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Differences of tensile strength distribution between mechanically high-grade and low-grade Japanese larch lumber II: Effect of knots on tensile strength distribution

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Abstract The tensile strength (TS) test results of Japanese larch (*Larix kaempferi*, Carriere) lumber of varying length have shown that the length effects on TS were different between high-grade (H) and low-grade (L) lumber. In this paper, we examined the effect of knots on the TS distribution by measuring the number of knots and the knot area ratio of each specimen. There were more knots in L than in H; and the knot area ratio in L distinctly increased as the length increased compared to that in H. The correlation coefficients between physical properties and TS indicated that knots were the most influencial factor for TS among several physical properties: annual ring width, distance from pith, density, dynamic Young's modulus, and knots. We attempted to estimate the length effect parameters by introducing the concept of assumed knot strength. We thought that the length effect parameters for 50th percentiles of TS could be estimated well with fitted 3P-Weibull, and that the parameters for 5th-percentiles could be estimated well with 2P-Weibull fitted to lower-tail 10% data by the likelihood method. The differences of length effect on TS between H and L should be governed by the presence of knots. The independent model based on the concept of assumed knot strength may express the TS of structural lumber of various lengths.

Key words Grouped knots · Edge knots · Weibull distribution · Assumed knot strength

Introduction

The design of structures with engineered wood products[1] such as glued laminated timber may be governed by the strength values of structural lumber in tension parallel to the grain. It is well known that as for the length effect, the tensile strength (TS) of lumber decreases as the length of each piece of lumber increases. Lam and Varoglu[2] reported the nonparametric length effect parameters for 5th percentiles in 2 × 4 inch spruce-pine-fir kiln-dried lumber: 0.19 in visual SS grade and 0.13 in visual no. 2 grade. Madsen[3] obtained 0.22 for the 5th percentile in commercial mixed species by the slope method using 25% of the data. Sugi, which is one of the most important species in Japan, was investigated by Hayashi et al.[4] to evaluate the length effect on TS. They conducted tensile tests on 32 × 140 mm lumber with 180, 240, and 300 cm length, 0.15 was obtained as the length effect parameter.

The length effect is dependent on species and grade. To evaluate the length effect parameter of Japanese larch (*Larix kaempferi*, Carriere) lumber we conducted tensile tests of mechanically graded lumber of various lengths, as shown in a previous paper.[5] The groups tested were of high grade (H) and low grade (L), and there average dynamic Young's moduli were 12.8 GPa in H and 7.5 GPa in L. The inverse values of the shape parameter of fitted two-parameter Weibull (2P-Weibull) were almost equal to the length effect parameters estimated by the nonparametric method except the 5th percentile in H. Although 2P-Weibull fitting to all data may be useful for estimating the length effect parameters, it is not negligible that most lumber fails owing to the presence of knots.

We then investigated the relation between TS and knots. To apply the weakest-link theory to this relation, we introduced assumed knot strength distribution and estimated the TS distribution of lumber tested with two gauge lengths (100 and 180 cm) using TS data of 60 cm gauge length lumber.

Experiment

Materials were sampled from Japanese larch lumber usually used for manufacturing glued laminated timber at a factory

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in Nagano Prefecture in Japan. The tested specimen was selected using the Japanese-made continuous mechanical grading machine.[6] The groups of tested grade were H and L lumber whose average dynamic Young's moduli (E_f) were 12.8 GPa in H and 7.5 GPa in L. The standard deviations of E_f were 1.21 GPa in H and 0.96 GPa in L. The spans of TS tests were with three gauge lengths: 60, 100, and 180 cm. There were six types of specimen: H060, H100, H180, L060, L100, and L180. The capital letter denotes the grade and the numerical value the length. The tests were done according to the Japanese Agricultural Standard for Structural Glued Laminated Timber[7] (JAS), and we recorded the TS for each specimen. The 50th and 5th percentiles of TS were calculated by the nonparametric method for each specimen, and the length effect parameters for the 50th and 5th percentiles were obtained by the slope method. More information can be found in a previous paper.[5]

The number of knots whose diameters were larger than 5 mm on wide and narrow faces of lumber were counted before the tests. At the same time, we measured the knots' area ratio (KAR) of grouped knots (GK) and edge knots (EK) for each specimen and recorded the locations and KAR values for the largest GK and EK knots separately. Though the requirement of GK and EK is set with a knot diameter ratio in JAS, we believe that KAR might be a simple indicator of the loss of the cross section of lumber that directly influences the TS. EK in kiln-dried surfaced lumber is defined by the specification for manufacturing structural glued laminated timber[8] that an EK is a knot whose part or whole is at the edge – not farther than 5 mm from the edge. After the tests, we checked whether the specimen failed at the recorded location and cut a small piece to measure the moisture content from the nearest portion of failure of each specimen. In addition to the moisture content, we measured annual ring width, density, and distance from pith with a transparent plastic board on which half-circles were drawn for each specimen.

Results and discussion

Relation between knots and TS

The number of knots on the wide and narrow faces of lumber are shown in Table 1. The number of knots increases as the span increases and the coefficients of variation decrease. It was expected that the influence of knots on TS in H grade lumber would be weaker than that of L grade lumber, as the numbers in H are smaller than in L. Figure 1

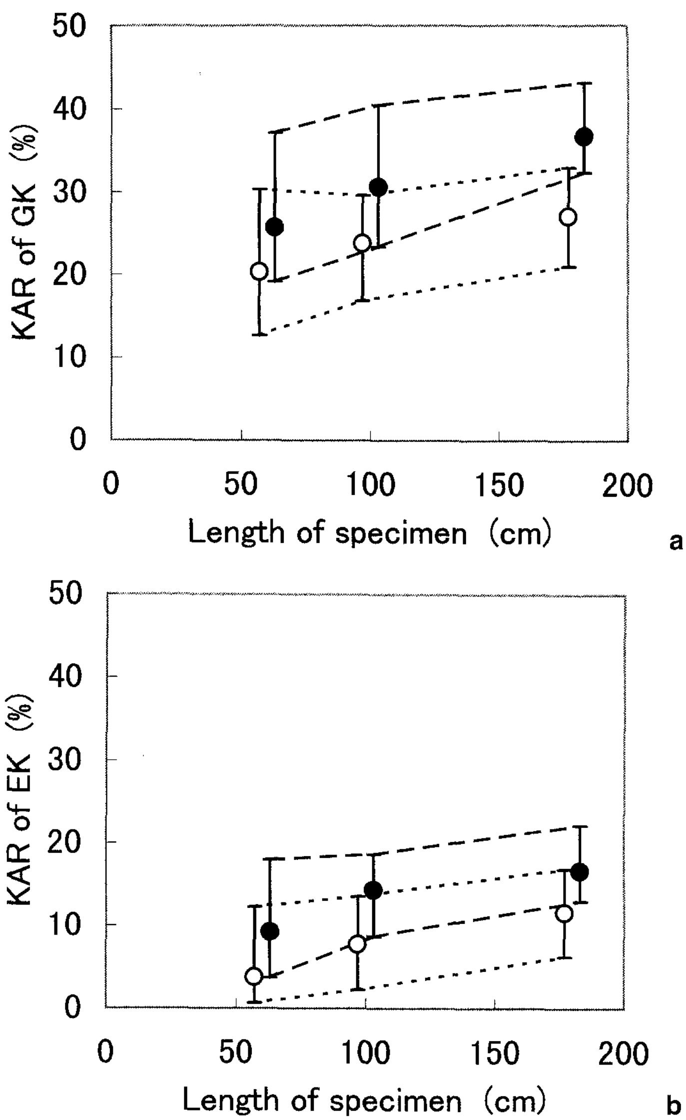


Fig. 1. Knot area ratio (*KAR*) of grouped knots **a** and edge knots **b** Plots and error bars denote medians and quartile ranges, respectively. *GK*, grouped knots; *EK*, edge knots; *open circles*, high-grade lumber; *filled circles*, low-grade lumber

Table 1. Number of knots on wide and narrow faces of lumber

Specimen	Wide faces				Narrow faces			
	Mean[a]	Maximum	Minimum	No./m	Mean[a]	Maximum	Minimum	No./m
H060	7.8 (42.9)	16	0	13.1	1.0 (93.2)	5	0	1.7
H100	13.5 (42.2)	31	1	13.5	1.6 (87.1)	7	0	1.6
H180	23.3 (35.9)	40	6	12.9	2.9 (65.4)	8	0	1.6
L060	10.8 (35.9)	21	2	18.0	1.6 (88.5)	10	0	2.7
L100	16.9 (32.0)	33	6	16.9	2.6 (62.2)	7	0	2.6
L180	31.6 (24.5)	54	9	17.6	4.4 (4.4)	10	0	2.5

The diameters of counted knots were larger than 5 mm
[a] Values in parentheses are coefficients of variation (%)

Table 2. Correlation coefficients for physical properties and tensile strength

Specimen	Annual ring width	Distance from pith	Density	Knot area ratio		E_f
				GK	EK	
H060	−0.340	0.263	−0.242	−0.555	−0.632	0.307
H100	−0.110	0.173	−0.127	−0.458	−0.510	0.482
H180	−0.137	0.186	−0.171	−0.540	−0.615	0.255
L060	−0.364	0.430	−0.000	−0.425	−0.751	0.213
L100	−0.374	0.256	0.120	−0.609	−0.571	0.359
L180	−0.316	0.246	0.190	−0.415	−0.511	0.363
Average of H	−0.196	0.207	−0.180	−0.518	−0.585	0.348
Average of L	−0.351	0.311	0.103	−0.483	−0.611	0.3125

GK, EK, grouped knots and edge knots; E_f, Young's modulus measured by the longitudinal vibration method

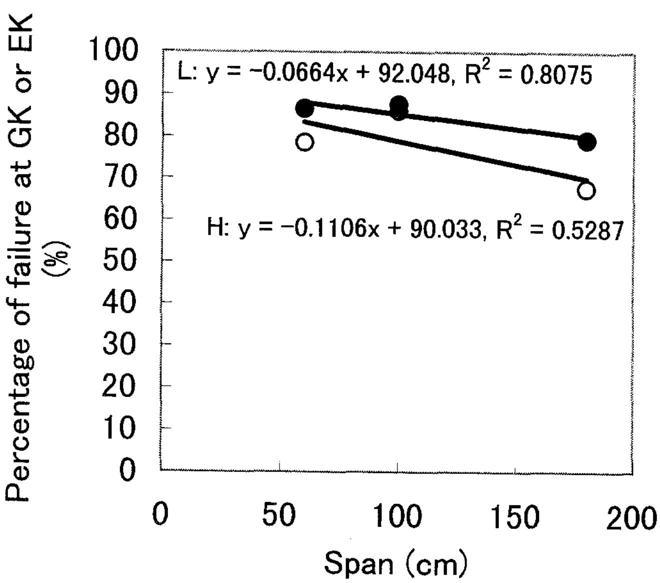


Fig. 2. Relation between spans and percentages of failure by GK or EK. R^2, coefficient of determination; *open circles*, high-grade lumber; *filled circles*, low-glade lumber

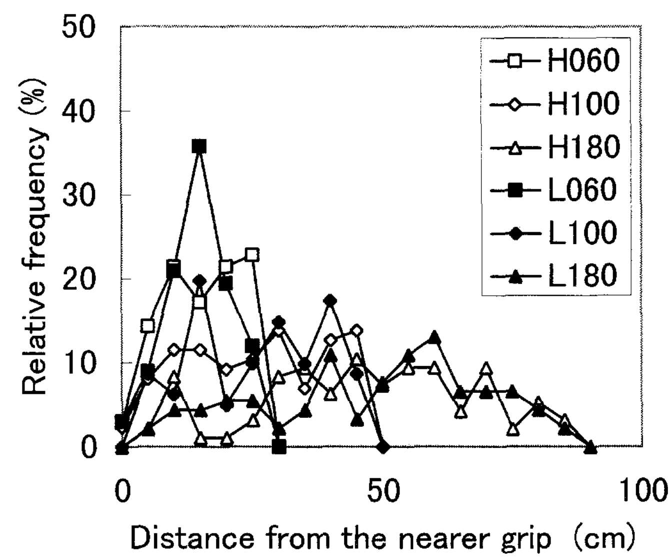


Fig. 3. Distribution of lengthwise failed location

shows the first quartiles, medians, and third quartiles of the KAR for GK and EK. The KAR of GK in L distinctly increases as the span increases, whereas a slight increase is observed in H, especially for the first quartiles. It might be expected that the length effect on TS in L should be stronger than that in H, as a larger KAR means a larger reduction of TS.

Correlation coefficients between physical properties and TS are shown in Table 2. These data indicated that the most effective factor acting on TS among these properties was knots, and there were small differences in the correlation coefficients between GK and EK. The relation between spans and percentages of failure at the GK or EK locations are shown in Fig. 2. The percentages ranged from 68% to 88%; and most of lumber failed at the knots. The percentages in L slightly decreased as the span increased compared to those in H.

The distribution of lengthwise locations of failure are shown in Fig. 3. The distribution in each specimen was almost identical to the distance from the nearer grip. The test span influences the failed location of lumber.

Concept of assumed knot strength distribution

The TS of lumber can be estimated by the strength of the weakest element of the chain-like link on the basis of the weakest link theory.[9] Suppose a knot-free member consists of n knot-free elements, as shown in Fig. 4a the cumulative distribution function (CDF) of TS of the member $F(x)$ can be expressed with the CDF of one-element TS as

$$F(x) = 1 - \left[1 - G_c(x)\right]^n \tag{1}$$

where n is number of elements, and $F(x)$ and $G_c(x)$ are the CDF of the strength of the member and a knot-free element.

In the case of commercial lumber, it may be presumed that the lumber consists of knot-free elements and knotty elements, as shown in Fig. 4b. When a member consists of

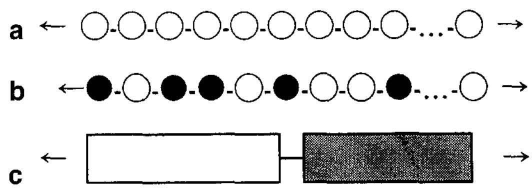


Fig. 4. Concept of assumed knot strength distribution. **a** Knot-free member. **b** Full-size member. **c** Assumed knot strength distribution. *Open circles*, knot-free element; *filled circles*, knotly element; *open bar*, strength of knot-free lumber; *filled bar*, assumed knot strength

$(n - k)$ knot-free elements and k knotty elements, the CDF is written as

$$F(x) = 1 - \left\{\left[1 - G_c(x)\right]^{n-k}\left[1 - G_k(x)\right]^k\right\} \tag{2}$$

where $G_k(x)$ is the CDF of TS of one knotty element, and k is the number of knotty elements.

It is impossible to determine the number, size, and strength of each element, so we presume that the member consists of two portions: a knot-free portion and a knotty portion. We call the strength of the knotty portion the "assumed knot strength" in the following sentences; it does not mean the strength of the knots. By substituting $[1 - F_c(x)]$ for $[1 - G_c(x)]^{n-k}$ and $[1 - F_k(x)]$ for $[1 - G_k(x)]^k$ in Eq. (2), Eq. (2) is rewritten as

$$F(x) = 1 - \left[1 - F_c(x)\right]\left[1 - F_k(x)\right] \tag{3}$$

where $F_c(x)$ and $F_k(x)$ are the CDFs of knot-free (clear) wood strength and assumed knot strength.

Once $F_c(x)$ and $F_k(x)$ of members with a certain length are determined, the TS distribution in any members with various length can be estimated from $F_c(x)$ and $F_k(x)$. It may be assumed that $F_c(x)$ is not affected by the length of members compared with $F_k(x)$ because the number of knots increased as the length of the lumber increased, as shown in Table 1, and knots caused failure in most of the lumber, as shown in Fig. 2. From Eq. (3), the CDF of TS in lumber with any length L can be expressed with $F_c(x)$ and $F_k(x)$ in L_0 (long lumber) as

$$F(x) = 1 - \left[1 - F_c(x)\right]\left[1 - F_k(x)\right]^{L/L_0} \tag{4}$$

where $F(x)$ is the CDF of TS in L-long lumber.

To obtain $F(x)$ in H, grade lumber, three-parameter Weibull CDF (3P-Weibull) was fitted to the original TS data in H060. 3P-Weibull is expressed as

$$F(x) = 1 - \exp\left[-\left(\frac{x - x_0}{m}\right)^k\right] \tag{5}$$

where k, m, and x_0 are the shape parameter, scale parameter, and location parameter of the 3P-Weibull, respectively. The location parameter x_0 was sought by the likelihood method and calculating correlation coefficients

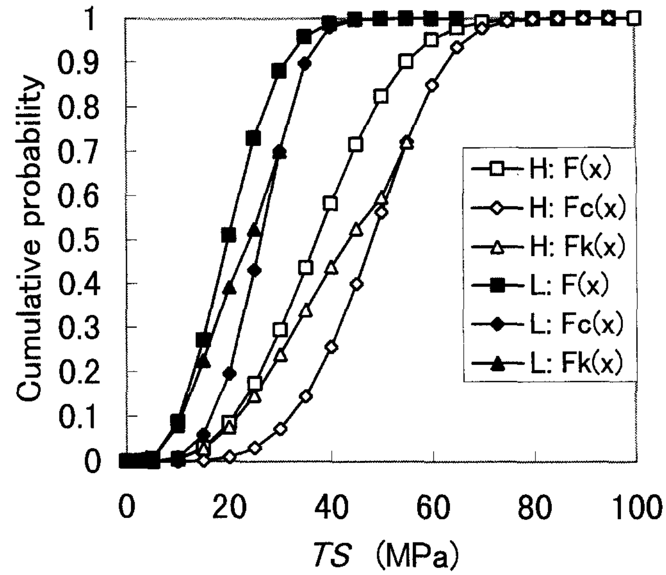


Fig. 5. Distribution of the assumed GK strength: $F_k(x)$. *GK*, grouped knots; $F(x)$, fitted 3P-Weibull function to data; $F_c(x)$, cumulative distribution function of tensile strength of knot-free lumber

with all data for H-grade lumber in advance, and the x_0 obtained was 4.1 MPa because the location parameter is defined as the characteristic smallest value and should be common to members of any length. The remaining parameters k and m were calculated using the likelihood method with TS data in H060.

To calculate $F_c(x)$ for H, we used the TS data for H060 in case of GK-affected failure. These TS data are expressed as $X_{(j)}$, with the subscript (j) meaning j-th data. With $X_{(j)}$ and $\mathrm{KAR}_{(j)}$ corresponding to $X_{(j)}$, we computed $X_{c(j)} = X_{(j)}/(1 - \mathrm{KAR}_{(j)})$. 3P-Weibull was fitted to $X_{c(j)}$ by the likelihood method, and we obtained $F_c(x)$ for H.

We calculated $F(x)$ and $F_c(x)$ for L with the location parameter of 2.6 MPa. The obtained parameters using GK data together with EK data are shown in Table 3. The parameters of $F_c(x)$ using GK data were close to the parameters of $F_c(x)$ using EK data. We obtained $F_k(x)$ in H060 and L060 from Eq. (4) by setting $L_0 = L = 60$ cm.

The $F(x)$, $F_c(x)$, and $F_k(x)$ are shown in Fig. 5. Because $F_k(x)$ was smaller than $F_c(x)$ in the ranges of TS > 51.6 MPa in H and TS > 27.6 MPa in L, $F_k(x)$ was set as the value of $F_c(x)$ in the ranges.

To compare the shape of $F(x)$ and $F_c(x)$ between H and L, the ratio of x in H to x in L at each percentage point were computed with the difference of E_f between H and L. Because it might be easier to estimate $F(x)$ and $F_c(x)$ in lumber with another E_f, we used the difference of E_f (= dE_0), which was 5.1 GPa [= 12.6 (in H060) − 7.5 (in L060)]. The multiplying factor per 1 GPa (= h_i) was expressed as

$$\frac{X_H(i)}{X_L(i)} = h_i \cdot dE_0 \tag{6}$$

where $X_{H(i)}$ and $X_{L(i)}$ are x in H060 and L060 at each i-th percentage point, respectively. The obtained h_i at the i-th

Table 3. Parameters of 3P-Weibull and 2P-Weibull used to estimate length effect

Specimen	Original CDF of TS: $F(x)$			Estimated CDF of knot-free lumber: $F_c(x)$		
	k	m	x_0	k	m	x_0
Likelihood method using 100% TS data				With GK data		
H	2.80	37.7	4.1	4.19	48.0	4.1
L	2.40	20.0	2.6	3.77	26.1	2.6
Likelihood method using 100% TS data				With EK data		
H	2.80	37.7	4.1	5.08	42.6	4.1
L	2.40	20.0	2.6	4.41	23.0	2.6
Likelihood method using lower 10% TS data				With GK data		
H	7.62	29.4	0	6.76	43.4	0
L	3.97	21.5	0	7.61	22.4	0
Regression method using lower 10% TS data				With GK data		
H	6.45	30.6	0	4.36	52.5	0
L	2.72	28.6	0	15.46	18.2	0

CDF, cumulative distribution function estimated using H060 and L060 data; k, m, x_0, shape, scale, and location parameters of 3P-Weibull, respectively; GK, grouped knots; EK, edged knots

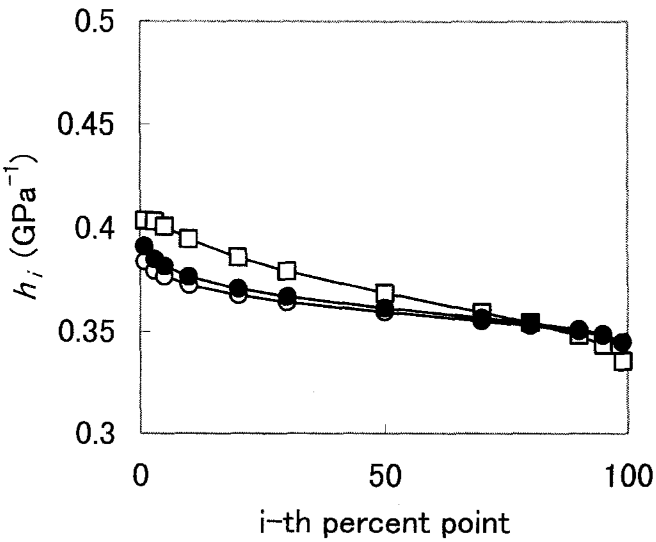


Fig. 6. Comparison of tensile strength (*TS*) distribution between original data and estimated knot-free strength. h_i, multiplying factor of TS per 1 GPa of E_f at i-th percentile: *Original* (*squares*) calculated from $F(x)$; C-GK (*open circles*) and C-EK (*filled circles*) calculated from $F_c(x)$ using GK and EK data, respectively

percent point is shown in Fig. 6. There were small differences among "Original", "C-GK," and "C-EK," with a slight decreasing trend with the i-th percentage points. The differences in the shape of distributions among these parameters may be small.

Estimation of the length effect on TS

We estimated the length effect on TS by computing TS distributions of 100 and 180 cm long specimens with data from a 60 cm long specimen. In the case of H, the TS distributions of H100 and H180 were calculated from Eq. (4) using the above-mentioned $F_c(x)$ and $F_k(x)$ from H060 data. The length effect parameters for the 50th and 5th percentiles were then obtained by the slope method with the 50th and 5th percentiles of TS distributions for each specimen. It was similar for L. The estimated values for the 50th and 5th percentiles are shown in Fig. 7. The experimental values for the 50th percentiles (NPM) and 5th percentiles (NPL) were also plotted in Fig. 7. Figure 7a shows that the plots of experimental data were on the estimated curves of the 50th percentiles, whereas Fig. 7b shows that the plots of experimental data were not on the estimated curves for the 5th percentiles. The estimated length effect parameters are shown in Table 4. Though the estimated parameters were almost identical to the experimental parameters using the nonparametric method for the 50th percentiles, it is necessary to find a method other than 3P-Weibull fitting for the 5th percentiles.

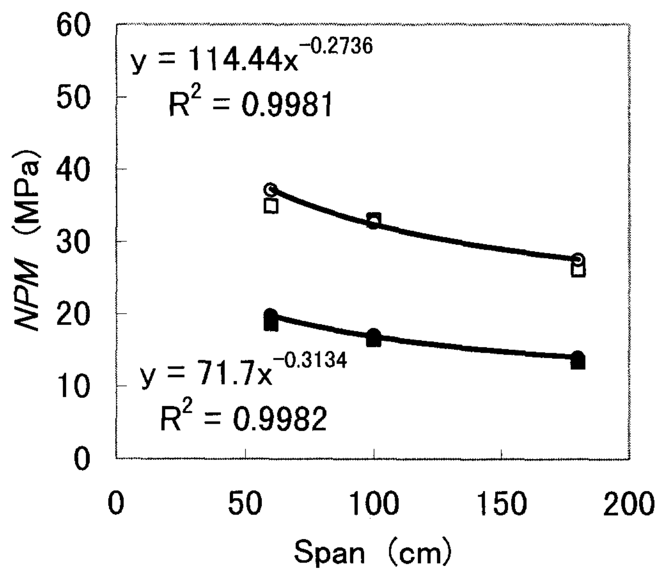
Durrans et al.[10] found that the 2P-Weibull model could yield good quantile estimates in the lower tail of the distribution, even when the true population was in 3P-Weibull. Therefore we attempted to estimate 5th percentiles of TS distribution using 2P-Weibull. They also suggested that the likelihood method is best in terms of bias and the regression method is best in terms of RMSE (root mean square error) for using lower-tail 10% data.[10] The parameters of 2P-Weibull are shown in Table 3, and the length effect parameters that were calculated with the fitted 2P-Weibull are shown in Table 4. It was clear that the estimated length effect parameters for 5th percentiles with 2P-Weibull by the likelihood method were the nearest to the experimental results using the nonparametric method among the estimated parameters. It is believed that 2P-Weibull by the likelihood method is better than 3P-Weibull to estimate the length effect parameter for 5th percentiles.

Conclusion

An experimental study was conducted to evaluate the effect of length on the parallel-to-grain TS of Japanese larch. The tensile test was conducted for each of three lengths (gauge lengths of 60, 100, and 180 cm) and two grades (H and L). We also investigated the effect of knots on TS and obtained the following results.

Table 4. Comparison of length effect parameters for the nonparametric method and estimation with assumed knot strength distribution

Specimen	Nonparametric method	Estimated with assumed knot strength distribution			
		100% GK data likelihood	100% EK data likelihood	Lower 10% GK data	
				likelihood	Regression
50th Percentiles					
H	0.268	0.274	0.275	–	–
L	0.304	0.313	0.312	–	–
5th Percentiles					
H	0.121	0.251	0.255	0.094	0.190
L	0.256	0.261	0.264	0.250	0.233



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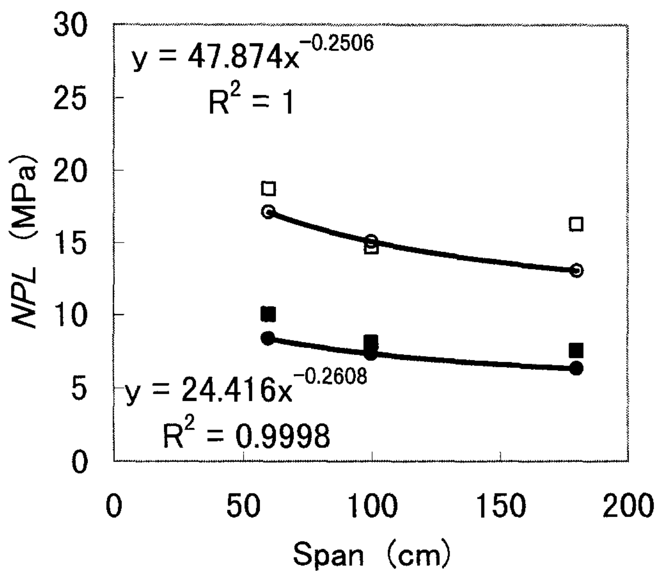

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Fig. 7. Estimation of (**a**) NPM (50th percentiles) and (**b**) NPL (5th percentiles). NPM and NPL are 50th and 5th percentiles of tensile strength distribution by a nonparametric method. Lines are regression curves of estimated values. R^2 is the coefficient of determination. *Open squares*, experimental H; *filled squares*, experimental L; *open circles*, estimated H; *filled circles*, estimated L

1. The number of knots in L was larger than in H; and the knot area ratio for L distinctly increased as the length increased compared to that for H.

2. The results of computing correlation coefficients between physical properties and TS indicated that knots were the most important influence on TS of all the physical properties, including the annual ring width, distance from pith, density, dynamic Young's modulus (E_f), and knots.

3. We attempted to estimate the length effect parameters by introducing the concept of assumed knot strength. We thought that the length effect parameters for the 50th percentiles of TS could be estimated with fitted 3P-Weibull and that the parameters for the 5th percentiles could be estimated with 2P-Weibull fitted to lower-tail 10% data by the likelihood method. The differences of length effect on TS between H and L should be governed by the presence of knots. The independent model based on the concept of assumed knot strength may express the distributions of TS of various-length lumber.

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