)
		g=20 mm			g=24 mm
λ_{ef}	30	43.71	45.19	48.69	48.87	55.06
k_c	0.948	0.857	0.844	0.810	0.808	0.737
n	1.0	1.106	1.123	1.170	1.173	1.285
(n-1)·100%	0%	10.6%	12.3%	17%	17.3%	28.5%

Table 3 Analysis of load-bearing capacity of columns N-truss $\lambda_{ef} = 50$

N-truss						
λ_{ef}=50	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
λ_{ef}	50	54.51	54.47	56.25	56.34	59.66
k_c	0.796	0.744	0.745	0.723	0.722	0.680
n	1.0	1.069	1.069	1.101	1.102	1.170
(n-1)·100%	0%	6.9%	6.9%	10.1%	10.2%	17%

Table 4 Analysis of load-bearing capacity of columns N-truss $\lambda_{ef} = 100$

N-truss						
λ_{ef}=100	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
λ_{ef}	100	97.67	97.50	98.34	98.38	100
k_c	0.305	0.318	0.319	0.314	0.314	0.305
n	1.0	0.959	0.956	0.970	0.971	1.0
(n-1)·100%	0%	-4.1%	-4.4%	-3%	-2.9%	0%

Table 5 Analysis of load-bearing capacity of columns N-truss $\lambda_{ef} = 150$

N-truss						
λ_{ef}=150	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
λ_{ef}	150	143.66	143.55	144.12	144.15	145.26
k_c	0.143	0.155	0.155	0.154	0.154	0.152
n	1.0	0.921	0.920	0.927	0.927	0.941
(n-1)·100%	0%	-7.9%	-8.0%	-7.3%	-7.3%	-5.9%

Table 6 Analysis of load-bearing capacity of columns V-truss $\lambda_{ef}=30$

V-truss					
λ_{ef}=30	1)	2)	3)	4)	5)
		g=20 mm			g=24 mm
λ_{ef}	30	39.37	39.88	39.98	40.06	44.03
k_c	0.948	0.891	0.887	0.887	0.886	0.854
n	1.0	1.063	1.068	1.069	1.069	1.109
$(n$-1)·100%	0%	6.3%	6.8%	6.9%	6.9%	10.9%

Table 7 Analysis of load-bearing capacity of columns V-truss $\lambda_{ef}=50$

V-truss						
λ_{ef}=50	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
λ_{ef}	50	51.68	51.92	51.97	52.01	53.90
k_c	0.796	0.777	0.774	0.774	0.773	0.751
n	1.0	1.024	1.028	1.028	1.029	1.059
$(n$-1)·100%	0%	2.4%	2.8%	2.8%	2.9%	5.9%

Table 8 Analysis of load-bearing capacity of columns V-truss $\lambda_{ef}=100$

V-truss						
λ_{ef}=100	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
λ_{ef}	100	96.23	96.34	96.36	96.37	97.24
k_c	0.305	0.327	0.326	0.326	0.326	0.321
n	1.0	0.933	0.935	0.936	0.936	0.951
$(n$-1)·100%	0%	-6.7%	-6.5%	-6.4%	-6.4%	-4.9%

Table 9 Analysis of load-bearing capacity of columns V-truss $\lambda_{ef}=150$

V-truss						
λ_{ef}=150	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
λ_{ef}	150	142.69	142.76	142.78	142.79	143.37
k_c	0.143	0.157	0.157	0.157	0.157	0.156
n	1.0	0.909	0.910	0.910	0.910	0.918
$(n$-1)·100%	0%	-9.1%	-9.0%	-9.0%	-9.0%	-8.2%

In Tables 2–9:

Item 1): calculations of columns, based on the formulae applied as per EN-1995 Eurocode 5 [6]

Items 2)–5): calculations with the formulae based on the theory presented in Timoshenko and Gere [5], where:

2) Calculations for columns with truss made of wood

3) Calculations for columns with truss made of plywood

4) Calculations for columns with truss made of fibreboard

5) Calculations for columns with truss made of particleboard

λ_{ef} as per formulae (1), (23) and (24), k_c instability factor, n load-bearing capacity of a column, $g=20$ mm and $g=24$ mm of truss elements thickness and the formulae based on the theory given in Timoshenko and Gere [5].

Comparative static analysis for the truss calculation

The comparative static analysis was conducted of maximum shear forces V_p, caused by longitudinal compressive force P, and maximum shear forces $V_{p,\max}$ a column can carry. The comparative analysis was based on the formulae (5) and (28)–(31). The formula (5) was used to determine shear forces in the column basing on the theory given in EN-1995 Eurocode 5 [6] which does not take into consideration the influence of shearing on critical load-bearing capacity. Formulae (28)–(31) were used to determine shear forces in the column basing on the theory given in Timoshenko and Gere [5] which takes into consideration the influence of shearing on critical load-bearing capacity.

Elasticity and strength values of the materials in use are presented in Table 1. In the analyses, columns of four slenderness ratios $\lambda_{ef}=30$, $\lambda_{ef}=50$, $\lambda_{ef}=100$ and $\lambda_{ef}=150$ were considered. The calculations were carried out for truss made in variants of wood, plywood, fibreboard and particleboard. In all cases, one compressive longitudinal force P was adopted, causing maximum compressive stresses in the flanges of the analysed column with the slenderness ratio of $\lambda_{ef}=150$ with calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The maximum initial curvature of the column a and the eccentric of the force e action were determined from the formulae (33) and (34). The results of the analyses are presented in Tables 10, 11, 12, 13, 14, 15, 16 and 17.

Experimental studies

The columns load-bearing capacity formulae, derived in the paper, can be verified experimentally by conducting comparative analysis for instability factors k_c and load-bearing capacity n. A description of non-destructive tests concerning instability factor k_c is given in the author's article Śliwka [7], while the destructive tests for load-bearing capacity consist in the determination of forces destroying an element under study, and comparing them with forces established through theoretical analysis.

Results and discussion

1. The analysis concerning the load-bearing capacity of columns with N-truss shows that the smallest differences between the compared static values of λ_{ef}, k_c, n, $(n-1)\cdot 100\%$ are observed in the case of the columns with the slenderness ratio of $\lambda_{ef}=100$ with the truss made of particleboard with the thickness of $g=24$ mm, and the greatest differences are observed for the columns

Table 10 Analysis of truss elements N-truss $\lambda_{ef}=30$

N-truss						
λ_{ef}=30	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
V_p^a	0.37	0.22	0.23	0.24	0.24	0.26
V_p^e		0.14	0.14	0.15	0.15	0.16
$V_{p,\max}^a$	3.50	3.47	3.67	4.15	4.17	5.06
$V_{p,\max}^e$		1.54	1.59	1.71	1.72	1.94

Table 11 Analysis of truss elements N-truss $\lambda_{ef}=50$

N-truss						
λ_{ef}=50	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
V_p^a	0.73	0.26	0.26	0.27	0.27	0.28
V_p^e		0.16	0.16	0.16	0.16	0.17
$V_{p,\max}^a$	5.84	4.98	4.98	5.23	5.24	5.69
$V_{p,\max}^e$		1.92	1.93	1.98	1.98	2.11

Table 12 Analysis of truss elements N-truss $\lambda_{ef}=100$

N-truss						
λ_{ef}=100	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
V_p^a	2.27	0.40	0.40	0.40	0.40	0.41
V_p^e		0.21	0.21	0.21	0.21	0.21
$V_{p,\max}^a$	7.00	7.44	7.44	7.43	7.43	7.41
$V_{p,\max}^e$		4.00	3.99	4.04	4.05	4.15

Table 13 Analysis of truss elements N-truss $\lambda_{ef}=150$

N-truss						
λ_{ef}=150	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
V_p^a	4.84	0.76	0.76	0.77	0.77	0.78
V_p^e		0.26	0.26	0.26	0.26	0.26
$V_{p,\max}^a$	7.00	6.27	6.27	6.26	6.26	6.22
$V_{p,\max}^e$		6.40	6.39	6.42	6.42	6.45

Table 14 Analysis of truss elements V-truss $\lambda_{ef}=30$

V-truss						
λ_{ef}=30	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
V_p^a	0.37	0.21	0.21	0.21	0.21	0.23
V_p^e		0.13	0.13	0.13	0.13	0.14
$V_{p,\max}^a$	3.50	2.93	2.99	3.00	3.01	3.51
$V_{p,\max}^e$		1.38	1.40	1.40	1.41	1.55

Table 15 Analysis of truss elements V-truss $\lambda_{ef}=50$

V-truss						
λ_{ef}=50	1)	2)	3)	4)	5)	
		g=20 mm				g=24 mm
V_p^a	0.73	0.25	0.25	0.25	0.25	0.26
V_p^e		0.15	0.15	0.15	0.15	0.16
$V_{p,\max}^a$	5.84	4.58	4.61	4.62	4.62	4.90
$V_{p,\max}^e$		1.82	1.83	1.83	1.83	1.90

with the slenderness ratio of $\lambda_{ef}=30$ with the truss made of particleboard with the thickness of $g=24$ mm.

1.1. In the case of the columns with the slenderness ratio of $\lambda_{ef}=100$ with the truss made of particleboard with the thickness of $g=24$ mm the percentage differences between the compared values are 0%.

1.2. In the case of the columns with the slenderness ratio of $\lambda_{ef}=30$ with the truss made of particleboard with the thickness $g=24$ mm the slender-

Table 16 Analysis of truss elements V-truss $\lambda_{ef}=100$

λ_{ef}=100	1)	2)	3)	4)	5)	5)
		g=20 mm				g=24 mm
V_p^a	2.27	0.40	0.4	0.40	0.40	0.40
V_p^e		0.21	0.21	0.21	0.21	0.21
$V_{p,\max}^a$	7.00	7.46	7.45	7.45	7.45	7.44
$V_{p,\max}^e$		3.91	3.92	3.92	3.92	3.98

Table 17 Analysis of truss elements V-truss $\lambda_{ef}=150$

λ_{ef}=150	1)	2)	3)	4)	5)	5)
		g=20 mm				g=24 mm
V_p^a	4.84	0.74	0.74	0.74	0.74	0.75
V_p^e		0.26	0.26	0.26	0.26	0.26
$V_{p,\max}^a$	7.00	6.30	6.29	6.29	6.29	6.28
$V_{p,\max}^e$		6.37	6.37	6.38	6.38	6.39

In Tables 10–17:

Item 1): calculations of columns, based on the formulae applied as per EN-1995 Eurocode 5 [6]

Items 2)–5): calculations with the formulae based on the theory presented in Timoshenko and Gere [5] where:

2) Calculations for columns with truss made of wood

3) Calculations for columns with truss made of plywood

4) Calculations for columns with truss made of fibreboard

5) Calculations for columns with truss made of particleboard

V_p^a maximum shear force, caused by compressive force P, for a column with its initial curvature described with a sinusoid, V_p^e maximum shear force, caused by compressive force P, for a column, where force P acts on eccentric e, $V_{p,\max}^a$ maximum shear force a column can carry, for a column with its initial curvature described with a sinusoid, $V_{p,\max}^e$ maximum shear force a column can carry, for a column, where force P acts on eccentric e, λ_{ef} slenderness ratio for a column

$g=20$ mm and $g=24$ mm of truss elements thickness

ness ratio determined from the formulae based on the theory given in Timoshenko and Gere [5] is by 83.53% greater than the slenderness ratio determined from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The instability factor k_c determined from the formulae based on the theory given in Timoshenko and Gere [5] is by 22.26% smaller than the instability factor determined from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The load-bearing capacity determined from the formulae based on the theory given in Timoshenko and Gere [5] is exceeded by 28.5% as compared to the load-bearing capacity of $n=1$ determined from the formulae based on the theory given in EN-1995 Eurocode 5 [6].

2. The analysis concerning the load-bearing capacity of the columns with V-truss shows that the smallest differences between the compared static values of λ_{ef}, k_c, n, $(n-1)\cdot 100\%$ are observed in the case of the columns with the slenderness ratio of $\lambda_{ef}=50$ with the truss made of wood, and the greatest differences are observed in case of columns with the slenderness ratio of $\lambda_{ef}=30$ with the truss made of particleboard with the thickness of $g=24$ mm.

   2.1. For columns with the slenderness ratio of $\lambda_{ef}=50$ with the truss made of wood, the slenderness ratio determined from the formulae based on the theory given in Timoshenko and Gere [5] is by 3.37% greater than the slenderness ratio determined from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The instability factor k_c determined from the formulae based on the theory given in Timoshenko and Gere [5] is by 2.39% smaller than the instability factor determined from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The load-bearing capacity determined from the formulae given in Timoshenko and Gere [5] is exceeded by 2.4% as compared to the load-bearing capacity of $n = 1$ determined from the formulae given in EN-1995 Eurocode 5 [6].

2.2. For the columns with the slenderness ratio of $\lambda_{ef} = 30$ with the truss made of particleboard with the thickness $g = 24$ mm, the slenderness ratio determined from the formulae based on the theory given in Timoshenko and Gere [5] is by 46.77% greater than the slenderness ratio determined from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The instability factor k_c determined from the formulae based on the theory given in Timoshenko and Gere [5] is by 9.92% smaller than the instability factor determined from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The load-bearing capacity determined from the formulae given in Timoshenko and Gere [5] is exceeded by 10.9% as compared to the load-bearing capacity of $n = 1$ determined from the formulae given in EN-1995 Eurocode 5 [6].

3. As a part of the analysis concerning the calculations for the N-truss and V-truss it was shown that the shear forces determined from the calculations based on the theory given in Timoshenko and Gere [5] are significantly smaller than the shear forces calculated on the basis of formulae given in EN-1995 Eurocode 5 [6]. The smallest differences between the values of these forces occur for the columns with the slenderness ratio $\lambda_{ef} = 30$ with the N-truss and V-truss made of particleboards with the thickness of $g = 24$ mm having the initial curvature described with a sinusoid. The greatest differences between the values of these forces occur in the case of columns with the slenderness ratio of $\lambda_{ef} = 150$ with the N-truss and V-truss made of wood, plywood, fibreboard and particleboard with the forces P acting at the eccentric e. In addition, it can be concluded from the analyses that the shear forces determined from the formulae based on the theory given in Timoshenko and Gere [5] do not differ in many considered cases from each other and in other cases they differ only slightly.
4. The shear forces calculated on the basis of the formulae based on the theory given in Timoshenko and Gere [5] are greater for the column with the initial curvature described with a sinusoid than in the case where the forces P act at the eccentric e.
5. It was also shown that the greatest shear forces $V_{p,\max}$ that the column can carry determined from the formulae based on theory given in Timoshenko and Gere [5] differed from the maximum shear forces derived from the calculations based on the formulae given in EN-1995 Eurocode 5 [6]. The smallest differences between the compared values in the case of the column with the N-truss occur with its slenderness ratio of $\lambda_{ef} = 30$, with the truss made of wood and the initial curvature described with a sinusoid. The smallest differences between the compared values in the case of the column with the V-truss occur with its slenderness ratio of $\lambda_{ef} = 30$, with the truss made of particleboard with the thickness of $g = 24$ mm and the initial curvature described with a sinusoid. The greatest differences between the compared values in the case of columns with the N-truss and V-truss occur with their slenderness ratio of $\lambda_{ef} = 50$, with the truss made of wood and with the forces P acting at the eccentric e. In addition, it can be concluded from the analyses that the maximum shear forces that the column can carry, determined from the formulae based on the theory given in Timoshenko and Gere [5] differ slightly in many cases.

Conclusion

1. The comparative static analysis of the column shows that the applied calculation method for the columns according to EN-1995 Eurocode 5 [6] and the method based on the theory given in Timoshenko and Gere [5] lead to partially divergent results.
2. In many cases considered, there are significant differences between the compared static values. Assuming that the presented theory given in Timoshenko and Gere [5] is correct and accurate, it can be stated that the calculation of columns based on the formulae used so far and given in the literature (EN-1995 Eurocode 5 [6]) may lead to significant design errors. Therefore, the author suggests that in the calculation of the columns, the formulae based on the theory given in Timoshenko and Gere [5] be used. The formulae presented in the paper are of practical importance in analyses concerning timber constructions.
3. The shear forces in the column depend directly on the initial curvature of the column described by the sinusoid a and alternatively on the eccentric e of forces application. Therefore, the author proposes to create the function of $a = f(\lambda_{ef})$, $e = f(\lambda_{ef})$ dependant on the slenderness ratio λ_{ef} of the analysed column described by the formulae (33) and (34).
4. It has been shown in the paper that in the analysis concerning the load-bearing capacity of columns, the largest differences between the compared values occur in the case of the columns with low slenderness ratios.

5. Browsing through the suitable literature the author came across the following articles concerning the buckling of lattice columns Timoshenko and Gere [5], Neuhaus [1], Porteous and Kermani [14], Mijailovic [15], Li and Li [16], Miller and Hedgepeth [17], Guo and Wang [18].
6. The static analysis of two-shaft columns spaced by gussets is given in the author's article Śliwka [12].

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