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ORIGINAL ARTICLE

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# Influence of span/depth ratio on the measurement of mode II fracture toughness of wood by end-notched flexure test

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**Abstract** The influence of the span/depth ratio when measuring the mode II fracture toughness of wood by end-notched flexure (ENF) tests was examined. Western hemlock (*Tsuga heterophylla* Sarg.) was used for the specimens. The ENF tests were conducted by varying the span/depth ratios; and the fracture toughness at the beginning of crack propagation $G_{IIc}$ was calculated by two equations that require the load-deflection compliance or Young's modulus. Additionally, the influence of the span/depth ratio on the load-deflection compliance was analyzed by Timoshenko's bending theory in which additional deflection caused by the shearing force is taken into account. The following results were obtained: (1) When the span/depth ratio was small, the fracture toughness calculated with the data of load-deflection compliance was large. In contrast, the fracture toughness calculated with the equation containing Young's modulus tended to be constant. (2) In the small span/depth ratio range, the load-deflection compliance was estimated to be larger than that predicted by Timoshenko's bending theory. (3) To obtain the proper fracture toughness of wood with a single load-deflection relation, the span/depth ratio should be larger than that determined in several standards for the simple bending test method of wood, 12:16.

**Key words** Fracture toughness · Mode II · ENF test · Span/depth ratio

## Introduction

In a previous report, the applicability of the end notched flexure (ENF) test to measurement of mode II fracture toughness of wood was examined.[1] In the ENF test, the equation for calculating fracture toughness is based on the elementary bending theory in which the extra deflection produced by the shearing force is often neglected. When the specimen has a small span/depth ratio, however, the effect of shearing force is marked, and the fracture toughness obtained would be seriously influenced by the span/depth ratio. Several studies have examined the influence of the span/depth ratio on the measurement of mode II fracture toughness, and the analyses conducted are based on Timoshenko's bending theory.[2–6] In these studies, however, finite element analyses were undertaken to examine the validity of Timoshenko's theory instead of the experimental work. According to previous work on the bending properties of wood without cracks, Timoshenko's theory is not sufficient for describing the effect of shearing force, and it is feared that the influence of the span/depth ratio cannot be described by Timoshenko's theory on the measurement of fracture toughness.[7] Here, ENF tests were conducted using wood specimens with various span/depth ratios to examine the influence of the span/depth ratio on measurements of mode II fracture toughness.

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## Theory

As shown in Fig. 1, an ENF specimen with a crack length of $a$ is forced by the load $P$ at the center of a span of $2L$. According to the elementary beam theory, the equation for flexure caused by the bending moment is given as follows:

$$\begin{cases} \dfrac{E_x I}{8}\dfrac{d^2 y_m}{dx^2} = -\dfrac{1}{4}Px & (0 \le x \le a) \\ E_x I \dfrac{d^2 y_m}{dx^2} = -\dfrac{1}{2}Px & (a \le x \le L) \\ E_x I \dfrac{d^2 y_m}{dx^2} = \dfrac{1}{2}Px - PL & (L \le x \le 2L) \end{cases} \quad (1)$$

where $y_m$ is the deflection caused by the bending moment, $E_x$ is Young's modulus in the long axis, and $I$ is the second

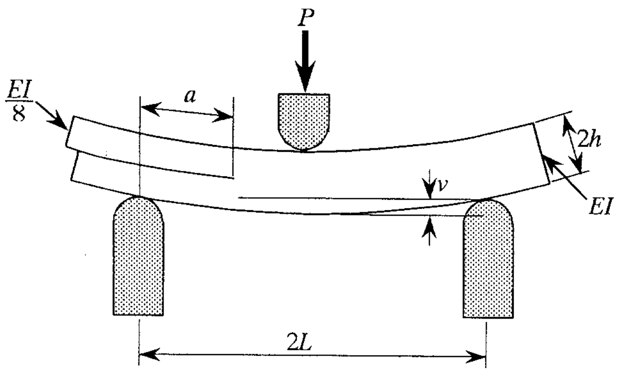


**Fig. 1.** End notched flexure (ENF) test

moment of cross-sectional area in crack-free region. Solving this equation, we can obtain the deflection at the loading point ($x = L$) caused by the bending moment, $v_m$, as follows:[5,6,8]

$$v_m = \frac{P(2L^3 + 3a^3)}{12E_x I} \tag{2}$$

When the beam has a small span/depth ratio, the effect of the shearing force should be taken into account when deriving the deflection and it is generally formulated by Timoshenko's bending theory. As mentioned above, although it is feared that the effect of the shearing force cannot be described sufficiently by Timoshenko's bending theory, the shearing force effect can be examined by Timoshenko's bending theory.[5,6,9] The slope caused by the shearing force, $dy_s/dx$, equals the shear strain at the neutral axis of the beam. It is derived as follows:

$$\begin{cases} \dfrac{dy_s}{dx} = \dfrac{sP}{2AG_{xy}} & (0 \le x \le a) \\ \dfrac{dy_s}{dx} = \dfrac{sP}{2AG_{xy}} & (a \le x \le L) \\ \dfrac{dy_s}{dx} = -\dfrac{sP}{2AG_{xy}} & (L \le x \le 2L) \end{cases} \tag{3}$$

where $A$ is the cross-sectional area of the crack-free region, $G_{xy}$ is the shear modulus, and $s$ is Timoshenko's shear factor, which equals 1.5. From Eq. (3), the deflection caused by the shearing force at the loading point, $v_s$, is represented as follows:

$$v_s = \frac{sPL}{2AG_{xy}} \tag{4}$$

Thus, the total loading-point deflection $v$ is

$$v = v_m + v_s = \frac{P(2L^3 + 3a^3)}{12E_x I} + \frac{sPL}{2AG_{xy}} \tag{5}$$

and the compliance $C$ is derived as follows:

$$C = \frac{v}{P} = \frac{1}{8E_x b}\cdot\left(2 + 3\frac{a^3}{L^3}\right)\cdot\left(\frac{2L}{2h}\right)^3 + \frac{s}{4G_{xy}b}\cdot\frac{2L}{2h} = C_m + C_s \tag{6}$$

where

$$\begin{cases} C_m = \dfrac{2L^3 + 3a^3}{12E_x I} = \dfrac{1}{8E_x b}\cdot\left(2 + 3\dfrac{a^3}{L^3}\right)\cdot\left(\dfrac{2L}{2h}\right)^3 \\ C_s = \dfrac{sL}{2AG_{xy}} = \dfrac{s}{4G_{xy}b}\cdot\dfrac{2L}{2h} \end{cases} \tag{7}$$

where $b$ and $2h$ are the width and depth of the crack-free region, respectively. Equation (7) shows that the crack length $a$ has no influence on $C_s$, and the energy release rate $G_{II}$ is given by using Young's modulus as:[5]

$$G_{II} = \frac{P^2}{2b}\cdot\frac{dC}{da} = \frac{9C_m P^2 a^2}{2b(2L^3 + 3a^3)} = \frac{9P^2 a^2}{16E_x b^2 h^3} \tag{8}$$

Another expression of $G_{II}$ is derived from Eqs. (6) and (7) as follows:

$$G_{II} = \frac{9CP^2 a^2}{2b(2L^3 + 3a^3)}\cdot\alpha_s \tag{9}$$

where

$$\alpha_s = \frac{C_m}{C} = \frac{C_m}{C_m + C_s} = \frac{(2L^3 + 3a^3)\cdot\left(\dfrac{2L}{2h}\right)^3}{(2L^3 + 3a^3)\cdot\left(\dfrac{2L}{2h}\right)^3\left[1 + \dfrac{2sE_x}{G_{xy}}\cdot\dfrac{L^3}{2L^3 + 3a^3}\cdot\left(\dfrac{2L}{2h}\right)^{-2}\right]} \tag{10}$$

When the span/depth ratio ($2L/2h$) is large enough, the effect of the shearing force is small; and the second term in the brackets of Eq. (10) can be ignored. Then the value of $\alpha_s$ in Eq. (9) can be regarded as 1; hence, $G_{II}$ of the specimen with large span/depth ratio can be written as follows:

$$G_{II} = \frac{9CP^2 a^2}{2b(2L^3 + 3a^3)} \tag{11}$$

## Experiment

### Materials

Western hemlock (*Tsuga heterophylla* Sarg.) with a density of 0.48 g/cm$^3$ was used for the specimens. Specimens were conditioned at 20°C and 65% relative humidity before and during the tests.

## ENF tests

Beam specimens were cut with a cross section of 15 mm (radial direction) × 15 mm (tangential direction); the lengths in the longitudinal direction varied, sometimes 30 mm longer than the span lengths mentioned below. The crack was first cut in the longitudinal-tangential (TL) plane by a band saw with a thickness of 1 mm as the supposed length; then it was extended by a razor blade to be one-quarter of the span. Thus, the ratio of the initial crack length $a$ to the half-span $L$ was 0.5. This ratio is often adopted for the ENF tests of advanced composites.[10] Two sheets of Teflon of 0.5 mm thickness were inserted between the crack surfaces to reduce the friction between the upper and lower cantilever beams. Specimens were supported by spans varying from 120, 150, 180, 210, 225, 300, 375, to 450 mm. A vertical load was applied at the center of the longitudinal-radial (LR) surface at a cross-head speed of 2 mm/min. The load-deflection compliance $C$ was measured by a dial gauge set below the loading nose. Similar to our previous study, the "critical load" $P_c$ was determined to be that at the intersection point between the two straight-line segments through the pre- and postlinear portions of the load-deflection curve. Figure 2 shows an example of the load-loading point displacement relation in which the determination method of $P_c$ is shown.

The fracture toughness at the beginning of crack propagation $G_{IIc}$ was calculated by substituting the critical load $P_c$ and Young's modulus $E_x$ into Eq. (8), which was the average obtained by the uniaxial-compression tests mentioned below, or by substituting $P_c$ and the compliance $C$ into Eq. (11). The fracture toughnesses obtained by these two procedures were compared, and the influence of the span/depth ratio was examined.

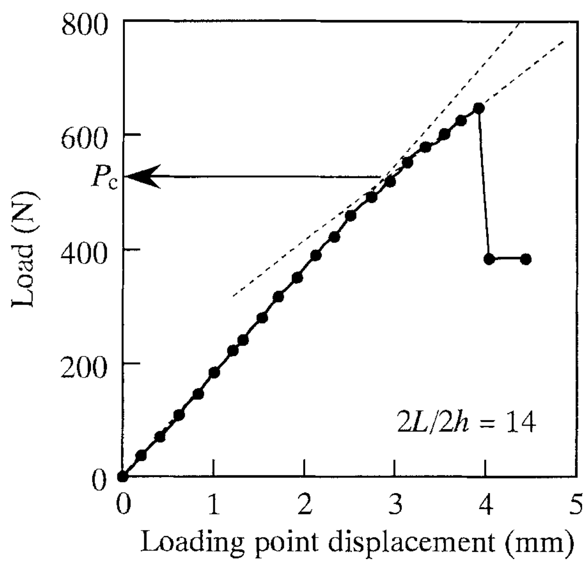


**Fig. 2.** Example of load-loading point displacement relation and determination method of critical load $P_c$

## Compression tests

To determine Young's modulus $E_x$ and the shear modulus $G_{xy}$, uniaxial-compression tests were conducted by the following procedure. A short-column specimen whose dimensions were 20 × 20 × 40 mm was prepared. When measuring $E_x$, the long axis of the specimen coincided with the longitudinal direction, whereas when measuring $G_{xy}$ the long axis coincided with the direction inclined at 45° with respect to the grain. Strain gauges were bonded at the centers of the specimen; and Young's modulus was obtained from the stress-strain relation. The shear modulus was determined from the following equation:

$$G_{xy} = \frac{E_{45}}{2(1 + \nu_{45})} \tag{12}$$

where $E_{45}$ and $\nu_{45}$ are Young's modulus and Poisson's ratio in the geometric symmetry, respectively.

## Results and discussion

From the uniaxial-compression tests, Young's modulus $E_x$ and the shear modulus $G_{xy}$ were determined to be 12.3 and 1.18 GPa, respectively. The values of $E_x$, $P_c$, and $C$ were substituted into Eqs. (8) and (11); the fracture toughness $G_{IIc}$ was calculated. Figure 3 represents the dependence of the fracture toughness calculated by each equation on the span/depth ratio $2L/2h$. The fracture toughness was not influenced by the span/depth ratio when it was calculated by Eq. (8), which does not contain the data for compliance $C$. This phenomenon suggests that the critical load $P_c$ is not influenced by the span/depth ratio. In contrast, the fracture toughness calculated by Eq. (11) in which compliance $C$ is involved was evaluated largely in the small span/depth ratio range. As mentioned above, the shearing force produced additional deflection, which increased the load-deflection compliance in the small span/depth ratio range.

The influence of the span/depth ratio on load-deflection compliance was examined by Eqs. (6) and (7). When the deflection behavior is described by the elementary bending theory, the effect of shearing force is neglected and

$$\frac{C}{\left(\frac{2L}{2h}\right)^3} = \frac{1}{8E_x b}\cdot\left(2 + 3\frac{a^3}{L^3}\right) \tag{13}$$

which is constant over the entire span/depth ratio range. In contrast, when the effect of shearing force is taken into account, $C/(2L/2h)^3$ is represented according to Timoshenko's bending theory as follows:

$$\frac{C}{\left(\frac{2L}{2h}\right)^3} = \frac{1}{8E_x b}\cdot\left(2 + 3\frac{a^3}{L^3}\right) + \frac{s}{4G_{xy}b\left(\frac{2L}{2h}\right)^2} \tag{14}$$

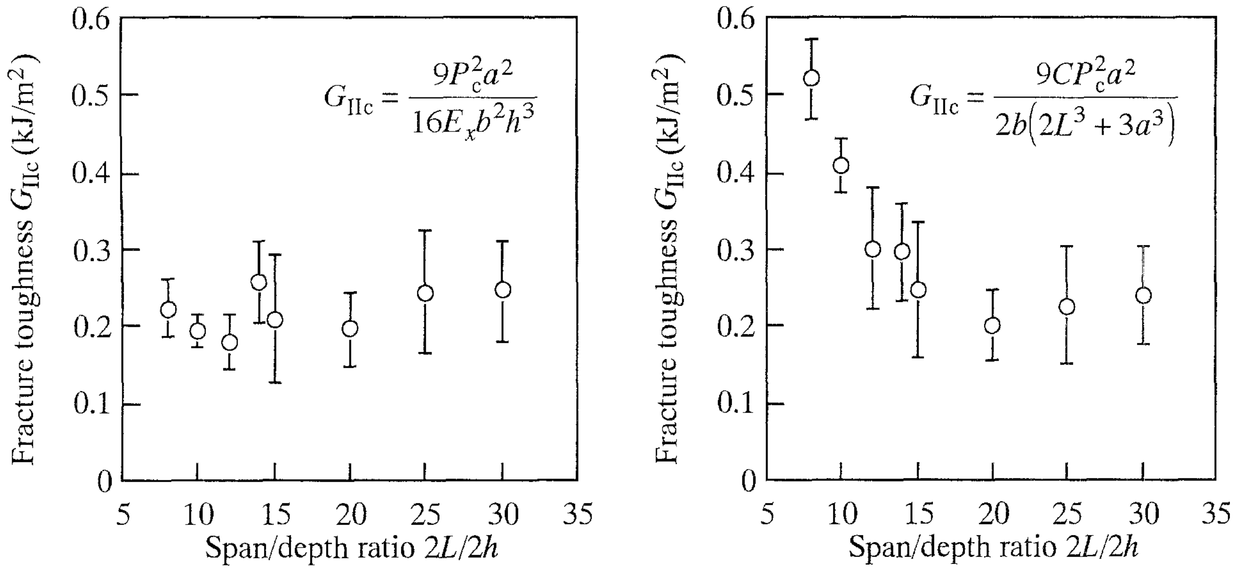


**Fig. 3.** Relation between the fracture toughness $G_{IIc}$ calculated by each equation and the span/depth ratio $2L/2h$. *Circles* and *horizontal bars* represent the mean and standard deviations, respectively

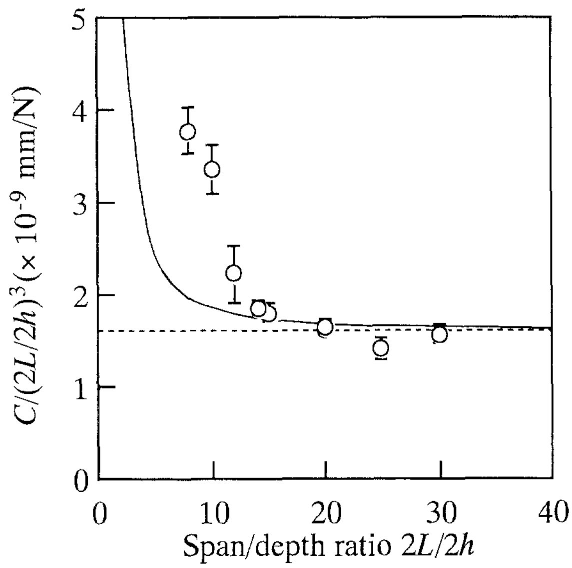


**Fig. 4.** Load-deflection compliance/cube of span/depth ratio corresponding to the span/depth ratio. *Dashed* and *solid lines* represent the predictions by the elementary bending theory and Timoshenko's bending theory, respectively. See Fig. 3 for further explanations

Figure 4 shows the dependence of the value of $C/(2L/2h)^3$ on the span/depth ratio $2L/2h$. The experimental values of $C/(2L/2h)^3$ increased with a decreasing span/depth ratio. In particular, when the span/depth ratio was smaller than 12, the experimental value exceeded that predicted by Timoshenko's bending theory. During the simple bending test of the specimen without cracks, the additional deflection, which cannot be predicted by Timoshenko's theory, is marked in the small span/depth ratio range; and it was suggested that Timoshenko's theory is not sufficient for representing the bending properties.[9] This phenomenon was applicable when obtaining the load-deflection compliance of the cracked specimen with a small span/depth ratio.

According to the ENF testing method of carbon fiber-reinforced plastics (CFRP) standardized in Japan Industrial Standard (JIS), the fracture toughness is calculated by the equation involving the load-deflection compliance because it is unnecessary to measure Young's modulus independently of the ENF tests. In the standard, the span/depth ratio is determined to be about 40, which coincides with that in the standard on the three-point bending test of CFRP.[10,11] As for wood, it is afraid that the ENF test would be conducted based on a span/depth ratio of 12–16, which is standardized on the three-point bending test of wood in JIS,[12] the American Society for Testing and Materials (ASTM),[13] and the International Organization for Standardization (ISO).[14] In this span/depth ratio range, however, the load-deflection compliance would vary markedly with the span/depth ratio, and it is feared that the fracture toughness cannot be obtained properly because of this variation in compliance. The additional deflection varies with the Young's modulus/shear modulus ratio ($E_x/G_{xy}$), and it is difficult to determine the proper span/depth ratio for obtaining a stable value for fracture toughness without considering the effect of $E_x/G_{xy}$. In general, CFRP has a large $E_x/G_{xy}$ ratio; and hence the span/depth ratio is determined to be 40.[10] In contrast, the $E_x/G_{xy}$ ratio of wood is smaller than that for CFRP, and the smaller span/depth ratio would be permissible. Additionally, the deflection behavior cannot be expressed properly by the elementary beam theory when the span/depth ratio is large.[2] Thus, it is thought that the span/depth ratio is should be at least more than 20 for measuring the fracture toughness of wood properly with a single load-deflection relation.

## Conclusions

The influence of the span/depth ratio when measuring the mode II fracture toughness $G_{IIc}$ of wood by end-notched flexure (ENF) tests was examined, and the following results were obtained.

1. When the span/depth ratio was small, the fracture toughness calculated with the data of load-deflection compliance was large. In contrast, the fracture toughness calculated with the equation containing Young's modulus tended to be constant.
2. In the small span/depth ratio range, the load-deflection compliance was estimated to be larger than that predicted by Timoshenko's bending theory.
3. To obtain the proper fracture toughness of wood with the load-deflection relation, the span/depth ratio should be larger than that determined in several standards for the simple bending test method of wood, which is 12:16.

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