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Measurement of the shear modulus of wood by static bending tests

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Abstract When measuring the shear modulus of wood by static bending tests, the basic theory is dependent on Timoshenko's bending theory. The shear modulus obtained by static bending is a much smaller value than that derived by other methods. We examined the applicability of Timoshenko's theory and propose an empirical equation that can derive the shear modulus properly. Three softwoods and three hardwoods were used for the tests. First, the Young's modulus and shear modulus were measured by free–free flexural vibration tests. Then the three-point static bending tests were undertaken, varying the depth/span ratios. Additionally, the bending tests were simulated by the finite element method (FEM). The shear moduli obtained by these methods were then compared. The deflection behaviors in static bending were not expressed by the original Timoshenko bending theory because of the stress distortion near the loading point. Based on the experimental results and numerical calculations, we modified the original Timoshenko bending equation. When using our modified equation the stress concentration must be carefully taken into account.

Key words Shear modulus · Static bending test · Flexural vibration test · Timoshenko's bending theory · Depth/span ratio

Introduction

During the bending of beam-shaped material, the load–deflection relation is influenced by the shear stress that occurs in the material. The effect of shear stress depends on the depth/span ratio of a beam; the effect of shear on the load–deflection relation is small when the beam has a small depth/span ratio, whereas it is marked when the depth/span ratio is large. In other words, the effect of shear stress can be controlled by varying the depth/span ratio. Several studies have measured the shear modulus of wood by static bending of the beams with various depth/span ratios.[1,2] The values of shear moduli obtained from the bending tests were smaller than those obtained by other testing methods, such as torsion tests and 45° off-axis compression tests.[3,4] In our previous work, the shear moduli obtained by torsion tests coincided well with those obtained by the 45° off-axis compression tests.[3] Therefore we think there is a serious drawback when measuring shear modulus by static bending. In this study we conducted static bending tests of several wood species and examined the method for measuring shear modulus by this manipulation.

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Theory

When measuring the shear modulus by bending, the method is dependent on Timoshenko's theory of bending, described below. When the load P is imposed at a center of the beam with a span of l, the deflection at the loading point caused by the bending moment (y_b) can be written as[5]

$$y_b = \frac{Pl^3}{48EI} \tag{1}$$

where E and I are Young's modulus and the second moment of cross-sectional area of the beam, respectively. When the beam is slender enough, the total deflection of the beam can be represented almost by (y_b). When the depth/span ratio of the beam is large, the effect of the shearing force should be taken into account. According to Timoshenko's theory, the deflection at the loading point caused by the shearing force (y_s) can be written as

$$y_s = \frac{sPl}{4GA} \tag{2}$$

where G and A are the shear modulus and cross-sectional area of the beam, respectively, and s is Timoshenko's shear factor. When the factor s is defined as the maximum/average shear stress ratio, it is 1.5 for a beam with a rectangular cross section. On the other hand, s is derived as 1.2 from calculation of strain energy.[5] Here we used the value 1.2 for the factor s. The shear force, which does not distribute homogeneously in the shearing plane, is leveled by this factor. The total flexural displacement at the center of the beam (y) is represented as

$$y = y_b + y_s = \frac{Pl^3}{48EI} + \frac{sPl}{4GA} \tag{3}$$

When the beam has a rectangular section whose depth is h, Eq. (3) can be given as

$$y = \frac{Pl^3}{48I} \cdot \left(\frac{1}{E} + \frac{s}{G} \cdot \frac{h^2}{l^2} \right) = \frac{Pl^3}{48E_s I} \tag{4}$$

where

$$\frac{1}{E_s} = \frac{1}{E} + \frac{s}{G} \cdot \left(\frac{h}{l} \right)^2 \tag{5}$$

The value of E_s, corresponding to the depth/span ratio h/l, is obtained from the P–y relation. Young's modulus (E) and the value of G/s can be obtained by varying the value of h/l.

Experiment

Specimens

Sitka spruce (*Picea sitchensis* Carr.), Western hemlock (*Tsuga heterophylla* Sarg.), akamatsu (Japanese red pine, *Pinus densiflora* D. Don), yellow poplar (*Liriodendron tulipfera* L.), shioji (Japanese ash, *Fraxinus spaethiana* Lingelsh.), and balsa (*Ochroma lagopus* Sw.) were used in the experiments. These specimens were conditioned at 20°C and 65% relative humidity before and during the tests.

Static bending tests

Beam specimens were cut with the dimensions 500 mm (longitudinal direction) × 30 mm (radial direction) × 10, 20, and 30 mm (tangential direction). Six specimens were used for each test condition. Specimens were supported by spans varying from 130 to 480 mm at 50-mm intervals. The vertical load, whose velocity was 5 mm/min, was applied to the center of the longitudinal-radial (LR) surface with a loading head whose radius was 15 mm. The load-deflection diagram was recorded by a X-Y recorder.

The value of E_s, corresponding to each h/l, was obtained from the linear segment of the load-deflection diagram. The values of E and G/s were separated from the $1/E_s$–h/l relations by the method of least squares.

Flexural vibration tests

Free–free flexural vibration tests are often used to measure shear modulus because this method has been established as an acceptable measuring method for this parameter. In our study the vibration tests were conducted as follows. The test beam was suspended by two threads at the nodal positions of the free–free vibration corresponding to its resonance mode. The specimen was excited in the direction of the thickness at one end by a hammer. The resonance frequencies whose mode was from the first to the fourth were measured by the fast frequency transform (FFT) digital signal analyzer. Young's modulus and the shear modulus were obtained from the Timoshenko-Goens-Hearmon method, whose details are described in several previous papers.[6] Young's moduli and the shear moduli obtained from the static bending tests and vibration tests were compared with, and the validity of Timoshenko's bending theory was examined.

FEM simulation of static bending

Static bending tests were simulated by the finite element method (FEM). By comparing simulation results with experimental ones, we also examined the validity of Timoshenko's boending theory.

The program used was MSC/NASTRAN Ver. 67, a library program of the Computer Center of the University of Tokyo. Figure 1 shows the finite element mesh and boundary conditions used in our study. The finite elements were divided by the dimensions 1 × 1 mm. Young's moduli in the longitudinal and tangential directions (E_L and E_T), the shear modulus (G_{LT}), and Poisson's ratio (ν_{LT}) used in the calculations are shown in Table 1. To examine the applicability of the bending theory to various species, we used the elastic moduli from an existing study for calculations independent of the experiment.[6] Supported points were variously changed in the same manner as for the static bending tests. A fixed displacement (0.5 mm = y_c) was established at the top of the center of the beam, and the nodal force at the displaced point (P) was obtained. E_s, corresponding to the depth/span ratio, was obtained by substituting the values for y_c and P into Eq. (4). The calculated results were compared with those obtained using the static bending tests.

Results and discussion

Flexural vibration tests

Table 2 shows Young's moduli and the shear moduli obtained from the vibration tests. As described in several previous reports, these values were adequate. Thus we ex-

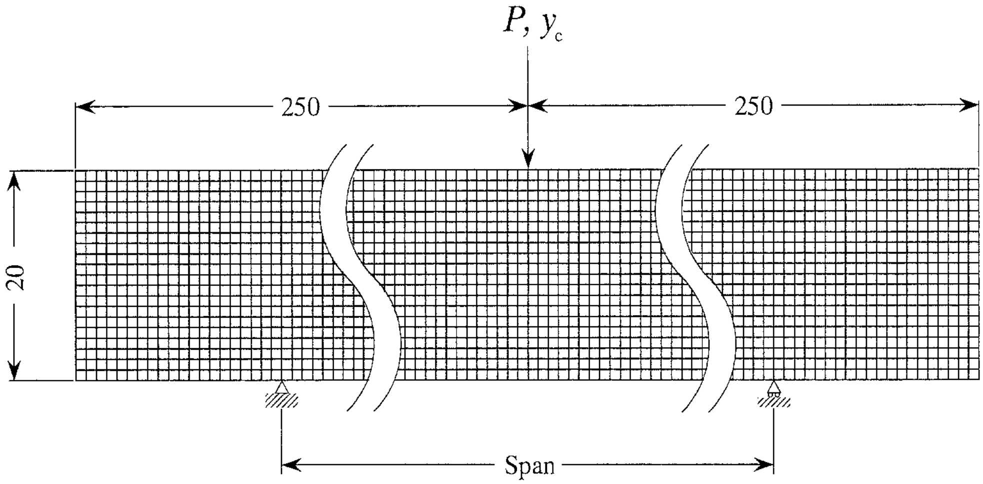

Fig. 1. Finite element mesh used for the numerical calculations (in millimeters). Meshes are uniformly divided to the dimensions of 1 × 1 mm. Span is 130 to 480 mm at 50-mm intervals

Table 1. Elastic constants used for finite element method analyses[3]

Spcies	Scientific name	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	ν_{LT}
Sitka spruce	*Picea sitchensis* Carr.	10.9	0.44	0.63	0.51
Sugi	*Cryptomeria japonica* D. Don	7.5	0.30	0.35	0.60
Douglas fir	*Pseudotsuga menziesii* Franco	16.0	0.80	0.90	0.45
Buna	*Fagus crenata* Bl.	12.5	0.60	0.65	0.50
Red lauan	*Shorea* sp.	13.2	0.52	0.49	0.61
Balsa	*Ochroma lagopus* Sw.	3.9	0.60	0.14	0.23

E_L, E_T, Young's moduli in longitudinal and tangential directions; G_{LT}, Shear modulus; ν_{LT}, Poisson's Ratio.

Table 2. Shear moduli (G^{v}_{LT}) and Young's modulus (E^{v}_{L}) obtained by flexural vibration tests

Species	G^{v}_{LT} (GPa)	E^{v}_{L} (GPa)	G^{v}_{LT}/E^{v}_{L}
Spruce	0.91	11.4	12.5
Western hemlock	0.75	13.6	18.1
Akamatsu	1.29	16.5	12.9
Yellow poplar	0.77	12.1	15.7
Shioji	0.85	12.0	14.1
Balsa	0.20	3.18	13.2

v, vibration test.

Table 3. Shear modulius (G^{b}_{LT}) and Young's modulus (E^{b}_{L}) obtained by the conventional Timoshenko bending theory

Species	G^{b}_{LT} (GPa)	E^{b}_{L} (GPa)	E^{b}_{L}/G^{b}_{LT}
Spruce	0.21	10.1	48.1
Western hemlock	0.32	14.5	45.3
Akamatsu	0.22	16.8	76.3
Yellow poplar	0.21	12.2	58.1
Shioji	0.15	13.2	88.0
Balsa	0.05	2.90	61.7

b, bending test.

amined the validity of the static bending tests by comparing their results with those obtained using the vibration tests.

Applicability of Timoshenko's bending theory

The $1/E_s$–h/l relation was regressed into Eq. (5). Young's modulus (E) and the shear modulus (G) were calculated and are shown in Table 3. As expected, the shear modulus was found to be much smaller than that obtained from the vibration tests for every species. In contrast to the small values for the shear modulus, Young's modulus was found to be higher than that obtained using the vibration tests.

Figure 2 shows the relation between $E/E_s - 1$ and h^2/l^2 for yellow poplar, where 0.77 GPa was derived as the shear modulus. From Eq. (5), this relation should be expressed by a linear function if the deflection behavior is subject to Timoshenko's theory. In the range where $h^2/l^2 > 0.01$ ($h/l > 0.1$), however, the experimental data deviate from the linear relation. Thus we thought that the applicable range should be limited to that where $h/l < 0.1$ if the conventional Timoshenko theory was used. Table 4 shows the shear moduli obtained from the specimens whose depth/span ratios were smaller than 0.1. Although the shear moduli were close to those obtained from the vibration tests, they were still smaller.

According to Dong and colleagues, stress concentration at the loading point produced the additional deflection.[7] Thus we thought that the shear modulus could be evaluated properly by Timoshenko's theory when the deflection was not measured at the loading point. Figure 3 shows the shear modulus (G) corresponding to the depth/span ratio ob-

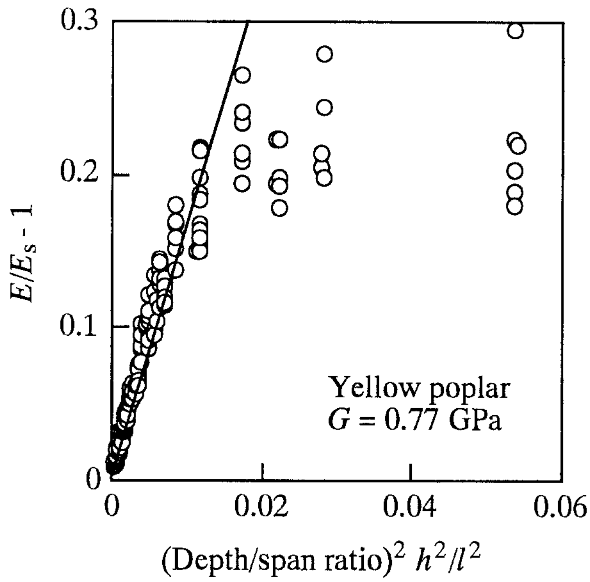

Fig. 2. Value of (E/E_s − 1) calculated by substituting the shear modulus obtained from the vibration tests into Timoshenko's bending theory corresponding to the squares of depth/span ratio

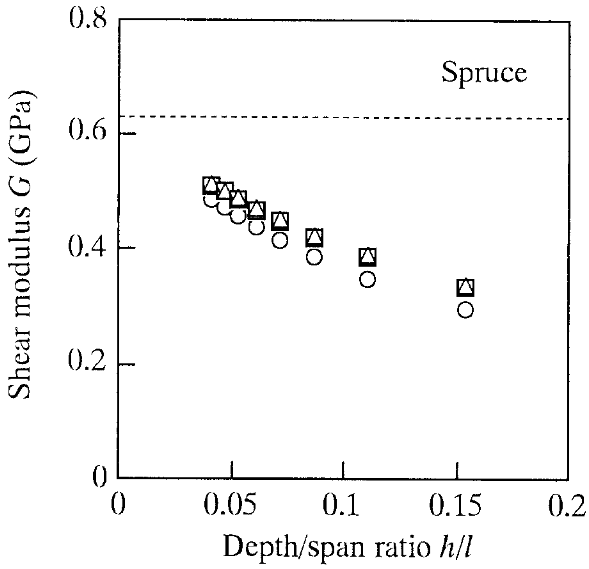

Fig. 3. Shear modulus corresponding to the depth/span ratio obtained from the finite element method (FEM) calculations when the grid point for measuring the deflection was varied. *Circles*, measured at the loading point; *squares*, measured at the middle of the depth; *triangles*, measured at the opposite side of the loading point; *broken line*, shear modulus given as a material parameter (G = 0.63 GPa for spruce)

Table 4. Shear moduli (G^b_{LT}) and Young's modulus (E^b_L) obtained by Timoshenko's bending theory with the data in the range where the depth/span ratio was less than 0.1

Species	G^b_{LT} (GPa)	E^b_L (GPa)	E^b_L/G^b_{LT}
Spruce	0.45	9.4	20.9
Western hemlock	0.54	13.3	24.6
Akamatsu	0.24	16.5	68.8
Yellow poplar	0.28	11.8	42.1
Shioji	0.15	13.3	88.7
Balsa	0.08	2.70	33.8

tained from the FEM calculations when the point used for measuring the deflection was varied. It is known that the shear modulus is calculated to be smaller when the deflection is measured at the loading point. However, the shear modulus, which should be independent of the depth/span ratio, decreases with the increase in the depth/span ratio for all cases. Thus it would be difficult to evaluate the shear modulus by the conventional Timoshenko theory when the deflection is measured at any point other than the loading point.

We undertook several trials, in vain, to obtain the shear modulus by the conventional Timoshenko theory. As pointed out by Uemura,[8] stress concentration near the loading point disturbs the stress distribution, and this distorted stress condition causes a low shear modulus. In fact, the flexural vibration method, in which no stress concentration exists, can provide a proper value for the shear modulus. The load–deflection relation cannot be predicted by the original Timoshenko theory when the stress concentration exists. From the results summarized above, we believe that the shear modulus cannot be predicted properly by the conventional Timoshenko bending theory.

New proposal for deriving a proper shear modulus value

It is true that we should solve a differential equation that can describe the distorted stress condition around the loading point when precisely determining the load-deflection relation. However, it is difficult to derive this equation, and the solution of the equation is complicated. Therfore we tried to modify the original Timoshenko bending theory to find one that can give the correct shear modulus using results from FEM calculations.

We thought that Timoshenko's shear factor (s), which is 1.2 (or 1.5) in the original Timoshenko theory, is dependent on the depth/span ratio. Thus we transformed Eq. (5) as

$$s = \frac{G\left(\dfrac{1}{E_s} - \dfrac{1}{E}\right)}{\left(\dfrac{h}{l}\right)^2} \qquad (6)$$

and examined the dependence of s on the depth/span ratio with the FEM calculation results. Timoshenko's shear factor (s) was calculated by substituting E and G (Table 1), h/l, and E_s obtained from the FEM calculation into Eq. (6). Figure 4 shows the relations between the shear factor (s) calculated by Eq. (6) and the depth/span ratio (h/l). According to the original Timoshenko theory, s should be a constant value of 1.2 or 1.5. Nevertheless, Fig. 4 suggests that the shear factor is represented by a linear function whose

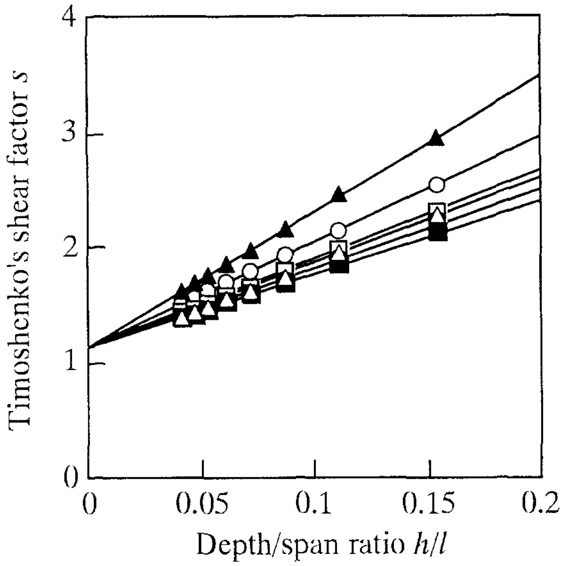

Fig. 4. Dependence of Timoshenko's shear factor on the depth/span ratio obtained from the FEM calculations. *Open circles*, spruce; *open squares*, Sugi; *open triangles*; Douglas fir *solid circles*, red melanti; *solid squares*, buna; *solid triangles*, balsa

Table 5. Shear moduli (G^{b}_{LT}) and Young's modulus (E^{b}_{L}) derived by our proposed method

Species	G^{b}_{LT} (GPa)	E^{b}_{L} (GPa)	E^{b}_{L}/G^{b}_{LT}
Spruce	0.89	9.3	10.5
Western hemlock	0.86	15.5	18.0
Akamatsu	0.95	15.5	16.3
Yellow poplar	0.97	11.3	11.6
Shioji	0.81	11.2	13.8
Balsa	0.17	2.44	14.1

intercept is 1.2. Thus we defined the shear factor dependent on the depth/span ratio as s' and supposed it to be derived as follows

$$s' = 1.2 + \alpha \frac{h}{l} \tag{7}$$

where α is a constant coefficient. Equation (7) shows that the deflection behavior is described by the original Timoshenko bending theory derived by calculating the strain energy ($s = 1.2$) when the depth/span ratio is small enough. With these adjustments, we modified Eq. (5) as follows:

$$\frac{1}{E_s} = \frac{1}{E} + \frac{s'}{G} \cdot \left(\frac{h}{l}\right)^2 \tag{8}$$

From the calculations, the values of αs were about 10. Nevertheless, the proper values of shear moduli were not obtained by substituting 10 into α; we then adjusted the value of α, and the proper shear moduli were obtained. Table 5 shows Young's moduli and the shear moduli obtained by the calculation results of $1/E_s$–h/l regressed into Eq. (8) when the values of αs were determined to be 35. The plausible values of shear moduli were obtained for the species tested here. As for spruce and balsa, Young's moduli are estimated to be smaller by Eq. (8) than those obtained from the vibration tests, although the shear moduli are close to those determined by the vibration tests. This phenomenon is because of the small values of E_s in the small depth/span ratio ranges where the influence of shearing force is negligible. To increase the accuracy of Young's modulus, the apparent Young's modulus in the small depth/span ratio range should be correct.

The disagreement of α between the FEM calculations and testing results is due to the difference of deformation around the loading point. We believe that the evaluation of the shear modulus is influenced by the shape of the loading head (whose radius was 15 mm in this experiment), and that the applicability of this method is limited by the testing condition. To obtain a plausible value for the shear modulus by static bending tests, the stress concentration by the loading point should be considered carefully. Otherwise, we should measure the shear modulus of wood by another method, such as static torsion tests,[3] 45° off-axis tests,[9] torsional vibration tests,[10] or the flexural vibration tests used here.

Conclusions

We examined the method for measuring the shear modulus of wood by static bending tests and obtained the following results.

1. Using the static bending test, the load–deflection behavior could not be predicted by the original Timoshenko bending theory. Hence we believe it is difficult to obtain the correct shear modulus of wood using the original Timoshenko theory.
2. By modifying Timoshenko's theory, we derived a method for measuring shear modulus by static bending.
3. When measuring the shear modulus using static bending tests, the stress concentration imposed by the loading point must be carefully considered.

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