Theory to estimate the stress due to moisture in wood using a transmissiontype microwave moisture meter

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Abstract In recent years a microwave transmission-type moisture meter has been developed in Japan. Its purpose is to measure the average moisture content of thick woods. Since its development I have realized that there is a negative correlation between the moisture content of wood and the power voltage of the meter. This realization suggests that an invisible stress has an effect on the attenuation constant of the wood. The presence of such a stress in the wood could easily be proven by the slicing technique. In this article a theory is presented to explain further the effect of this stress on the attenuation constant. The theory was applied to softwood specimens in various states of moisture. It was concluded that the calculated strain distributions of the various specimens approximated those of the experimental results. Thus, the proposed theory presented herein has validity or adaptability with regard to qualitatively understanding the stress. Future research efforts would also be expected to detect the stress in wood due to moisture.

Key words Moisture content \cdot Microwave moisture meter \cdot Estimation of stress \cdot Stress distribution \cdot Detection of stress

Introduction

A new type of moisture meter utilizing the transmission power of microwaves¹ appeared in Japan several years ago. It is generally used to measure the average moisture content of a thick board or lumber below the fiber saturation point. However, the meter also seems to have the capacity to measure moisture distribution in wood, similar to X-ray scanners.

There have been many reports on the uses of X-ray computed tomography (CT) in the field of wood drying. Early in 1985 Hattori and Kanagawa² conducted an experiment that measured the moisture distribution of tropical woods with a medical X-ray CT scanner. Lindgren³ studied X-ray absorption coefficients and the CT numbers of woods. Antti and Perré⁴ suggested the possibility of moisture content profiling of wood during drying by their scanner, and we⁵ reported the CT images of the moisture distribution of softwood lumbers based on the data from two-dimensional projection of the microwave. In the aforementioned study it must be noted that the accuracy of the obtained images was not perfect, especially when the moisture content was above the fiber saturation point, but it increased up to 1%-2% at around 10% moisture content. During the course of this study I recognized the negative correlations between the moisture content of wood and the degree of attenuation of the microwave and thus formulated a theory based on this realization. My theory is presented here, and its validity or adaptability is investigated through a comparison of the calculated and observed results.

Theory

Absorption of microwaves by wood

According to the physical principle, the intensity of a microwave decreases with its entry into a medium. This phenomenon can be expressed by the equation

$$I = I_{o} \exp\left(-\int \mu ds\right) \tag{1}$$

where I_{o} and I are the intensities of the microwave prior to reaching the medium and immediately after passing it, respectively; μ is the attenuation constant; and *s* is the passage of the microwave. Taking the logarithm of Eq. (1) yields

$$p = \ln(I_o/I) = \int \mu ds \tag{2}$$

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where p is the projection datum. Generally, μ depends on the properties of the medium, and its value changes with the variation in local properties of this passage. Equation (2) can also be expressed as

$$p = \underline{\mu}l \left(\underline{\mu} = \int \mu \, ds / \int ds\right) \tag{3}$$

where $\underline{\mu}$ is the average of μ over the passage, and l is the total length of the passage. In the following equation $\underline{\mu}$ is replaced simply by μ , and p/l is replaced by p for convenience. Generally, wood can be assumed to be composed of dry wood and water. Taking this into consideration, μ would be composed of these components; hence

$$p = \mu = \mu_{\rm o} + \mu_{\rm w} \tag{4}$$

where μ_o and μ_w are the attenuation constants of dry wood and water, respectively. Furthermore, μ_w might be proportional to the moisture content of the wood. For the actual moisture measurment, the *p* in Eq. (4) can be replaced with the power voltage *v* of the moisture meter. Then, Eq. (4) should be rewritten as

$$v = \mu_{o} + \lambda u \quad (\lambda > 0) \tag{5}$$

where u is the average moisture content of a slight layer of the wood corresponding to the passage of the microwave. Then, the following equation can be obtained

$$u = av + b \quad (a = 1/\lambda, \quad b = -\mu_o/\lambda)$$
 (6)

where *a* and *b* are parameters corresponding to the species of wood, its density, moisture content, temperature, and so on. The linear relation between *u* and *v* in Eq. (6) is commonly recognized in the area of the moisture content that is below the fiber saturation point of the wood. However, if the wood is subjected to forces, the above linear relation would be affected to some degree by the stress due to these forces because the constant μ of the wood inevitably would change its value higher or lower. Taking this into consideration, Eqs. (4) and (5) should be modified as follows

$$p = \mu' = \mu'_0 + \mu'_w \tag{7}$$

$$v = \mu_0' + \lambda' u' \tag{8}$$

where the prime (') is the effect of the stress, so u' corresponds to the moisture content of the wood under that stress. The equation also means that the value of v given by Eq. (8) is different from that of v given by Eq. (5) even if u' = u. The reverse of this can be expressed by the equation

$$u' = a'v + b'$$
 $(a' = 1/\lambda', b' = -\mu'_o/\lambda')$ (9)

The difference of u' and u yields Eq. (10) and (11).

$$u'' = u' - u = a''v + b'' \qquad (a'' = a' - a, \ b'' = b' - b)$$
(10)

$$u' = u + u'' = (a + a'')v + (b + b'')$$
(11)

These equations suggest that the difference u'' is an index representing the effect of stress σ on the moisture content uor on the voltage v. Conversely, the moisture difference u''would be accompanied by a volume change in the wood, which would inevitably induce stress σ' within it. Then it might be that the stress σ under consideration was the same as the stress σ' itself, provided there were no external forces acting on the wood. The stress σ' would be proportional to the volume change of the wood (i.e., the amount of the moisture change u''). Thus, the strain ε corresponding to the stress σ would be proportional to the u''. These observations led to the following equation utilizing Hooke's law.

$$|\varepsilon| = \alpha |u''| = |\sigma|/E \qquad (\alpha > 0) \tag{12}$$

where α is the average shrinkage or swelling per unit change of the moisture content of the wood, and *E* is Young's modulus of the wood. Equation (12) suggests the possibility of estimating the stress σ or strain ε in the wood from the reading of the voltage v on the moisture meter. For the moisture difference, u'' is again a linear function of the voltage v, as shown in Eq. (10).

Solutions for moisture-induced stress

Although Eq. (12) includes the stress, it does not, at this stage, reveal the solution of the stress σ . To find the solution we must note here again the meaning of the moisture difference u''. If the value of u'' is always zero, the stress given by Eq. (12) is always zero. This, however, is not the solution we are pursuing and can be avoided provided the linear equations for u and u' [i.e., Eqs. (6) and (9)] intersect with each other in the u-v plane. The moisture difference u'' in this case is not always zero but finite, and it usually determines the finite value of the stress except the intersection point C. Figure 1 shows examples of such cases where fine and bold lines u and u' represent Eqs. (6) and (9), respectively. The coordinates (v_c , u_c) of the intersection point C can be easily determined from the condition u'' = 0.

$$u'' = a''v + b'' = 0; v = v_{c} = -b''/a''$$

$$u = u_{c} = av_{c} + b = u'_{c} = a'v_{c} + b'$$
(13)

Using the v_c given by Eq. (13) the u'' can be rewritten as follows

$$u'' = a''V \qquad \left(V = v - v_{\rm c}\right) \tag{14}$$

where the value of u'' depends on the parameter a'', with the variable V varying over a wide range depending on its negative and positive values. Of course the parameter a'' is composed of two parameters, a and a'. Hence, the value of u'' is also dependent on the various combinations of both a and a', as detailed in Table 1, where both a = a'and a < 0 are omitted as they are not important for our purposes.

Lines u and u' detailed in the diagrams of Fig. 1 can also be regarded as lines v and v' in the v-u plane, respectively. Hence: Fig. 1a–c. Models showing the relation between moisture content u(u') and power voltage v for a case 1, b case 2 and c case 3 in Table 1. Lines u' and u denote the application and nonapplication of stress, respectively



$$v = cu + d$$
 $(c = 1/a, d = -b/a)$ $(\sigma = 0)$ (15)

$$v' = c'u + d' \quad (c' = 1/a', d' = -b'/a') \quad (\sigma \neq 0)$$
 (16)

The intersection point C is ostensibly the same as before, but the moisture content u_c , at point C, can be determined again as follows:

$$v'' = v' - v = c''u + d'' \quad (c'' = c' - c, d'' = d' - d)$$
(17)

$$= c''(u - u_{c}) = c''U \quad (u_{c} = -d''/c'', U = u - u_{c})$$
(18)

where U can also be rewritten using the relations in Eqs. (13) and (14).

$$U = a(v - v_{\rm c}) = aV = au''/a''$$

Hence the v'' in Eq. (18) becomes

$$v'' = c'' a u'' / a'' = (1/a' - 1/a) a u'' / a'' = -u'' / a' = -c' u''$$
(18)

$$u'' = -a'v'' = -v''/c'$$
(19)

Here we consider the fact that, under pressure, polar media increase the dielectric constants with increasing pressure.⁶ This suggests that the application of stress to such a medium has the effect of increasing its dielectric constant.⁷ Thus, the value of v in our case would increase with increasing stress. This might agree with the assumption

$$\operatorname{Sign} v'' = \operatorname{Sign} \sigma \tag{20}$$

Using Eq. (19), therefore, the following relations can be deduced.

Sign
$$u'' = -\operatorname{sign} v''/c' = -\operatorname{sign} v'' = -\operatorname{sign} \sigma \quad (c' \text{ or } a' > 0)$$

Sign $u'' = -\operatorname{sign} v''/c' = \operatorname{sign} v'' = \operatorname{sign} \sigma \quad (c' \text{ or } a' < 0)$

$$(21)$$

Thus, the solution for the stress or the strain in Eq. (12) is as follows:

$$|\varepsilon| = |\sigma|/E = \alpha |u''| = \alpha |a'v''| = \alpha |v''/c'|$$
(22)

Table 1. Parameters a and a', and their effect on a'' and u''

Case	Parameters	<i>a</i> "	$u'' \ (V > 0)$	u'' (V < 0)	Types of stress
1 2	$\begin{array}{l} a > a' > 0 \\ a' > a > 0 \end{array}$	a'' < 0 a'' > 0	$u'' < 0 \\ u'' > 0$	u'' > 0 u'' < 0	Type 1 $(c' > c > 0)$ Type 2 $(c > c' > 0)$
3	a>0>a'	a'' < 0	u'' < 0	u'' > 0	Type 3 $(0 > c')$

Types of stress: see text; a'': a' - a; u'': moisture difference (u' - u)

Table 2. Types of stress due to the effect of moisture and their characteristics

Stress	Type 1 ($c' > c > 0$)		Type 2 ($c > c' > 0$)		Type 3 $(c' < 0)$	
	A	В	A	В	A	В
<i>u</i> ″	+	_	_	+	+	_
Stress σ u'	_ u'(A) <	+ < u'(B)	$^{+}_{u'(A)} <$	- u'(B)	+ u'(A) 2	-> u'(B)
Portion	In	Out	Out	In	In	Out
Wood-water state	Adsorp	tion	Early dry	ving	Late di	rying

A and B, triangular regions in Fig. 1; u'(A) and u'(B), moisture content u' within regions A and B, respectively; In and Out, inward and outward portions of each specimen; c' and c, parameters (c' = 1/a', c = 1/a); + and -, positive and negative signs of u'' or σ

$$\varepsilon = -\alpha u'' = \alpha a' v'' = \alpha v''/c' = \sigma/E \quad (a',c'>0)$$

$$\varepsilon = \alpha u'' = -\alpha a' v'' = -\alpha v''/c' = \sigma/E \quad (a',c'<0)$$
(23)

When the replacement $\alpha u'' = \beta$ is used for simplicity, Eq. (23) can be expressed more simply as:

$$\sigma = -E\beta, \ \varepsilon = -\beta \ (\beta = \alpha u'') \qquad (a' > 0)$$

$$\sigma = E\beta, \ \varepsilon = \beta \qquad (a' < 0)$$
(24)

The stress in wood is often assessed from the corresponding strain, as measured with the slicing technique (where the strain is a sort of spring-back). Accordingly, the strain in the said slicing technique ε_s is related to the usual strain ε as follows

$$\varepsilon_{s} = -\varepsilon = \beta \qquad (a' > 0) \\ = -\beta \qquad (a' < 0)$$
(25)

Types of stress and their relation to the wood-water state

As seen in Table 1 the moisture difference u'' is closely related to parameters *a* and *a'*. Therefore, the difference in u'' plays an important role in characterizing the stress σ . Then, the stress σ can be divided into three types by combining the parameters in Table 1:

Type 1:
$$\sigma = -E\beta$$
 (case 1: $c' > c > 0$, or $a > a' > 0$)
Type 2: $\sigma = -E\beta$ (case 2: $c > c' > 0$, or $a' > a > 0$)
Type 3: $\sigma = -E\beta$ (case 3: $c > 0 > c'$, or $a > 0 > a'$)
(26)

The characteristics of each type in the above can be easily derived from Eq. (26) and the relation $\beta = \alpha u''$, where the value of u''(=u' - u) varies with that of v, as seen in Fig. 1. Here the region of u'' in each case is represented by two triangular regions (A and B), noted by a positive (+) or negative (-) sign. A brief explanation follows of the main characteristics derived, as summarized in Table 2.

Type 1. The stress σ in Fig. 1a is tensile (+) in region B (-; i.e., u'' < 0) and compressive (-) in region A (+; i.e., u'' > 0), where the moisture content u'(A) is lower than the moisture content u'(B) of region B. Such a distribution of the moisture content would occur during the adsorption process of the wood. Hence, the high moisture region B denotes the outer portion of the wood. In contrast, the low moisture region A is the inner portion. The tensile stress in the outer portion and the compressive stress in the inner portion are also reasonable. It must be noted that for the process of adsorption to take place a water molecule must enter a minute hole in the wood's structure. Thus, the tensile stress would appear in the outer portion; and in reaction to this there would be compressive stress in the inner portion. Therefore, type 1 might be termed an adsorption type.

Type 2. In contrast, tensile stress occurs in low moisture region A and compressive stress in high moisture region B. These circumstances coincide with those of the wood early in the course of drying. Hence, regions A and B would be in the outer and inner portions of the wood, respectively.

Type 3. Tensile stress occurs in high moisture region A and compressive stress in low moisture region B. These circumstances are well recognized late in the course of drying. Hence, the outer and inner portions are reversed compared to those of type 2. Therefore, types 2 and 3 might be called early-drying and late-drying types, respectively. Thus, it is proved that the type of stress has a close relation to the wood-water state.

Calculations and discussion

To determine the validity of this theory, the strain is calculated as follows, and the results are compared with the experimental results. The experiments carried out are briefly described in the Appendix.

Calculation of one-dimensional stress distribution

Based on Eq. (24) it is simple to calculate the uniaxial stress distribution of the wood provided the lines u and u' in Fig. 1, Young's modulus E, and the average shrinkage α are known as well as data for the power voltage v for each specimen. Figure 2 shows plots of the data obtained for v and u' of each of the five specimens plotted against the

Fig. 2. a Voltage and b moisture distributions of each of the specimens along its width. x, abscissa from the center of the width of each specimen



Fig. 3. Comparison of the calculated strains and the observed strains with each of the specimens. ε_s and ε_s^* , observed and calculated strains, respectively; *x*, abscissa from the center of the width of each specimen

abscissa x from the center of the width of the specimen. The numeral used as part of the specimen number in Fig. 2 represents an approximate value for the average moisture content of the specimen. Further information regarding individual specimens is detailed in Table 3. The data for u' and v in Fig. 2 yields the following regression line u' for each specimen.

No. 50:
$$u' = 45.839v - 23.960^{**}$$
 (%) (early drying)
No. 14: $u' = -2.8224v + 15.711^{**}$ (late drying)
No. 18: $u' = 2.7837v + 14.447^{*}$ (adsorption)
No. 9b: $u' = -1.0423v + 11.754$ (air-dry)
No. 22: $u' = 16.962v - 5.493^{**}$ (early drying)
(27)

Where * and ** denote significance levels for the line beyond 0.05 and 0.01, respectively. In the strict sense it is

difficult to attain the regression line u. Thus a temporary line was determined from the data of nine specimens (which were dried as slowly as possible) for experiment A in Table 3.

$$u = 9.6408v + 6.0521 * *(\%) \quad (9 < u < 30) \tag{28}$$

The data from specimen 36 were excluded as its moisture content was over the fiber saturation point (approximately 30%). The data for u and v were for the middle layer of the width of each of the specimens. After this detailed process the average shrinkage α could be assumed to be as follows.⁸

$$\alpha = 0.01 \left(\%\right) \tag{29}$$

Strain ε can then be calculated using Eqs. (27) and (28).

Specimen no.	Moisture con	tent (%)	Correlation	Wood-water	
	Average	Middle	coefficient	state	
Experiment A ^a					
36	36.23	49.65	0.9519**	Drving	
22	21.66	28.41	0.9486**	Drying	
20	20.00	22.60	0.8355**	Drying	
31	17.85	17.36	-0.2081	Drying	
15	15.16	21.26	0.6153	Drying	
12	12.20	17.12	0.5239	Drying	
11	11.37	13.44	-0.3421	Drying	
4	10.79	10.63	-0.2468	Air-dry	
5	10.06	9.87	-0.2536	Air-dry	
9	9.26	10.39	-0.8810 **	Drying	
Experiment B ^b				, ,	
69	68.59	83.31	0.8691**	Drying	
50	50.22	69.84	0.9560**	Drying	
14	13.49	13.92	-0.8332 **	Air-dry	
18	17.82	17.32	0.6402*	Adsoption	

11.59

-0.3113

Table 3. Specimen numbers utilized and their regression lines between moisture content u' and power voltage v within each specific specimen

Middle, a middle slice of the cross section of each specimen

11.05

* and **, significance levels beyond 0.05 and 0.01, respectively

^a Experiment for regression

9b

^bExperiment for regression and the slicing technique

No. 50: $\varepsilon = -(36.1982)\alpha(v - 0.8291)$	(%) (type 2)	
No. 14: $\varepsilon = (-12.4632)\alpha(\nu - 0.7750)$	(type 3)	
No. 18: $\varepsilon = -(-6.8571)\alpha(v - 1.2234)$	(type 1)	<i></i>
No. 9b: $\varepsilon = (-10.6831)\alpha(\nu - 0.5337)$	(type 3)	(30)
No. 22: $\varepsilon = -(7.3212)\alpha(v - 1.5769)$	(type 2)	

where ε and v are the functions of abscissa x, where the projected point on the specimen can be expressed as $x = x_i$. Hence

$$\varepsilon = \varepsilon_i = \varepsilon(v_i), v = v_i = v(x_i)(i = 1, 2, \dots, n)$$
(31)

where *n* is the number of observed points of each of specimens 9–11; the data for $v(x_i)$ are shown in Fig. 2a. After calculating the data for strains ε , they are then transformed into strains ε_s and plotted for each specimen in Fig. 3 together with the observed strains, where ε_s^* and ε_s are the calculated and observed strains, respectively. The asterisk * of ε_s denotes also the reverse projection of the calculated ε_s against the vertical axis $y = \varepsilon_s$, as we recognize that the reverse projection is effective for yielding the normal image.

Comparison of theoretical results and experimental results

The lines in each diagram of Fig. 3, as a whole, tend to coincide. Therefore, the theory seems to be adaptable to the experimental results. However, the lines in the diagram of each specimen also exhibit a particular shape or feature. It can be noted that the lines for specimen 14 are well separated in the middle, which can be attributed to the low

accuracy of the observed strains ε_s , as the ε_s plots are scattered widely, whereas the calculated ε_s^* plots exhibit a relatively smooth alignment. For specimen 9b, the plot of ε_s shows discontinuation in the middle. The reason for this was cracking in that region and thus a lack of data. However, the calculated ε_s^* tends to agree with that of the experimental ε_s . With regard to the bottom diagram of Fig. 3, the circumstances are somewhat complicated. The average moisture content of specimen 50 was approximately 50%, so most of the calculated ε_s^* values were outside the parameters of the diagram; hence their plots are not shown. However, the calculated ε_s^* for specimen 22 (while in the early drying period) was used as a substitute for specimen 50. Despite these circumstances, the graphs are in close accord with each other. This is a rather interesting observation.

Air-dry

Overall, the ε_s^* and ε_s in each specimen could be said to be a close match to each other. The types of stress each specimen exhibits can be easily derived from the values of parameters *a* and *a'* in Eq. (27) and the subsequent information of the wood-water states described in the same equation. The results obtained are shown in parentheses at the right-hand side of Eq. (30) and at the top of each of the diagrams in Fig. 3. These results also coincide with the features of the moisture and strain distributions shown in Figs. 2 and 3.

Conclusions

It was concluded from this study that the present theory agrees with the experimental results with respect to the qualitative understanding of stress due to moisture so long as the necessary conditions are satisfied. Further research on the true line of u under no stress is needed for quantitative estimation of stress, together with more detailed and further experiments related to this subject. If such experiments were undertaken, the detection of stress due to moisture could be formulated further.

Appendix: outline of the experiments

The material used in the experiments was Japanese sugi (*Cryptomeria japonica*). As shown in Table 3, there were 15 specimens used in the study. The specimen parameters were as follows: size: $10.5 \pm 12.0 \times 10.5 \pm 12.0$ cm (cross section) $\times 20$ –45 cm (length, parallel to the fiber direction); density (air-dried): 420–440 kg/m³; pith: one near the center of the cross section; states of wood-water system: three sorts of drying, adsorption, and air-drying.

The specimens were divided into two groups: The first group (a) consisted of 10 specimens for determining a regression line between the moisture content u' and the voltage v among the specimens. The second group (b) consisted of five specimens for measuring the strains ε_s with regard to the slicing technique. The specimens of group (a) were dried intermittently in both a room and a small kiln at approximately <40°C, from their green state to the targeted moisture content (they were dried as slowly as possible). The specimens in group (b) were humidified in three ways. Specimens 50 and 69 were dried intermittently in a kiln with a humidifier, (60%–80% relative humidity) at temperatures similar to the above. Specimens 18 and 14 were humidified from the air-dried condition to the targeted moisture contents of about 18% and 14%, respectively, in desiccators filled with the saturated salt-water vapors of 93% and 75% relative humidity at room temperature. The final specimen (9b) was specimen 9 of the first group (a) that was used again 6 months after the end of the first experiment for the slicing technique. The moisture content for all the specimens was determined by the oven-drying method after cutting the specimen into small blocks or thin slices 0.5-1.0 cm thick. With the slicing technique the length was measured using a digital caliper with an accuracy of ± 0.01 mm.

The microwave moisture meter used was type MM-94L (Kawasaki Kiko). The measuring points were at 1 cm intervals along the width of the specimen. The electric field was parallel to the axial direction of the specimen; and the receiving window for projected microwaves was 4.0×4.0 cm.

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