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Effective sampling method for estimating bending strength distribution of Japanese larch square-sawn timber

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Abstract Information on the strength distribution of timbers and other wood products seems to have become more important for users and producers after revision of the Japan architectural standard in 1998, which emphasizes the performance requirements of structures. Because there is no way other than expensive destructive tests to collect strength data, many researchers have proposed many inspecting methods for predicting strength by nondestructive evaluation. The most popular method for structural timber is the mechanical grading method based on the relation between Young's modulus (E) and strength (σ) with some linear regression models. On the other hand, it is well known that the proof loading test is superior for obtaining information on the lower tail of σ distribution. If the E distribution of the objective timbers is known approximately, selecting timbers nearest to the projected E values saves timbers for destructive tests. We examined the alternative sampling method using the reported E-σ data sets of Japanese larch square-sawn timber. The simulated results showed that the estimated lower tail of the bending strength distribution by the alternative method was a better approximation of the experimental distribution than that derived from the conventional linear regression model.

Key words Modulus of elasticity · Modulus of rupture · Compressive strength · Sampling method · Bivariate frequency distribution

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Introduction

Square-sawn timbers are usually used for posts of "jikugumi"[1] (framing) of Japanese modern-style houses, and among structural timbers in Japan they are consumed most often. Many researchers have investigated their mechanical properties. For example, Iijima and Nakai[2] reported that the tolerance limits of square-sawn timbers were dependent on the species even if the averages of their Young's modulus values were almost same. During the late 1980s the concept of engineered wood was gradually beginning to spread among users and manufacturers who needed a guarantee of timber strength in Japan, as pointed out by Hayashi.[3] The Japan architectural standard[4] was revised in 1998, and one of the principal revised points was to clarify performance requirements. With the revision, the information on strength distribution of timbers and other wood products seems to be more important for users and producers. To collect strength data, there is no way other than the many expensive destructive tests. To save the costs of these tests or for other purposes, researchers have proposed alternative inspection methods for predicting strength to enable a nondestructive evaluation.[5]

The most popular method[2] for structural timber is the mechanical grading method based on the relation between Young's modulus (E) and strength (σ). This method can be used for any model to predict σ using E data, and the models are usually based on the linear regression relation between E and σ. Though the regression method has many advantages, it is somewhat easier to express the lower tail of the σ-distribution and depends on the σ-distribution at the E-level. Hayashi[6] attempted a linear regression method with conditional variance[7] for predicting of wood laminates. This method, however, does not reflect the nature of timber in that timber with low E values generally has more defects, *e.g.* knots, than timber with a high E level; and such defects should decrease the timber strength. This tendency was observed in Japanese larch lumber for structural glued laminated timber.[8–10]

In the scope of the lower tail of the σ-distribution, Madsen[11] suggested that "If you want to know how strong a sample of material is, you need to test all the pieces to destruction. On the other hand, if you only need to know how weak the material is, [the] proof loading approach can be used successfully in which case only the weak portion of the material is destroyed." This approach gives us little information on the relation between E and σ for the whole material.

In this paper we propose an alternative method: that σ-distributions of timbers at particular E levels are obtained by sampling specimens with E values nearest to projected E values. Hereafter we call this method the target-E method. The whole σ-distribution is estimated by using the obtained σ-distributions at each target-E level. We examined the method using data sets on mechanical properties of Japanese larch (*Larix kaempferi* Carriere) square-sawn timber.

Experiment

The scheme of the experiment was projected by Prof. Shigematsu,[12,13] now deceased, for investigating the relation between wood quality and growth in Japanese larch trees. The experiment on the mechanical properties of Japanese larch square-sawn timbers were completed at Shinshu University, Forestry and Forest Products Research Institute, and Nagano Prefectural General Forestry Research Center in Japan. The static destructive bending tests were done with span lengths of 270 cm by third-point concentrated loading. The dimensions of the bending specimens were nominally 12.0 cm thick, 12.0 cm wide, and 300 cm long. The modulus of elasticity (MOE) and modulus of rupture (MOR) were measured and recorded for each specimen. After the bending tests, short columns for compression tests parallel to the grain were cut from the nonfailure portion of each bending specimen. The length of the column was 54 cm, and the slenderness ratio was 17. The compressive strength (CS) for each column was obtained using static compression tests. The details of the experiments are found in the previous report;[14] the data sets for MOE, MOR, and CS for each square-sawn timber were used for the following analysis. We focused on the MOR distribution, and the CS data were used for comparing the MOR data.

Results and discussion

Experimental results

The mechanical properties of specimens are shown in Table 1. The mean and standard deviation (SD) for the MOE were 9.44 GPa and 1.54 GPa, respectively. Figure 1 shows the distribution of MOE by Gauss probability paper. It may be assumed that the MOE distribution can be expressed as a normal distribution, as we reported.[15] Thus the MOE values of almost 70% of specimens should be within the range from the mean plus the SD to the mean minus the SD.

Table 1. Basic statistics of mechanical properties of specimens

Properties	No.	Mean	SD	CV
MOE (GPa)	287	9.44	1.54	16.3
MOR (MPa)	287	44.6	11.6	26.1
CS (MPa)	287	11.6	4.9	15.9

MOE, modulus of elasticity; MOR, modulus of rupture; CS, compressive strength; No., number of specimens; SD, standard deviation; CV, coefficient of variation (%)

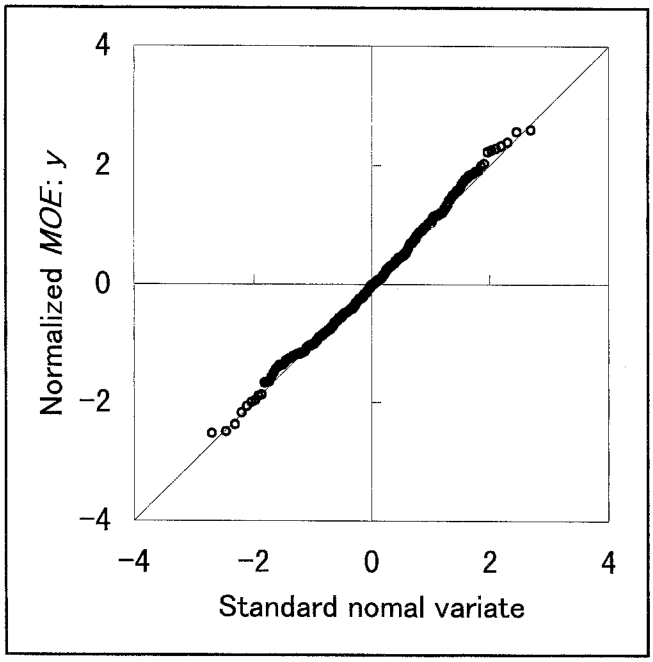

Fig. 1. Distribution of modulus of elasticity (MOE) by Gauss probability paper. *y*, normalized MOE

The relation between MOE and MOR is shown in Fig. 2a. Note that the regression line of MOR on MOE was also described with 5% and 95% limit lines. The regression line was expressed as MOR = 4.42 MOE + 2.86 (n = 287, r = 0.585, SE = 9.45), where n is the number of specimens, r is the correlation coefficient, and SE is the standard error. The additional axes denote normalized MOE (y) and MOR (b), respectively. The y values was simply defined as y = (MOE data – mean MOE)/(SD of MOE); b as similar to y. Most of the plots were within a range of ±2 of y and b. The horizontal broken line denotes the 15th percentile level of the MOR distribution. Foschi et al.[16] used 2P-Weibull fitted to the test data truncated at the 15th percentile level for reliability evaluation of wood structures. It may be presumed that their chief concern about MOR distribution is within the range below the line in Fig. 2a. Similar to MOR, Fig. 2b shows the CS distribution; c is the normalized CS; regression line, CS = 1.70 MOE + 15.12 (n = 287, r = 0.529, SE = 4.20).

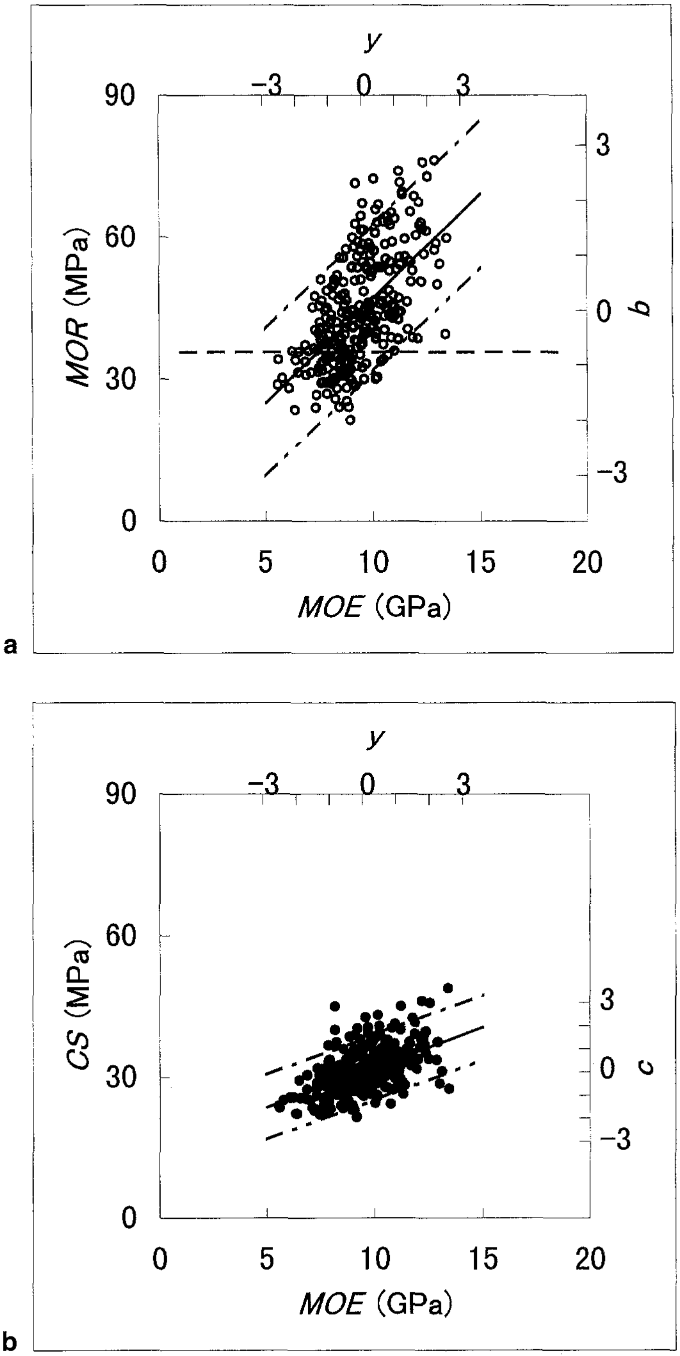

Fig. 2. Relation between MOE and strength. **a** Modulus of rupture (*MOR*), *open circles*. **b** Compressive strength (*CS*): *filled circles*, *y*,*b*,*c*, normalized MOE, MOR, CS, respectively. *Dash-dot lines*, 95% limit lines; *dash-dot-dot lines*, 5% limit lines; *horizontal broken line* (**a**), 15th percentile level

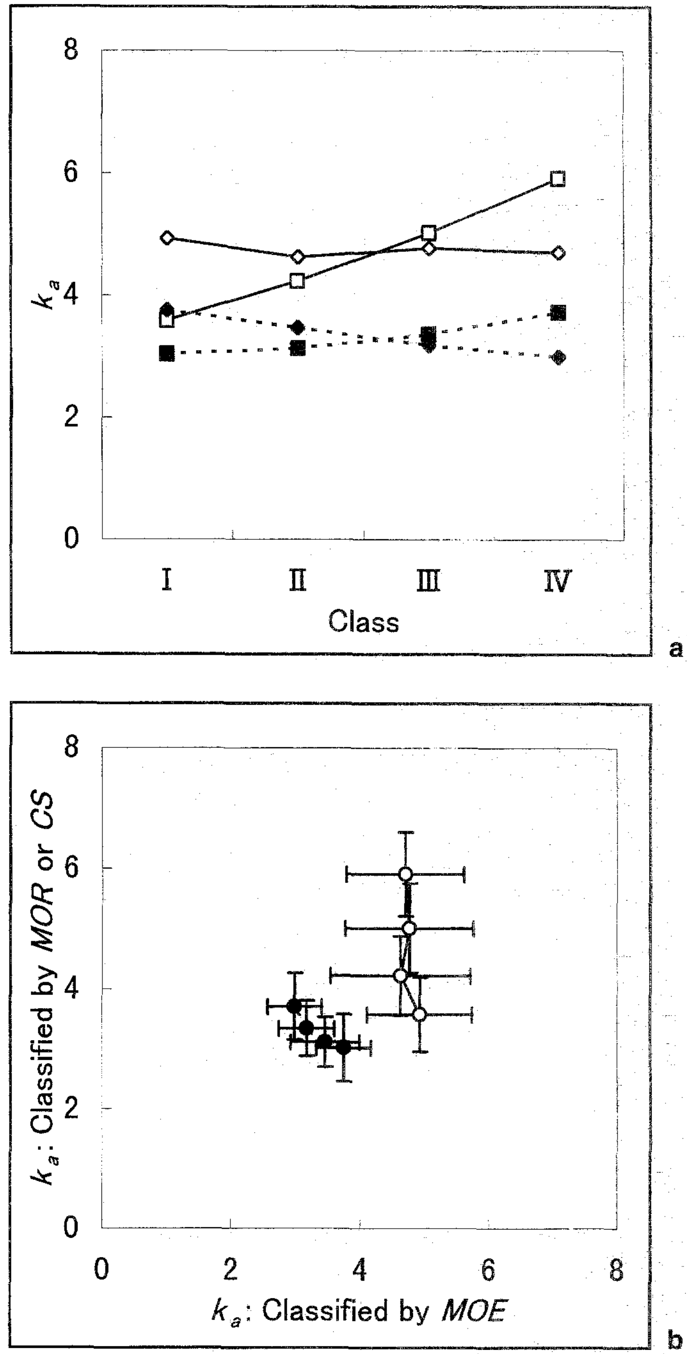

Fig. 3. Ratio (k_a) of MOR and CS to Young's modulus (MOE) of specimens classified by Young's modulus and strength. Class boundaries: $p = 0.15, 0.5, 0.85$, respectively; p, cumulative probability, k_a, classified MOE (*diamonds*) and strength (*squares*); *open and filled plots*, MOR and Cs, respectively. **b** Effects of classification: *blank and filled circles*, MOR and CS, respectively

Formerly, the strength/elasticity ratio[17] was usually adopted for evaluating mechanical properties of commercial timbers defined as $k_a = \sigma_a/E_a/1000$, where σ and E denote strength and elasticity, and the subscript a means average. We attempted to compare MOR and CS with the ratio (k_a) at each class of MOE. The four MOE classes were divided by the three boundaries with cumulative probabilities of 0.15, 0.50, and 0.85, respectively. In addition, k_a values at each MOR and CS class were obtained similar to the MOE classes. If the correlation between MOE and MOR were strong, the difference between two k_a values classified by MOE and MOR should be small at each class. Figure 3a shows the k_a values at each class. It was clear that k_a values at each MOE class in MOR was almost constant compared to the k_a values in CS. The relations between k_a values at MOE and strength classes are shown in Fig. 3b. The differences between MOR and CS were obvious. In other words, the dependence of strength distribution on Young's modulus levels may vary between bending and compression due to failure types: brittle or ductile fracture.

Application of target-E method

The concept of the target-E method is simple. When you know the approximate distribution of Young's modulus in your objective materials, sampling for destructive tests limits materials at some particular E-levels. We expected that the whole strength distribution estimated by this method might reflect the dependence of strength distribution on Young's modulus. Figure 4 illustrates the difference in sampling among the regression model, the proof loading approach, and the target-E method. The proof loading approach and the target-E method need smaller sampling sizes than the regression model. We examined the efficiency of the target-E method for estimating strength distribution through simple simulation.

At first, four target-E levels were set as $y = -1.5, -0.5, 0.5$, and 1.5; they were intended to represent the four above-mentioned MOE classes. We selected 10 specimens with MOE values nearest to each target-E: $E_1 = 7.1$, $E_2 = 8.7$, $E_3 = 10.2$, and $E_4 = 11.8$ GPa. When two specimens had the same MOE value, both were selected; then the selected numbers at E_1, E_2, E_3, and E_4 were 10, 11, 12, and 11, respectively.

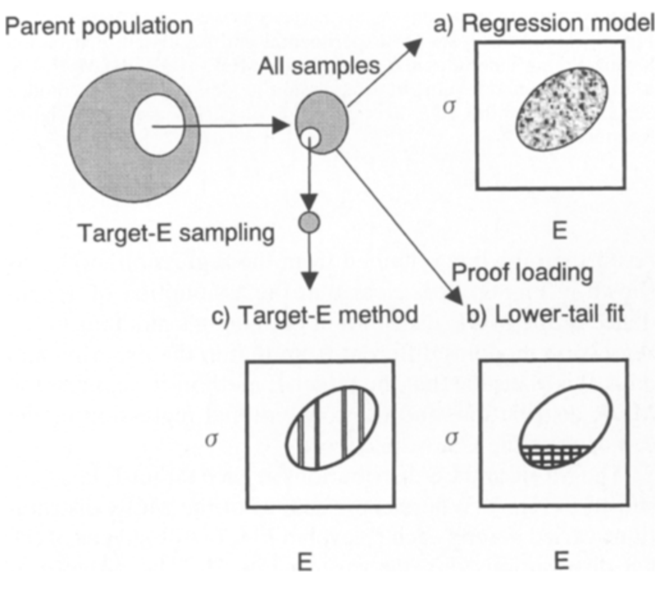

Fig. 4. Concept of target-E method comparing to other models

Next, these data were fitted to 3P-Weibull, and the obtained parameters are shown in Table 2. The relation between MOE and the parameters of 3P-Weibull was expressed approximately as exponential equations for each parameter.

Finally, simple simulation was done as following: Each replication was repeated 5000 times, and we obtained 5000 data sets for MOE·MOR.

1. A standard normal random number was chosen to calculate MOE.
2. A uniform random number (r_1) from 0 to 1 was chosen to calculate strength.
3. In the regression model, r_1 was transformed to the standard normal random number (r_2), and the MOR value was obtained by linear regression with the above MOE value and SE $\times$ r_2 using the above mentioned experimental regression equation.
4. For the target-E method, the three parameters of 3P-Weibull were calculated with the approximate equations as shown in Table 3 using the MOE value in item 1 (above). The MOR value was obtained by the 3P-Weibull with r_1 in item 2 (above).

Data sets for MOE·CS were obtained similar to MOE·MOR.

Comparing experimental data and simulated results

Basic statistics for the simulated results are shown Table 4. The values in this table seem almost equal to the experi-

Table 2. Estimated parameters of 3P-Weibull at each target-E

Target E	$1/k$	m (MPa)	x_0 (MPa)	Average MOE (GPa)
MOR				
E_1	0.296	24.2	13.8	7.2
E_2	0.284	27.6	16.8	8.7
E_3	0.352	32.6	15.4	10.2
E_4	0.256	33.8	22.9	11.7
CS				
E_1	0.196	14.9	12.9	7.2
E_2	0.150	15.2	15.3	8.7
E_3	0.263	21.1	14.4	10.2
E_4	0.176	19.6	18.3	11.7

k, m, x_0, shape, scale, and location parameters of 3P-Weibull, respectively; CS, compressive strength

Table 3. Parameters of interpolating 3P-Weibull between adjacent target-E

Parameter	$1/k$			m			x_0		
	C_1	C_2	r^2	C_1	C_2	r^2	C_1	C_2	r^2
MOR	0.340	−0.015	0.048	14.1	0.078	0.948	6.9	0.094	0.721
CS	0.166	0.016	0.017	8.6	0.075	0.693	8.1	0.066	0.768

Each parameter of 3P-Weibull was expressed as $1/k$, m, or $x_0 = C_1 e^{C_2 X}$; x, MOE; r^2, determination coefficients

Table 4. Basic statistics of simulated results

Propert	No.	Mean	SD	CV
MOE (GPa)	5000	9.44	1.50	15.9
MOR (MPa)				
Reg.	5000	44.3	11.6	26.3
Tar.	5000	43.4	10.4	23.9
CS (MPa)				
Reg.	5000	31.0	5.0	16.0
Tar.	5000	30.8	6.2	20.0

Reg. and Tar. denote simulation results by regression model and target-E method, respectively

Table 5. Comparison of 50th, 10th, and 5th percentiles of simulated results with experimental data

Strength and percentiles	Data	Reg.	Tar.
MOR (MPa)			
50th	43.1	44.3 (1.03)	42.8 (0.99)
10th	30.5	29.1 (0.95)	30.2 (0.99)
5th	28.9	25.1 (0.88)	27.4 (0.95)
CS (MPa)			
50th	30.4	31.0 (1.02)	30.5 (1.01)
10th	25.5	24.6 (0.96)	23.0 (0.90)
5th	23.4	22.9 (0.98)	21.3 (0.91)

Reg. and Tar. denote simulation results by regression model and target-E method, respectively. Values in parentheses are the ratios of simulated values/experimental data

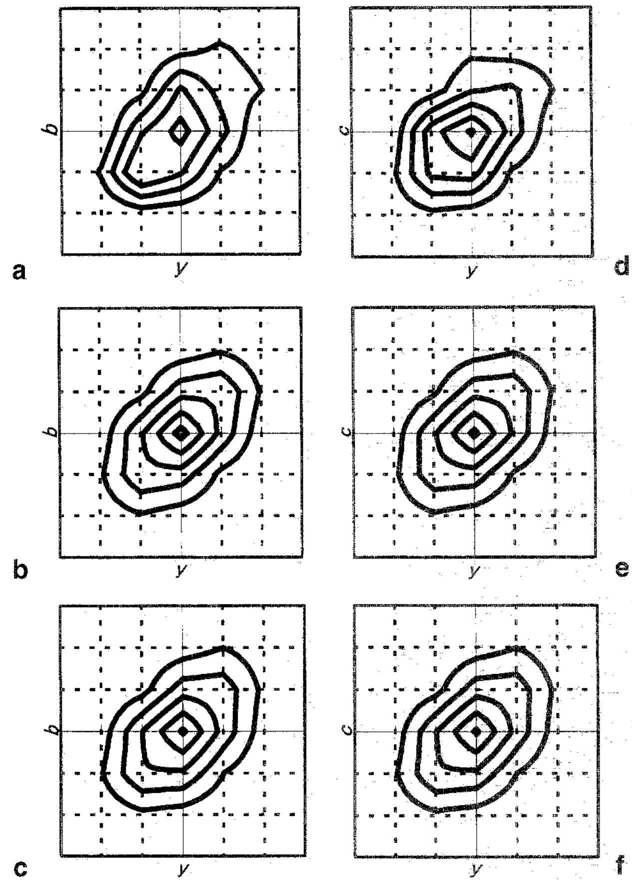

Fig. 5. Contour ellipses for experimental and simulated distribution having Young's modulus and strength. **a–c** MOE-MOR. **d–f** MOE-CS; **a**, **d** experimental results; **b**, **e** regression model; **c**, **f** target-E model. Scales are 1 of the *y*,*b*,*c* axis; intervals of curves are at a relative frequency of 5%

mental data shown in Table 1. The obtained regression lines by the regression model were MOR = 4.46 MOE + 2.24 (r = 0.575, SE = 9.53) and CS = 1.71 MOE + 14.84 (r = 0.520, SE = 4.23). The lines by the target-E method were MOR = 3.72 MOE + 8.30 (r = 0.539, SE = 8.74) and CS = 2.20 MOE + 10.03 (r = 0.538, SE = 5.19). The linear regression lines obtained by simulation are near the experimental regression lines.

We then compared the 50th, 10th, and 5th percentiles of MOR and CS obtained from simulation to the experimental data (Table 5). For MOR, the 5th percentile by the target-E method was nearer the experimental data than the regression model. On the other hand, the regression model made a better approximation than did the target-E method.

We drew contour ellipses[18] to express a bivariate frequency distribution of Young's modulus and strength (Fig. 5). For the axes in the figures we used normalized MOE, MOR, and CS. Though few differences among the six maps are seen, careful observation might lead one to appreciate the difference in the lower-tail portions of the MOR distribution among the three maps. Figure 6a shows the 15% lower tail of MOR distributions from experimental data and simulated results. The figure shows that the target-E method gets a better approximation than the regression model. To clarity the differences of the E-σ relation at the 15% lower tail among these cases, we calculated the regression line for each case. The regression line at the 15% lower tail from the target-E method was nearer the experimental result than the line obtained from the regression model, as shown in Fig. 6b. It is clear that the assumption of dependence of the MOR distribution on Young's modulus in the regression model is different from that in the experimental data. It is plausible that the target-E method can express the MOR distribution, and the conventional regression model can express the CS, respectively.

The MOR and CS distributions at each target-E level are shown in Fig. 7. Whereas the shapes of the MOR distributions varied among each E level in Fig. 7a, the shapes of the CS distributions did not vary in Fig. 7b. The differences between MOR and CS could be caused by the difference of failure modes of MOR and CS. Timber often contains knots in the tension zone under the bending test, and brittle failure often occurs. Compression, on the other hand, has a ductile failure mode, and it is not sensitive to defects such as knots. In general, timber with a high MOE value has fewer knots than low-grade timber; thus the MOR distribution is affected more strongly by the MOE level than by the CS. Thus characterization of timber by a few or many knots may make a difference among species in terms of the dependence of MOR distribution on. Young's modulus. This questions can be answered by comparing other species, pos-

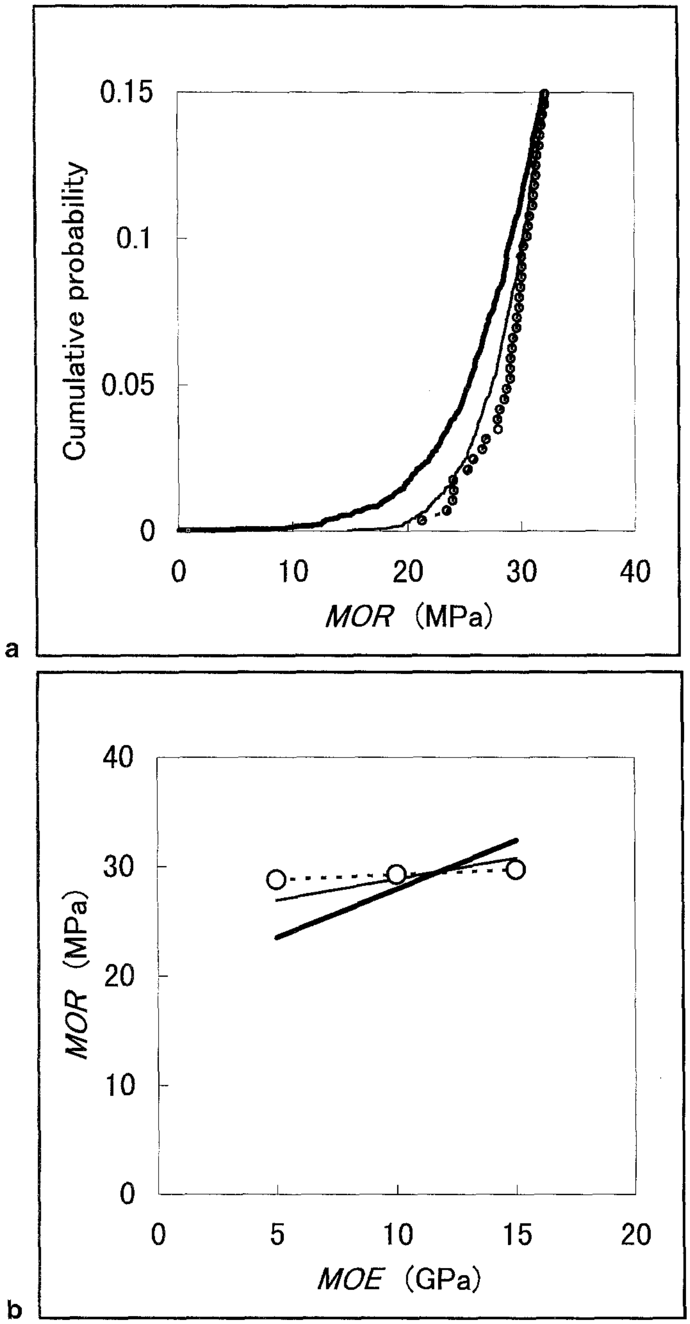

Fig. 6. MOR distributions truncated at the 15th percentile (**a**) and regression lines of MOR on MOE in the lower tail (**b**). *Circles*, experimental; *heavy curves*, regression model; *thin curves*, target-E model

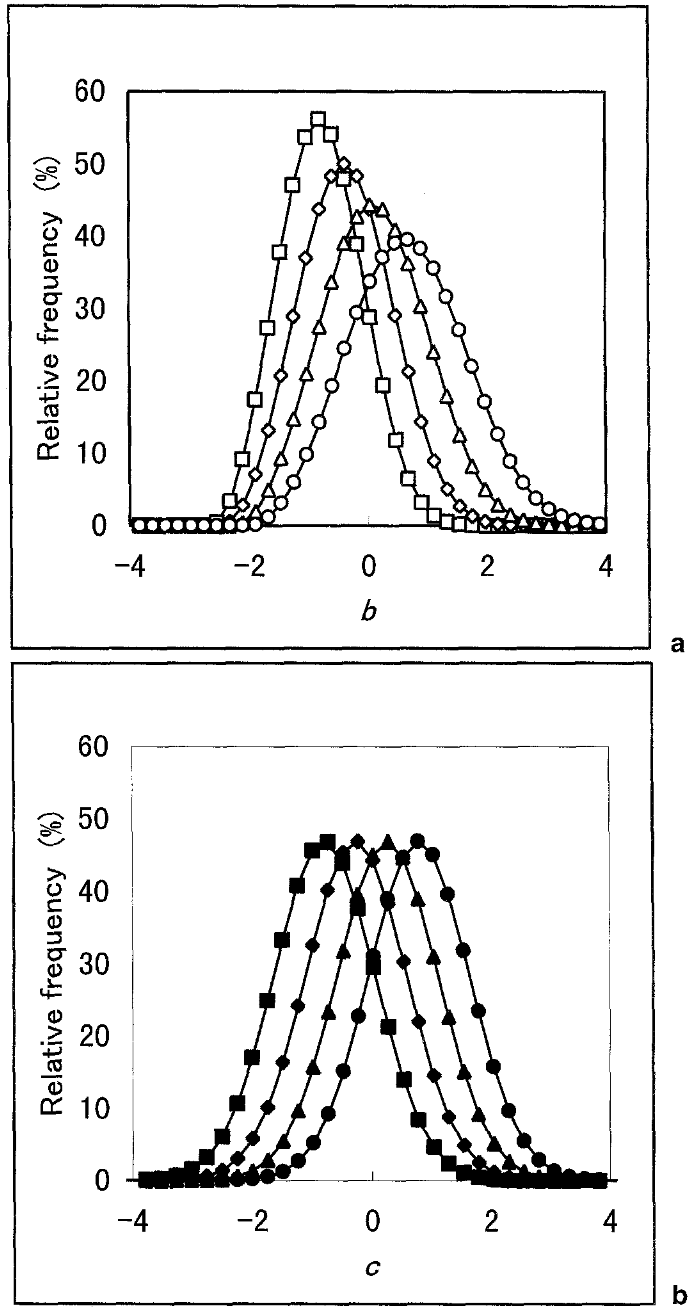

Fig. 7. Strength distribution at each target-E level. **a** Normalized MOR (y). **b** *Filled plots*, normalized CS (*c*); *squares*, E_1; *diamonds*, E_2; *triangles*, E_3; *circles*, E_4; see Table 2

sibly with a more effective method than the target-E method for each species.

The target-E method is successful for estimating MOR distribution in Japanese larch timber, although some problems remain for its practical use, for example social machinery for collecting more MOE data, a more effective setting of target-E, a more accurate interpolation method of MOR distributions between each target-E, and so on. It should be noted that only 15% of the data from all specimens was needed for the target-E method.

Conclusions

We propose the target-E method for estimating MOR distribution in Japanese larch timber as an alternative method to the conventional regression model. The comparison of simple application of a regression model and the target-E method to experimental data led to the following results. We also compared MOR and CS for contrast.

1. The MOE-MOR or MOE-CS regression lines obtained by simulation with a regression model and the target-E

method were near the regression lines obtained from experimental results.

2. The lower tail of MOR distribution estimated by the target-E method was a better approximation of the experimental data than that obtained by the regression model, although only 15% of the data of all specimens was needed for the target-E method estimation.
3. The regression model, on the other hand, expressed CS distribution more accurately than the target-E model.

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