ORIGINAL ARTICLE Open Access

Yoshitaka Kubojima^{1*}, Satomi Sonoda² and Hideo Kato¹

Abstract

A vibration test to measure the mass of a specimen without weighing it using the diference between the resonance frequency with an additional mass and that without it (vibration method with additional mass, VAM) was applied to Japanese cedar flat square lumber with dimensions of 210 mm × 135 mm × 3000 mm both with and without a pith in the process of air-drying. The longitudinal and bending vibration tests were performed on the specimens with and without an additional mass. The accuracy of VAM that was expressed by the estimated mass by VAM to the measured mass (M_{VAM}/M₀) was sufficiently high throughout the air-drying process. The accuracy of VAM for the longitudinal vibration was higher than that for the bending vibration. The moisture content based on the estimated mass by VAM was in the expected range.

Keywords Additional mass, Flat square lumber, Mass, Stack, Vibration test

Introduction

The ratio of old-age class forests to artificial forests has increased year by year in Japan. The ratio of forests over 50 years old has reached 50%. For the sustainable use of forest resources, the efective use of timber from mature artifcial forests and reforestation are important.

The age classes of the logs produced from the conifer artifcial forests have changed with time, and wood demand has infuenced the required qualities of lumber. Since the appropriate uses and efective processing methods for large-diameter logs whose supply will increase are

² Polytechnic University, 2-32-1 Ogawa-Nishimachi, Kodaira-Shi,

not established, they are not sufficiently used based on the quantitative and qualitative characteristics [\[1](#page-8-0)].

Flat square lumber is expected to be one of the uses of large-diameter logs. If the qualities of the fat square lumber are clarifed before shipping, the yield of products will improve. If the quality variation can be reduced, the added value of the fat square lumber will increase.

Young's modulus, which positively correlates with strength [\[2](#page-8-1)], is helpful for an index before shipping. Since a vibration test is a non-destructive and simple testing method for measuring Young's modulus, it is expected to be promising for classifying the fat square lumber using Young's modulus. However, Young's modulus calculation based on the vibration theory needs the density (or mass) of the fat square lumber.

Flat square lumber is usually stored as a stack. Since it is large and heavy, handling it is difficult, and determining the output of a scale is time-consuming. When the stack is dismantled, space is required to place the fat square lumber.

© The Author(s) 2024. **Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit [http://creativecommons.org/licenses/by/4.0/.](http://creativecommons.org/licenses/by/4.0/)

^{*}Correspondence:

Yoshitaka Kubojima

kubojima@ffpri.affrc.go.jp

¹ Forestry and Forest Products Research Institute, 1 Matsunosato, Tsukuba, Ibaraki 305-8687, Japan

Tokyo 187-0035, Japan

Since flat square lumber is stored as a stack, measuring the mass of each fat square lumber removed from the stack is inefficient. Therefore, the measuring method for the stacked fat square lumber is required.

The mass of a specimen can be measured without weighing the specimen using the diference between the resonance frequency with an additional mass and that without it $[3-8]$ $[3-8]$. If the mass (or density) can be obtained, Young's modulus can be calculated using the usual method shown in the following Eqs. $(1a)$ $(1a)$ and $(1b)$ $(1b)$. This method is called the "vibration method with additional mass (VAM)" in this study.

It is possible to measure the mass of the cross beam for the timber guardrail without removing the cross beam from the support using VAM $[9]$ $[9]$. This result will lead to a simple investigation method to assess the deterioration of the cross beam although the large amount of labor required for the investigation was one of the factors that hindered the widespread use of timber guardrails.

For a stacked squared lumber model and actual stacked round bars for the timber guardrail, the vibration restraint could be reduced by changing the position and materials of stickers. Consequently, the estimation accuracy of VAM was sufficiently high $[10, 11]$ $[10, 11]$ $[10, 11]$ $[10, 11]$.

In this study, the vibration method with additional mass was applied to measure the mass of each stacked flat square lumber during air-drying process.

Theory

Vibration method with additional mass

Here, longitudinal and bending vibrations under free– free conditions are considered. The effects of shear defection and rotatory inertia on the bending vibrational defection of a thin beam with a constant cross-section are negligible, and the Euler–Bernoulli elementary theory of bending can be applied to the bending vibration.

The resonance frequency, denoted by f_{n0} (*n* refers to the resonance mode number and 0 refects the value without the additional mass), can be expressed as follows:

$$
f_{n0} = \frac{m_{n0}}{2\pi l} \sqrt{\frac{E}{\rho}} \text{(Longitudinal vibration)} \tag{1a}
$$

$$
f_{n0} = \frac{1}{2\pi} \left(\frac{m_{n0}}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \text{(Bending vibration)} \qquad (1b)
$$

where *l*, *E*, *ρ*, *I*, and *A* are the specimen length, Young's modulus, density, the second moment of area, and the cross-sectional area. m_{n0} is a constant depending on vibration modes and can be expressed as follows:

$$
m_{n0} = n\pi \text{(Longitudinal)} \tag{2a}
$$

$$
m_{10} = 4.730, m_{20} = 7.853, m_{30} = 10.996,
$$

$$
m_{n0} = \frac{1}{2}(2n+1)\pi(n>3))
$$
(Bending) (2b)

The resonance frequency is experimentally reduced by attaching the additional mass while the dimensions, density, and Young's modulus are not altered. Hence, it can be said that m_{n0} changes to m_n . Thus, the resonance frequency after attaching the additional mass is expressed as follows:

$$
f_n = \frac{m_n}{2\pi l} \sqrt{\frac{E}{\rho}} \text{(Longitudinal)}
$$
 (3a)

$$
f_n = \frac{1}{2\pi} \left(\frac{m_n}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \text{(Bending)}\tag{3b}
$$

From Eqs. $(1a)$ $(1a)$, $(1b)$ $(1b)$, $(2a)$ $(2a)$ $(2a)$ and $(2b)$,

$$
m_n = \frac{f_n}{f_{n0}} m_{n0} \text{(Longitudinal)} \tag{4a}
$$

$$
m_n = \sqrt{\frac{f_n}{f_{n0}}} m_{n0} \text{(Bending)}
$$
 (4b)

The frequency equation for the free–free vibration with concentrated mass *M* placed at position *x*=*al* (*x*: distance along the bar, $0 \le a \le 1$ $0 \le a \le 1$, $a+b=1$) on a bar (Fig. 1) can be expressed as follows:

$$
\begin{aligned}\n\sin m_n + \mu m_n \cos a m_n \cosh m_n &= 0(\text{Longitudinal}) \\
(5a) \\
(\cos m_n \cosh m_n - 1) - \frac{1}{2} \mu m_n \{(\cos a m_n \cosh a m_n + 1) \\
(\sin b m_n \cosh b m_n - \cosh m_n \sinh b m_n) + (\cosh m_n \cosh b m_n + 1) \\
(\sin a m_n \cosh a m_n - \cos a m_n \sin h a m_n)\} &= 0(\text{Bending})\n\end{aligned}
$$
\n
$$
(5b)
$$

where μ is the ratio of the concentrated mass to the mass of the bar and can be defned as follows:

$$
\mu = \frac{M}{\rho A l} \tag{6}
$$

To calculate m_n , the measured resonance frequencies $(f_{n0}$ and f_n) are substituted in Eqs. ([4a](#page-1-4)) and ([4b\)](#page-1-5). The calculated m_n is substituted in Eqs. [\(5a\)](#page-1-6) and [\(5b](#page-1-7)) to calculate μ . The specimen mass and density can be obtained by substituting the calculated *μ*, concentrated mass, and dimensions of a bar in Eq. (6) (6) . The Young's modulus can be calculated by substituting the estimated density, resonance frequency without the concentrated mass and dimensions of a bar in Eqs. $(1a)$ $(1a)$ and $(1b)$ $(1b)$ [\[3](#page-8-2)–[8\]](#page-8-3). The specimen mass is not required for these calculations.

Fig. 1 Beam with an additional mass

These calculations are the VAM procedure. In this study, the estimation accuracy of VAM was expressed by the ratio of the specimen mass estimated by VAM (M_{VAM}) to the measured specimen mass $(M₀)$. The estimation accuracy of VAM is considered sufficiently high at $0.9 \leq M_{VAM}/M_0 \leq 1.1$ in this study.

The moisture content based on the mass estimated by the VAM

The moisture content based on the measured mass (MC_0) is expressed as follows:

$$
MC_0 = \frac{100(M_0 - M_{\rm OD})}{M_{\rm OD}} [\%]
$$
 (7)

where $M_{\rm OD}$ is the oven-dried mass.

The moisture content based on the mass estimated by VAM (MC_{VAM}) is expressed as follows:

$$
MC_{VAM} = \frac{100(M_{VAM} - M_{OD})}{M_{OD}} [\%]
$$
 (8)

When M_{VAM} can be written as kM_0 , using Eqs. [\(7](#page-2-1)) and ([8\)](#page-2-2), the accuracy of the moisture content based on the mass obtained by VAM (d*MC*) can be expressed as follows:

$$
dMC = MCVAM - MC0 = \frac{100(MVAM - M0)}{MOD} = \frac{100M0(k - 1)}{MOD} = (k - 1)(MC0 + 100)[%]
$$
\n(9)

Fig. 2 Schematic diagram of the experimental setup for the free–free longitudinal and bending vibration tests

From Eq. (9), when
$$
0.9 \le k \le 1.1
$$
,
-0.1MC₀ - 10 \le dMC $\le 0.1MC_0 + 10[\%]$ (10)

Materials and methods

Specimens

Japanese cedar (*Cryptomeria japonica*, D. Don) green logs with a diameter of about 400 mm were used. Five fat square lumbers with a pith with cross-sectional dimensions of 210 mm $(T) \times 135$ mm (T) and with a length of

Table 1 The initial oven-dried moisture content, density and Young's modulus in the last longitudinal vibration test

Flat square lumber	Initial moisture content [%]	Density [kg/ $m3$]	Young's modulus [GPa]
	With a pith		
5	84	415	8.09
$\overline{4}$	129	363	6.53
3	115	417	8.55
$\overline{2}$	103	531	9.85
1	182	381	7.13
	Without a pith		
5	130	430	11.52
$\overline{4}$	175	382	6.55
3	146	461	12.65
\mathfrak{D}	142	459	11.95
1	195	385	6.90

3000 mm (L) and fve fat square lumbers without a pith with cross-sectional dimensions of 210 mm $(T) \times 135$ mm (R) and with a length of 3000 mm (L) were made from the logs.

The flat square lumbers were placed in a room without controlling the temperature and relative humidity for a year until the masses became constant. The flat square lumbers were stacked in five layers on the floor in the room. The position of the stickers were the nodal positions of the free–free bending vibration corresponding to its first resonance mode as shown in Fig. [2.](#page-2-4) The stickers were composed of a Japanese cedar bar with crosssectional dimensions of 30 mm × 30 mm and a length of 500 mm, and rubber sheet in order to reduce the vibration restraint caused by the mass of the upper fat square lumbers [[10](#page-8-5)[–12](#page-8-7)].

All the following vibration tests were conducted in the room on the day the fat square lumbers were delivered and 2 weeks, 1 month, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 months after delivering. After the vibration tests, each fat square lumber was removed from the stack and its mass was measured, and then the lumber was stacked again. Flat square lumber was cut into 150 small pieces with dimensions of 40 mm \times 25 mm \times 500 mm and they were oven-dried at 105 °C.

Free–free longitudinal vibration test

The longitudinal vibration tests were conducted on the fat square lumbers with and without a steel plate, which was used as the concentrated mass (dimensions:

Fig. 3 Temporal change in moisture content based on the measured mass of the fat square lumbers during the air-drying process

70.10 mm×70.10 mm×19.05 mm, mass: 728 g) and the resonance frequency of the first mode was measured. The range of the mass ratio (measured concentrated mass/ measured specimen mass) was 0.00843–0.0237. The steel plate was bonded on the RT-plane using a set of four wood screws through the holes of the plate.

The free–free longitudinal vibration tests were conducted according to the following procedure as shown in Fig. 2 [\[11\]](#page-8-6). The test bar was placed at the nodal positions of the free–free bending vibration corresponding to its first resonance mode. The longitudinal vibration was initiated by hitting the RT-plane of the fat square lumber at one end using a hammer with an iron head, whereas the bar motion was monitored using a microphone (MI-1235, Ono-Sokki, Co., Ltd., Yokohama, Japan) at the same end. The direction of the microphone was parallel to the L-direction. The signal was processed using a fast Fourier transform (FFT) digital signal analyzer (Multi-Purpose FFT Analyzer CF-9200, Ono-Sokki, Co., Ltd., Yokohama, Japan) to obtain high-resolution resonance frequencies of the frst resonance mode. When the specimens were on the stickers at such positions, the accurate longitudinal resonance frequency of the frst mode can be obtained [[11,](#page-8-6) [13](#page-8-8)].

Free–free bending vibration test

The bending vibration tests were conducted on the flat square lumbers with and without a steel plate, which was used as the aforementioned concentrated mass and the resonance frequency of the first mode was measured. The steel plate was bonded on the RT-plane using a set of four wood screws through the holes of the plate as shown in Fig. [2.](#page-2-4)

The free–free bending vibration tests were conducted according to the following procedure $[11]$ $[11]$. The test bar was placed at the nodal positions of the free–free vibration corresponding to its first resonance mode. The bending vibration was initiated by hitting the RT-plane, the short side with a length of 135 mm and the long side with a length of 210 mm of the fat square lumbers at one end using the hammer, whereas the bar motion was monitored using a microphone at the same end. The bending vibration generated by tapping the 135 mm \times 3000 mm plane and that by tapping the 210 mm \times 3000 mm plane are referred to as "edgewise" and "fatwise", respectively in this study. The microphone's direction was adjusted so that the edgewise and fatwise bending vibrations could be detected. The signal was processed using the aforementioned FFT digital signal analyzer to obtain high-resolution resonance frequencies. According to the previous paper [\[13](#page-8-8)], the bending vibration can be generated by hitting the RT-plane.

Measurement of the oven‑dried mass

After the last vibration tests, the flat square lumbers were divided into $5 \times 5 = 25$ small bars with dimensions of about 40 mm \times 25 mm \times 3000 mm and the short small bars with dimensions of about 40 mm \times 25 mm \times 600 mm were made from the 3000 mm small bars. The short small bars were heated at 105 ◦C and the oven-dried masses were measured. Using the sum of the oven-dried mass of each short small bar, the oven-dried mass of the fat square bar was calculated, supposing that the moisture content of the short small bar, that of the small bar, and that of the fat square lumber after the last vibration test were the same [\[14](#page-8-9)].

Results and discussion

The initial oven-dried moisture content, density and Young's modulus in the last longitudinal vibration test are shown in Table [1.](#page-3-0)

Figure [3](#page-3-1) shows the temporal change in moisture content based on the measured mass of the fat square lumbers. The moisture content became stable during this experimental process.

Figure [4](#page-5-0) shows the temporal change in the accuracy of VAM. There was not a significant difference between at the initial higher moisture content stage and at the later lower moisture content stage. This tendency was observed regardless the vibration mode and did not depend on the presence or absence of a pith. This result is proper because VAM can be applied in the moisture content range of air-dried moisture content (12%) to high moisture content (134% and 238%) [[15,](#page-8-10) [16](#page-8-11)].

The factors that affect the accuracy of VAM are discussed. The accuracy of VAM depends on the constant m_n in Eqs. ([4a](#page-1-4)) and [\(4b\)](#page-1-5), that is to say, the measured resonance frequency. The constant m_n decreases monotonically with μ as shown in Fig. [5](#page-6-0). [\[5](#page-8-12), [7\]](#page-8-13). The value of μ is susceptible to m_n when the absolute value of the slope of the curve shown in Fig. 5 is small. The change in m_1 was 3.11534≥*m*1≥3.0692 in the range of 0.00843≤ μ \leq 0.0237 for the longitudinal vibration, which was calculated by solving Eq. $(5a)$ using Mathematica 12.3 J software (Wolfram Research Co., Ltd.). Consequently,

Fig. 4 Temporal change in the accuracy of VAM during the air-drying process

Table 2 Number of cases $0.9 \leq M_{VAM}/M_0 \leq 1.1$

Longitudinal vibration	Specimen with a pith	68
	Specimen without a pith	64
Bending vibration	Specimen with a pith	Edgewise 64 Flatwise 63
	Specimen without a pith	Edgewise 60 Flatwise 66

 M_{VAM} : estimated mass by the vibration method with additional mass, M_0 : measured mass

the absolute ratio of the change in m_1 to that of μ $(dm_1/d\mu)$ was 3.305. In the same way, the change in m_1 was 4.69199≥*m*1≥4.63132 in the range of 0.00843≤ $\mu \leq 0.0237$ for the bending vibration, which was calculated by solving Eq. $(5b)$. Consequently, the absolute dm_1 / $dμ$ was 3.976. Since $dm_1/dμ$ for the longitudinal vibration was smaller than $dm_1/d\mu$ for the bending vibration, it was expected that the accuracy of VAM was lower for the longitudinal vibration than for the bending vibration. However, there were more cases of $0.9 \leq M_{VAM}/M_0 \leq 1.1$ for the longitudinal vibration than the bending vibration as shown in Table [2](#page-6-1). This result was caused by the larger diference between the resonance frequency without the concentrated mass and that with it $(f_{10} - f_1)$ for the longitudinal vibration than $f_{10} - f_1$ for the bending vibration as shown in Table $3: f_{10} - f_1$ $3: f_{10} - f_1$ decreased with the decrease in f_{10} . Therefore, the longitudinal vibration is more suitable for estimating the mass of each stacked fat square lumber with the dimensions in this study. The reduction in the accuracy of VAM due to the deviation of the resonance frequency from the true value can be improved: averaging several frequencies near the true resonance frequency is efective [\[17](#page-8-14)].

Figure [6](#page-7-0) shows the moisture content based on the estimated mass by VAM. Since the result of $0.9 \leq M_{VAM}/M_0 \leq 1.1$ was obtained for almost all of the cases, the moisture content was in the range of Eq. ([10](#page-3-2)) for almost all the cases.

Fig. 5 Relationship between the constant m_1 and the mass ratio μ

Conclusions

VAM estimated the mass and moisture content of the stacked fat square lumber in the air-drying process, and the following results were obtained:

- 1) The accuracy of VAM was sufficiently high for all the processes of natural drying.
- 2) The accuracy of VAM for the longitudinal vibration was higher than the bending vibration for the fat square lumber with dimensions in this study.
- 3) The moisture content based on the estimated mass by VAM was in the expected range.

Table 3 Resonance frequency without additional mass and diference between resonance frequency without additional mass and that with it

		f_{10} [Hz]	$f_{10} - f_1$ [Hz]
Longitudinal vibration	Specimen with a pith	638.8 (74.3)	11.3(3.6)
	Specimen without a pith	675.1 (122.6)	11.5(4.2)
Bending vibration	Specimen with a pith	Edgewise 91.2 (10.2) Flatwise 59.4 (6.6)	Edgewise 2.8 (0.8) Flatwise 1.9 (0.6)
	Specimen without a pith	Edgewise 91.7 (14.4) Flatwise 60.0 (10.8)	Edgewise 2.7 (0.8) Flatwise 1.8 (0.7)

Mean (standard deviation), *f*₁₀: Resonance frequency of the first mode without an additional mass, *f*₁: Resonance frequency of the first mode with an additional mass

Fig. 6 Moisture content based on the estimated mass by VAM

Abbreviations

- VAM Vibration method with additional mass
- L Longitudinal direction
R Radial direction
- Radial direction
- T Tangential direction
- FFT Fast Fourier transform

Acknowledgements

This work was supported by MAFF Commissioned project study on "Development of a stable supply system for domestic lumber by utilizing large-diameter logs" Grant Number JPJ012040. This study was presented in part at the 41st Annual Meeting of Wood Technological Association of Japan, Fukuoka, October 2023.

Author contributions

All the authors designed the experiments. YK performed the experiments, analyzed the data, and was a major contributor in writing the manuscript. All the authors contributed to interpretation and discussed results, and read and approved the fnal manuscript.

Funding

This work was supported by MAFF Commissioned project study on "Development of a stable supply system for domestic lumber by utilizing large-diameter logs" Grant Number JPJ012040.

Availability of data and materials

All data generated or analyzed during this study are included in this published article.

Declarations

Competing interests

The authors declare that they have no competing interests.

Received: 12 June 2024 Accepted: 21 September 2024

References

- 1. Matsumura Y (2017) Wood properties and issues of sawmilling for largediameter Sugi Logs. Abstracts of Mokuzai to Mizu Kenkyukai. 1–7
- 2. Nakayama Y (1968) Studies on the dynamic Young's modulus of wooden beam (I) The dynamic Young's modulus and the bending strength. Bull Kochi Univ For 2:17–32
- 3. Skrinar M (2002) On elastic beams parameter identifcation using eigen frequencies changes and the method of added mass. Comput Mater Sci 25:207–217
- 4. Türker T, Bayraktar A (2008) Structural parameter identifcation of fxed end beams by inverse method using measured natural frequencies. Shock Vib 15:505–515
- 5. Kubojima Y, Sonoda S (2015) Measuring Young's modulus of a wooden bar using longitudinal vibration without measuring its weight. Eur J Wood Wood Prod 73:399–401
- 6. Matsubara M, Aono A, Kawamura S (2015) Experimental identifcation of structural properties of elastic beam with homogeneous and uniform cross section. Trans JSME.<https://doi.org/10.1299/transjsme.15-00279>
- 7. Kubojima Y, Kato H, Tonosaki M, Sonoda S (2016) Measuring Young's modulus of a wooden bar using fexural vibration without measuring its weight. BioResources 11:800–810
- 8. Matsubara M, Aono A, Ise T, Kawamura S (2016) Study on identifcation method of line density of the elastic beam under unknown boundary conditions. Trans JSME. <https://doi.org/10.1299/transjsme.15-00669>
- 9. Kubojima Y, Sonoda S, Kato H (2018) Application of the vibration method with additional mass to timber guardrail beams. J Wood Sci 64:767–775
- 10. Kubojima Y, Sonoda S, Kato H (2020) Mass of piled lumber estimated through vibration test. J Wood Sci 66:32
- 11. Kubojima Y, Sonoda S, Kato H (2021) Mass of piled round bar estimated through vibration tests. Wood Fib Sci 53:216–227
- 12. Kubojima Y, Sonoda S, Kato H (2017) Practical techniques for the vibration method with additional mass: efect of crossers' position in longitudinal vibration. J Wood Sci 63:147–153
- 13. Kubojima Y, Sonoda S, Kato H (2018) Practical techniques for the vibration method with additional mass: bending vibration generated by tapping cross section. J Wood Sci 64:16–22
- 14. Kubojima Y, Suzuki Y, Kato H, Kobayashi I, Matsumoto H, Ishida Y (2021) Temporal change in moisture gradient in fat square lumber produced from large-diameter log during natural drying. Wood Ind 76:540–544
- 15. Kubojima Y, Sonoda S, Kato H (2017) Practical techniques for the vibration method with additional mass: effect of specimen moisture content. J Wood Sci 63:568–574
- 16. Kubojima Y, Sonoda S, Kato H (2021) Measuring density and Young's modulus of a log through the vibration test without measuring its weight. J Wood Sci 67:6
- 17. Sonoda S, Kubojima Y, Kato H (2017) Application of the vibration method with additional mass to lumber in structural sizes—application to lumber for column use. Abstracts of the 67th Annual Meeting of Japan Wood Research Society, D17-07-1515.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional afliations.