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ORIGINAL ARTICLE

Application of the vibration method with additional mass to timber guardrail beams

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Abstract

This study examined whether or not weight, density, and Young's modulus can be accurately measured by vibration test without weighing beams for timber guardrails. Bending vibration tests with and without the concentrated mass were performed on small clear round bars without the pith of spruce and cedar and on actual size round bars with the pith of cedar for the timber guardrail. The following results were obtained. The vibration method with additional mass could be applied to the round bar as well as the rectangular bar. It is possible that resonance frequency was decreased by the sawn split in the horizontal tapping. It is believed that the free ends condition is easier to realize than the fixed ends condition for cross beams for timber guardrails. The weight of the cross beam for the timber guardrail could be accurately estimated by the vibration method with additional mass under several testing conditions.

Keywords Beam · Bending vibration · Cross section · Timber guardrail · Vibration method with additional mass

Introduction

Since establishment of the "Protection Fence Installation Standard" by the Ministry of Construction in 1998, there has been an increase in the examples of installation of timber guardrails as road facilities [1, 2]. For safety control, a simple investigation method to assess the deterioration of the cross beam is required, and the flexural vibration test is expected to be promising. However, it is a difficult task in practice to measure the weight by removing the cross bar from the support in the investigation. The size of the rod reaches a diameter of 200 mm and a length of 2 m, and the weight is 25 kg. It is necessary to take a weighing scale for the test in the field.

Adding mass to a bar decreases its resonance frequencies due to longitudinal and bending vibrations. Using this approach, the weight, the density, and the Young's modulus of a bar can be calculated without weighing it [3–8]. This testing method is referred to as the vibration method with additional mass in this study, and this method enables to simply obtain the properties of each piled lumber and each beam of timber guardrails.

Several test conditions have been investigated to apply the vibration method with additional mass to actual cases. For example, the suitable mass ratio (additional mass/specimen) and the connection way between the additional mass and the specimen [9], the effect of the crosser's position used for piled lumber on longitudinal vibration [10], the effect of moisture content on the estimation accuracy of the vibration method with additional mass [11], and the effect of a method for generating bending vibration on the accuracy of the vibration method with additional mass [12] have been investigated.

The purpose of this study is to obtain basic data for applying the vibration method with additional mass to the cross beam of the timber guardrail. For this purpose, we examined whether or not the weight of a small clear round bar can be accurately obtained from the vibration method with additional mass because the vibration method with additional mass has not been applied to round bars. Then, the weight of

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an actual size round bar for the timber guardrail with knots and several processing obtained from the vibration method with additional mass was investigated.

Vibration method with additional mass

In the case of a thin beam, the effects of shear deflection and rotary inertia involved in the bending vibrational deflection are negligible, and the Euler–Bernoulli elementary theory of bending can be applied to the vibration.

The Young's modulus using the bending vibration E of a rectangular bar with length l is expressed as follows:

$$E = \left(\frac{l}{m_n}\right)^2 \frac{\rho A}{I} \omega_n^2, \tag{1}$$

where ρ, ω, A, and I are the density, angular frequency ($\omega = 2\pi f$, f: resonance frequency), cross-sectional area, and the moment of inertia of the cross section, respectively. The value of m_n is a constant obtained from the following frequency Eq. (2).

The frequency equation for the free–free bending vibration with concentrated mass M placed at $x = al$ (x: distance along the bar, $0 \le a \le 1$, $a + b = 1$) of a rectangular bar (Fig. 1) is expressed as follows [12]:

$$\begin{aligned}(\cos m_n \cosh m_n - 1) - \frac{1}{2}\mu m_n\{&(\cos a m_n \cosh a m_n + 1)\\ &(\sin b m_n \cosh b m_n - \cos b m_n \sinh b m_n) + (\cos b m_n \cosh b m_n + 1)\\ &(\sin a m_n \cosh a m_n - \cos a m_n \sinh a m_n)\} = 0,\end{aligned} \tag{2}$$

where μ is the ratio of the concentrated mass to the mass of the bar and is written as:

$$\mu = \frac{M}{\rho A l}. \tag{3}$$

The suffix n is the resonance mode number.

If $\mu = 0$, Eq. (2) becomes

$$\cos m_{n0} \cosh m_{n0} - 1 = 0, \tag{4}$$

where the suffix 0 represents the value without the concentrated mass.

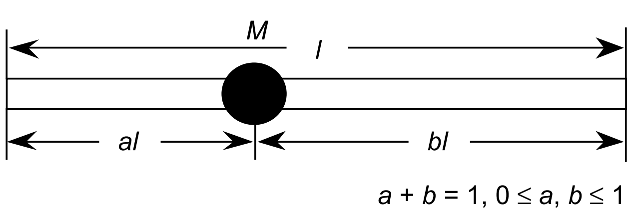


Fig. 1 A beam with additional mass

For a bar without a concentrated mass, Eq. (4) gives

$$m_{10} = 4.730, \quad m_{20} = 7.853, \quad m_{30} = 10.996, \quad m_{n0} = \frac{1}{2}(2n + 1)\pi \ (n > 3). \tag{5}$$

The density and the Young's modulus are the same before and after the concentrated mass is bound to a specimen. Thus, using Eq. (1) results in:

$$m_n = \sqrt{\frac{f_n}{f_{n0}}} m_{n0}. \tag{6}$$

The value of μ can be calculated by substituting m_n from Eq. (6) into Eq. (2). The weight and density can be obtained by substituting the calculated μ, the concentrated mass, and the dimensions of a bar into Eq. (3). The Young's modulus can be calculated by substituting the density from Eq. (3) and the resonance frequency without the concentrated mass into Eq. (1) [3–8]. This procedure is referred to as the "vibration method with additional mass" in this study.

Materials and methods

Specimens

Sakhalin spruce (*Picea glehnii* Mast.) and Japanese cedar (*Cryptomeria japonica* Do. Don) were used as the sample specimens in this study. Small clear round bars without the pith of spruce and cedar and actual size round bars with the pith of cedar for the timber guardrail were made. The small clear round bars had a diameter of 30 mm and a length of 300 mm, and the actual size round bar had a diameter of 200 mm and a length of 1980 mm. Two small clear round bars were made for each wood species, and three actual size round bars were made for cedar. They were conditioned at 20 °C and 65% relative humidity. The tests for the small round bar were conducted under the same conditions, and those for the large round bars were carried outdoors.

Cross beam for timber guardrail

The timber guardrail used in this study was based on the type of Wood Technological Association of Japan shown in Fig. 2 and was located in Forestry and Forest Products Research Institute. The round bar for the timber guardrail was attached to the post through a steel flat plate using a bolt of 16 mm in diameter at a position of 125 mm from both ends. In the round bar for the timber guardrail, a 20-mm diameter through hole for bolt fastening was drilled at 125 mm from both ends, and a hole of 45 mm in diameter and 25 mm in depth for the nut was opened at the tip of the through hole. In

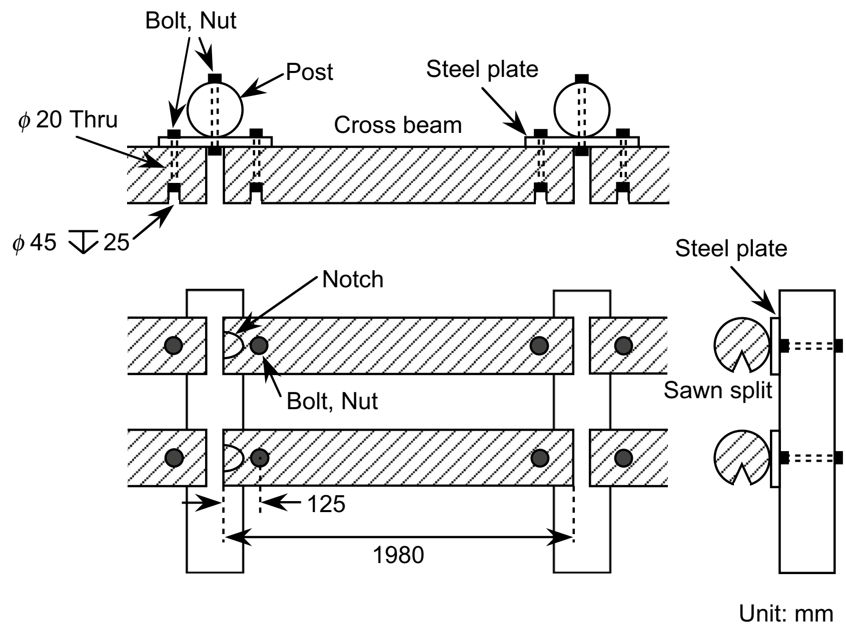


Fig. 2 Cross beam for the timber guardrail

addition, a round bar for the timber guardrail was provided with a sawn split, and a notch for alleviating the impact during a vehicle collision was formed at one end.

Bending vibration test

Small clear round bar

The bending vibration tests for the small clear round bars were conducted under the free–free condition with and without the concentrated mass by the following procedure. An iron wood screw (3.87 g) having a diameter of 3 mm and a length of 60 mm was used as the concentrated mass, and it was inserted into the bottom surface of the small clear round bar at $x = 0.5\ l$. The test bar was suspended by two threads at the nodal positions of free–free vibration corresponding to its first resonance mode, and then the bending vibration was generated by tapping the specimen in the vertical direction at $x = 0.5\ l$ using a wooden hammer, while the bar motion was detected by a microphone at $x = 0.5\ l$. The direction of the microphone was the same as that of tapping. The signal was processed through the fast Fourier transform (FFT) digital signal analyzer to yield high-resolution resonance frequencies (Fig. 3). The values of μ were 0.0420–0.0449 as calculated using the masses of the concentrated mass and the specimen.

Actual size round bar

The bending vibration tests for the actual size round bars were conducted under the following three end conditions (boundary conditions) with and without the concentrated mass. The iron concentrated mass for the actual size round bar is shown in Fig. 4 and it was attached on the top and side surfaces of the round bar at $x = 0.5\ l$ using a rubber two-sided adhesive tape and four wood screws. Since the concentrated mass dropped from the specimen, it was impossible to attach the concentrated mass to the side of the specimen using the two-sided adhesive tape.

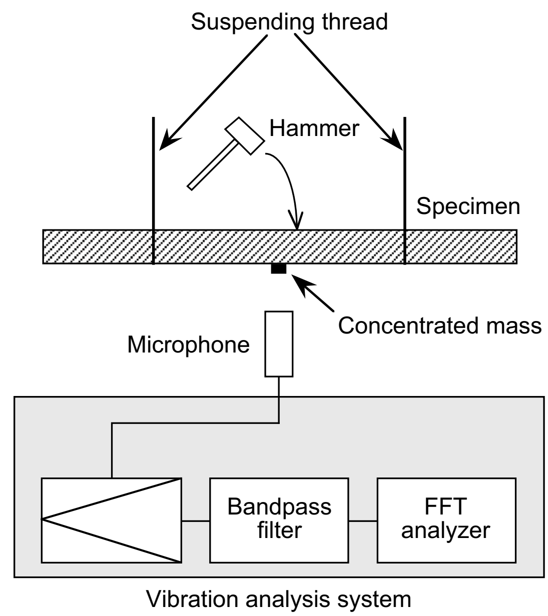


Fig. 3 Schematic diagram of the experimental setup of the bending vibration tests for the small clear round bar. The vibration analysis system consisted of an amplifier, a bandpass filter, and a fast Fourier transform (FFT) analyzer

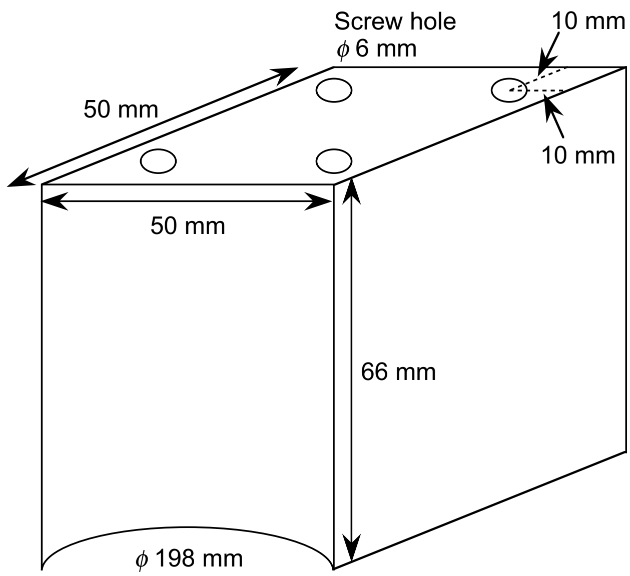


Fig. 4 The concentrated mass for the actual size round bar

The first end condition was the above mentioned free–free condition. The test bar was suspended by two threads at the nodal positions (444 mm from the end of the test bar) of the free–free vibration corresponding to its first resonance mode (Fig. 5a). An iron round bar with a diameter of 16 mm and a length of 500 mm was penetrated through two holes drilled in the actual size round bar, and the iron round bar and the wooden round bar for the timber guardrail were supported by the jig under the second end condition (Fig. 5b). The specimen was attached loosely or tightly to the post of the timber guardrail under the third end condition. The nut was loosely tightened using fingers to the bolt connecting the cross beam of the timber guardrail and the steel plate, whereas it was tightened as much as possible using a ratchet wrench to the bolt (Fig. 5c). The second end conditions was investigated as the model with loosened screw completely.

The bending vibration was generated by the vertical and horizontal tapping at $x = 0.5\ l$ using an iron hammer, while the bar motion was detected by a microphone at $x = 0.5\ l$. The direction of the microphone was the same as that of tapping. The signal was processed through the FFT digital signal analyzer to yield high-resolution resonance frequencies (Fig. 5).

The weight of the concentrated mass and the two-sided adhesive tape was 1282 g, and that of the concentrated mass and the four wood screws was 1310 g. The values of μ were 0.0519–0.0580 (two-sided adhesive tape) and 0.0531–0.0592 (four wood screws) as calculated using the masses of the concentrated mass and the specimen.

Results and discussion

The density obtained using the weight and volume of the specimen and the Young's modulus using free–free longitudinal vibration without the concentrated mass are shown in Table 1. The estimation accuracy of the vibration method with additional mass are expressed by the ratio of the weight calculated by the vibration method with additional mass to that measured by a platform scale.

Small clear round bar

The estimation accuracies of the vibration method with additional mass were 0.92 and 0.96 for spruce, and 0.92 and 0.98 for cedar. Hence, the vibration method with additional mass could be applied to round bars as well as the rectangular bars, as described in previous studies as well [5, 7, 9–12].

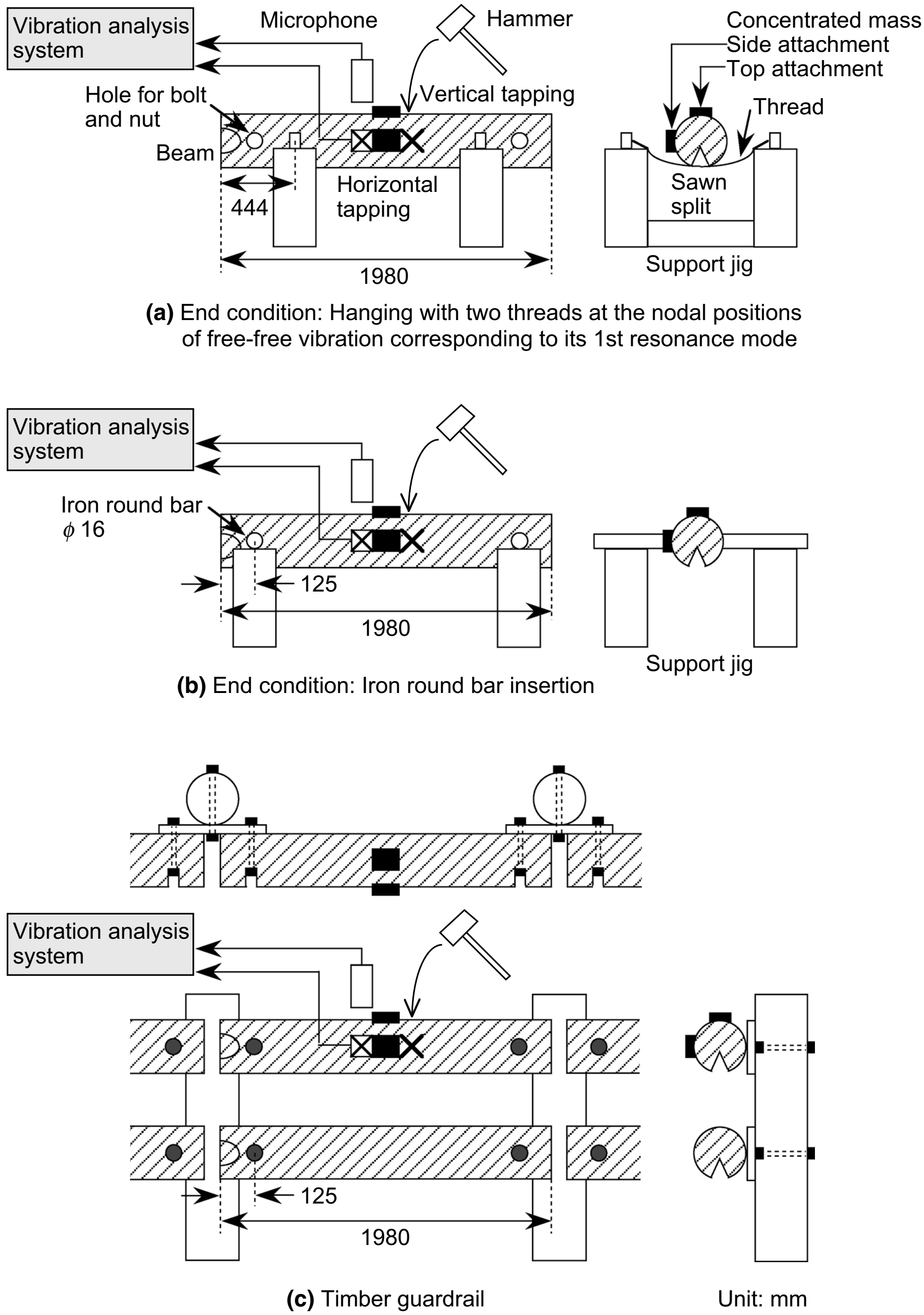
Actual size round bar

Results without the concentrated mass

Table 2 shows the results without the concentrated mass. First of all, the end condition of "hanging with two threads" will be discussed as the condition closest to the free ends. The resonance frequency in the horizontal tapping was lower than that in the vertical tapping for the three specimens. Regarding the cause of the change in the resonance frequency, the influence of the sawn split and the heterogeneity of the specimen may be considered; however, since the tendencies of all specimens were similar, the influence of the sawn split common to all specimens is considered to be stronger.

Next, the effect of the various end conditions on the resonance frequency will be discussed. Regarding the vertical tapping, the significant change in the resonance frequency was not observed for all specimens under the end conditions of hanging with two threads, iron round bar insertion, and loose attachment to the post. In other words, the resonance frequencies measured under the end conditions of iron round bar insertion and loose attachment to the post were similar to that under the end condition closest to the free ends. This shows that the influences of the iron round bar with a diameter of 16 mm and the bolt of the timber guardrail were small since they acted as a "point" in the vibration direction. On the other hand, the resonance frequency increased with the tight attachment to the post. This was because the end condition shifted from free ends to fixed ends, and this trend was similar to that reported in the previous studies [13, 14]; however, it was far from the perfect fixed ends condition. The resonance frequency under the free ends and that under the fixed ends condition

Fig. 5 Schematic diagram of the experimental setup of the bending vibration tests for the actual size round bar



are both expressed by Eq. (1). For free ends, l in Eq. (1) is the specimen length, and for fixed ends, it is the distance between the two holes for the bolt. Since the length of the actual size round bar is 1980 mm and the distance between the two holes for the bolt is 1730 mm, the resonance frequency under the fixed ends can be estimated by multiplying the measured resonance frequency under the free ends by $(1980/1730)^2$. Hence, the tight attachment to the post is semi-rigid. It is difficult to mathematically express the semi-rigid. Since the vibration method with additional mass is based on the assumption that the end condition can be clearly expressed mathematically, we believe that the tight attachment to the post that is semi-rigid should not be used. It is believed that the free ends condition is easier to realize than the fixed ends condition for testing cross beams for timber guardrails in the field.

Regarding the horizontal tapping, because vibration waveforms became flat for all specimens and the peak was

Table 1 The density obtained using the weight and volume of the specimen and the Young's modulus using free–free bending vibration without the concentrated mass

Specimen	Density (kg/m^3)	Young's modulus (GPa)
Small clear round bar		
Spruce1	417	11.38
Spruce2	418	11.13
Cedar1	429	7.16
Cedar2	400	7.06
Actual size round bar		
Cedar3	409	7.30
Cedar4	374	7.09
Ceadr5	394	8.82

unclear, the resonance frequency could not be measured for all specimens when the iron round bar was penetrated. The significant change in the resonance frequency was not observed in the other end conditions (hanging with two threads, and loose and tight attachments to the post). It is believed that the vibration was restrained because the iron round bar with a diameter of 16 mm acted as a "line" in the vibration direction. In contrast to this, the bolts of the timber guardrail were mounted in a cantilevered state on the post. Details are subject for future study.

From the results of vertical and horizontal vibrations, the estimation accuracy of the vibration method with additional mass was expected to be high except for the testing condition of the "tight attachment to the post—vertical vibration".

Results with the concentrated mass

Table 3 shows the results with the concentrated mass. Testing conditions are shown in parentheses.

The estimation accuracy of the vibration method with additional mass was expected to be high except for the "tight attachment to the post—vertical vibration" condition, i.e., the following three testing conditions of "top attachment—tape fastening—tight attachment to the post—vertical vibration [testing condition: (4,1)]", "top attachment—wood screw fastening—tight attachment to the post—vertical vibration [testing condition: (8,1)]", and "side attachment—wood screw fastening—tight attachment to the post—vertical vibration [testing condition: (12,1)]" as described above. However, it was not efficiently high excluding the following five conditions of "top attachment—tape fastening—hanging with thread—vertical tapping [testing condition: (1,1)]", "top attachment—tape fastening—loose attachment to the post—vertical tapping [testing condition: (3,1)]", "top attachment—wood screw tightening—hanging with thread—vertical tapping [testing condition: (5,1)]", "top attachment—wood screw tightening—loose attachment to the post—vertical tapping [testing condition: (7,1)]", and "side attachment—wood screw tightening—hanging with thread—horizontal tapping [testing condition: (9,2)]". The low estimation accuracy is considered to be due to the semi-rigid end condition, imperfect round bar and sliding of the concentrated mass.

Regarding that the specimen was not a perfect round bar, the testing condition (5,1) that is closest to the testing condition for the small clear specimen will be discussed. Since the estimation accuracy under testing condition (5,1) was

Table 2 Resonance frequency (Hz) measured without the concentrated mass of the actual size round bar

Specimen	End condition	Vertical vibration	Horizontal vibration
Cedar3	Hanging with two threads	189.200	185.400
	Iron round bar insertion	188.000	–
	Loose attachment to the post	191.650	180.250
	Tight attachment to the post	209.840	185.680
	Fixed ends	247.833	242.855
Cedar4	Hanging with two threads	192.800	186.200
	Iron round bar insertion	195.360	–
	Loose attachment to the post	195.200	180.820
	Tight attachment to the post	228.460	186.220
	Fixed ends	252.549	243.903
Cedar5	Hanging with two threads	213.000	207.160
	Iron round bar insertion	210.480	–
	Loose attachment to the post	213.280	200.580
	Tight attachment to the post	230.280	204.660
	Fixed ends	279.009	271.359

Resonance frequency under fixed ends condition was estimated based on the measured frequency under free ends condition

Table 3 Results with the concentrated mass for the actual size round bar

Attaching surface of the concentrated mass	Attaching method of the concentrated mass	End condition	Specimen	Vertical vibration	Horizontal vibration
Top	Two-sided adhesive tape	Hanging with two threads	Cedar3	0.89 (1,1)	1.45 (1,2)
			Cedar4	0.91 (1,1)	− 4.37 (1,2)
			Cedar5	1.06 (1,1)	− 5.96 (1,2)
Top	Two-sided adhesive tape	Iron round bar insertion	Cedar3	1.04 (2,1)	− (2,2)
			Cedar4	0.72 (2,1)	− (2,2)
			Cedar5	1.20 (2,1)	− (2,2)
Top	Two-sided adhesive tape	Loose attachment to the post	Cedar3	1.02 (3,1)	− 3.40 (3,2)
			Cedar4	1.08 (3,1)	− 458.80 (3,2)
			Cedar5	1.06 (3,1)	− 5.40 (3,2)
Top	Two-sided adhesive tape	Tight attachment to the post	Cedar3	1.99 (4,1)	− 2.71 (4,2)
			Cedar4	2.35 (4,1)	− 7.96 (4,2)
			Cedar5	1.78 (4,1)	− 10.25 (4,2)
Top	Wood screw	Hanging with two threads	Cedar3	0.99 (5,1)	0.81 (5,2)
			Cedar4	0.95 (5,1)	0.57 (5,2)
			Cedar5	1.05 (5,1)	0.51 (5,2)
Top	Wood screw	Iron round bar insertion	Cedar3	1.37 (6,1)	− (6,2)
			Cedar4	0.72 (6,1)	− (6,2)
			Cedar5	1.26 (6,1)	− (6,2)
Top	Wood screw	Loose attachment to the post	Cedar3	1.07 (7,1)	0.94 (7,2)
			Cedar4	1.07 (7,1)	0.63 (7,2)
			Cedar5	0.96 (7,1)	0.53 (7,2)
Top	Wood screw	Tight attachment to the post	Cedar3	0.56 (8,1)	0.87 (8,2)
			Cedar4	0.55 (8,1)	0.74 (8,2)
			Cedar5	0.77 (8,1)	0.59 (8,2)
Side	Wood screw	Hanging with two threads	Cedar3	0.84 (9,1)	1.03 (9,2)
			Cedar4	0.69 (9,1)	0.87 (9,2)
			Cedar5	− 0.55 (9,1)	0.97 (9,2)
Side	Wood screw	Iron round bar insertion	Cedar3	0.96 (10,1)	− (10,2)
			Cedar4	0.55 (10,1)	− (10,2)
			Cedar5	− 0.52 (10,1)	− (10,2)
Side	Wood screw	Loose attachment to the post	Cedar3	0.79 (11,1)	0.97 (11,2)
			Cedar4	0.54 (11,1)	0.80 (11,2)
			Cedar5	− 0.53 (11,1)	1.24 (11,2)
Side	Wood screw	Tight attachment to the post	Cedar3	0.41 (12,1)	0.95 (12,2)
			Cedar4	0.30 (12,1)	1.19 (12,2)
			Cedar5	− 0.48 (12,1)	1.46 (12,2)

The estimation accuracy of the vibration method with additional mass are expressed by the ratio of the weight calculated by the vibration method with additional mass to that measured by a platform scale. Testing conditions are shown in parentheses

efficiently high, the effects of the sawn split, the holes for the bolts, and the notch were small for the vibration method with additional mass.

Regarding the semi-rigid condition and sliding of the concentrated mass on the surface of the specimen, the following four testing conditions of "top attachment—vertical tapping [testing condition: (1,1), (2,1), (3,1), (4,1), (5,1), (6,1), (7,1), (8,1)]", "top attachment—horizontal tapping [testing condition: (1,2), (3,2), (4,2), (5,2), (7,2)", (8,2)], "side attachment and vertical tapping [testing condition: (9,1), (10,1), (11,1). (12,1)]", and "side attachment and horizontal tapping [testing condition: (9,2), (11,2), (12,2)]" will be discussed.

1. Top attachment and vertical tapping [testing condition: (1,1), (2,1), (3,1), (4,1), (5,1), (6,1), (7,1), (8,1)]

   For the "tight attachment to the post [testing condition: (4,1), (8,1)]", the estimation accuracy of the vibration method with additional mass was low because the end conditions was semi-rigid as the result without the concentrated mass.

The estimation accuracy was efficiently high under the testing conditions of "top attachment—tape fastening—hanging with thread—vertical tapping [testing condition: (1,1)]", "top attachment—tape fastening—loose attachment to the post—vertical tapping [testing condition: (3,1)]", "top attachment—wood screw tightening—hanging with thread—vertical tapping [testing condition: (5,1)]", and "top attachment—wood screw tightening—loose attachment to the post—vertical tapping [testing condition: (7,1)]". There was no force acting to slide the concentrated mass horizontally from the specimen. In these cases, it is obvious that the concentrated mass does not slide in the vertical direction. Therefore, the estimation accuracy was the highest.

The estimation accuracy under the testing conditions of "top attachment—tape fastening—iron round bar insertion—vertical tapping [testing condition: (2,1)]" and "top attachment—wood screw tightening—iron round bar insertion—vertical tapping [testing condition: (6,1)]" was not so high. The iron round bar was not mounted in a cantilevered state on the post as the bolts of the timber guardrail. This may relate to the low accuracy and this is an interesting subject for future study.

2. Top attachment and horizontal tapping [testing condition: (1,2), (3,2), (4,2), (5,2), (7,2), (8,2)]

   There is a possibility that the concentrated mass slips horizontally with respect to the specimen. Therefore, the estimation accuracy was considered to be low.

3. Side attachment and vertical tapping [testing condition: (9,1), (10,1), (11,1), (12,1)]

   For the "side attachment—wood screw tightening—tight attachment ot the post—vertical vibration [testing condition: (12,1)]", the estimation accuracy of the vibration method with additional mass was low because the end conditions was semi-rigid as the result without the concentrated mass.

   Even if the specimen is not vibrating, there is a possibility that the concentrated mass slides off the specimen due to the action of gravity. Furthermore, there is a possibility that the concentrated mass will slide off from the specimen even by vertical vibration of the specimen. Therefore, the estimation accuracy was low under the testing conditions of (9,1), (10,1), (11,1), and (12,1).

4. Side attachment and horizontal tapping [testing condition: (9,2), (11,2), (12,2)]

   Even if the specimen is not vibrating, there is a possibility that the concentrated mass slides off the specimen due to the action of gravity. On the other hand, the horizontal vibration of the specimen is not heavily involved in sliding down the concentrated mass from the specimen. Therefore, the estimation accuracy was lower than that with (1), and higher than that with (2) and (3), and there was a condition with efficient high accuracy [testing condition: (9,2)].

Conclusions

The applicability of the vibration method with additional mass to the beam of timber guardrail was examined. The following results were obtained:

1. The vibration method with additional mass could be applied to the round bar.
2. It is possible that the resonance frequency was decreased by the sawn split.
3. It is believed that the free ends condition is easier to realize than the fixed ends condition for the cross beams for the timber guardrails.
4. The testing conditions of the vibration method with additional mass to accurately estimate the weight of the cross beam for the timber guardrail could be found.

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