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Mechanism for deformation of wood as a honeycomb structure II: First buckling mechanism of cell walls under radial compression using the generalized cell model*

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Abstract Coniferous woods were modeled as honeycomb cellular solids consisting of hexagonal-prism tracheids to examine the mechanism for radial compression. Because of the abrupt breaks of radial cell walls, it was assumed that the first break followed Euler's equation of buckling. The nominal stress at the buckling of the radial cell wall was theoretically obtained based on this assumption, and the actual nominal stress was obtained experimentally. The theoretical stress was found to correspond almost to the experimental value. This finding suggests that the abrupt first break that occurs in wood under radial compression can be mainly attributed to the buckling of radial cell walls.

Key words Porous structure · Radial compression · Buckling · Image analysis · Cell wall

Introduction

Wood demonstrates excellent ductility when it undergoes transverse compression. This results from two characteristics of wood. First, wood consists of a honeycomb of cellular solids in which the cells are arranged in either a longitudinal or a radial direction. Second, the cell walls become much closer to each other after the hollow part of wood is broken, which enables it to endure more transverse compression.

These characteristics of the wood subjected to transverse compression are important when wood is used as a construction material. In Japan, buildings must be designed to withstand the force of an earthquake, that is, repeated loads as earthquakes are often experienced in Japan. Energy absorption is an efficient way to resist the repeated loads. To improve earthquake resistance for wooden buildings, it is essential to enhance the energy-absorbing capacity of the structure.

Attempts have been made to make a model for the hollow cells of wood and to frame a theory for the process of their elastic transverse deformation.[1–10] Ohgama and Yamada[3,5] and Kanaya and Yamada[1,2] succeeded in modeling the wood as porous solids consisting of cell-wall substance and the cell lumen. They carried out theoretical studies to examine how the porous structure of wood contributes to the modulus of elasticity and verified their validity through experimentation. Moreover, Ohgama and colleagues[6,7] examined the tracheid of hinoki wood when a tensile force was applied in a tangential direction. They carried out a numerical analysis by a finite element method to obtain the distribution of stress within a cell wall[6] and the modulus of elasticity of cell-wall substances,[6,7] taking the shape of the cell lumen into consideration. Liu et al.[10] studied experimentally the relation between stress and strain under radial compression, considering the porous structure of wood. However, no studies have analyzed an abrupt event, that is, the first break of the wood due to the breaks in radial cell walls under compression in a radial direction observed in our previous study.[11]

The mechanism for radial compression of coniferous wood was discussed in our previous report.[11] The first break occurred simultaneously in one tangential row of earlywood tracheids because of abrupt breaks of radial cell walls just after the load–displacement diagram exceeded the proportional limit. This abrupt break is seems to be an outstanding feature of wood under radial compression.[11–15] In this study, coniferous wood was modeled as cellular solids consisting of hexagonal-prism cells (tracheids). Theoretical analysis was carried out to determine if the first break due to abrupt breaks of the radial cell walls was mainly attributed to buckling of the cell walls. The validity of the analysis was then verified by experimentation.

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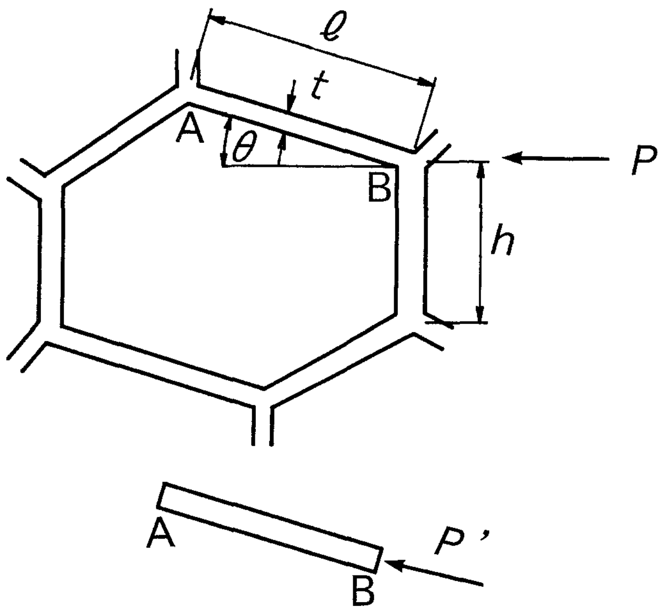

Fig. 1. Cell model shape under radial compression for earlywood tracheid of coniferous wood. *l*, *h*, lengths of the radial and tangential cell wall, respectively; *t*, thickness of the cell wall; *P*, compressive load

Theory

In the previous study on radial compression of coniferous woods,[11] the deformation behavior of the first broken cells was analyzed and the following evidence was obtained. First, the cells had shear deformation in a radial direction until the occurrence of the first break. Second, the shearing strain by the shear deformation of cells was positively related to the compressive load. Finally, the first break occurred abruptly because of the abrupt breaks in the radial cell walls. This abrupt break is considered to be an outstanding feature of wood under radial compression. We think there is no linear relation between the progress of the first break and the compressive load.

Coniferous wood was used as the model for the theoretical analysis in this report. About 90% of coniferous wood tissues are made up of tracheids. The tracheids have variously shaped cross sections, and in this study many hexagonal sections were observed in the earlywood. The woods, therefore, were modeled as honeycomb cellular solids consisting of hexagonal-prism tracheids. The following analysis was carried out.

The model of a cell used for this analysis is shown in Fig. 1. With the assumption that the stress at an infinite point (nominal stress) (σ_R) is loaded over the side section (tangential section) of the model shown in Fig. 1, the load (P) produced on radial wall AB in a radial direction can be written as

$$P = \sigma_R(h + l\sin\theta)b \tag{1}$$

where l is the length of radial wall AB, b is the width of the radial wall, and h is the length of the tangential wall. Then, the compressive load was assumed to be applied on the radial wall AB in a direction vertical to it. For this compressive load (P'), this model had the form

$$P' = \frac{P}{\cos\theta} \tag{2}$$

No evidence was found that the radial wall was deformed by bending before the first break. The first break was considered to be an abrupt event. Therefore, it was assumed that radial wall AB suffered the buckling fracture when P' reached the P_{euler} of Euler's buckling load. This assumption ignored a minor change of θ that occurred just prior to the buckling of the radial wall. P_{euler} was given by the following equation.[16]

$$\sigma_e = \frac{n^2\pi^2}{12}\frac{t^3\cos\theta}{l^2(h + l\sin\theta)}E_S \tag{3}$$

In this equation, n is the coefficient of fixity, I is the moment of inertia of the area, and E_S is the modulus of elasticity for cell wall substances. When P' is equal to P_{euler}, the stress in a radial direction at an infinite point (σ_e) had the following form based on Eqs. (1), (2), and (3).

$$\sigma_e = \frac{n^2\pi^2}{12}\frac{t^3\cos\theta}{l^2(h + l\sin\theta)}E_S \tag{4}$$

where t is the thickness of the radial wall. The coefficient of fixity (n) was given by the following equation.[16]

$$n = \frac{2\beta^*}{\pi} \tag{5}$$

where β^* is the result calculated with the following equation.

$$\tan\beta = \frac{2l}{h\beta} \tag{6}$$

E_S was obtained by extrapolating the specific gravity of cell wall substances into an equation showing the relation between the specific gravity of wood and the modulus of elasticity in a radial direction.[3]

Experiment

Hinoki (*Chamaecyparis obtusa* Endl.), sugi (*Cryptomeria japonica* D. Don), and kuromatsu (*Pinus thunbergii* Parl.) were used as specimens. The shape and size were described in a previous report.[11] The section of the specimen (cross section) to be observed was obtained with a microtome. The average specific gravities and moisture contents for hinoki, sugi, and kuromatsu were 0.38 and 1.8%, 0.37 and 3.0%, and 0.52 and 2.5%, respectively.

The procedure for the compressive test was the same as reported in the previous study.[11] The equipment was made by incorporating a machine for material tests into a wet-type scanning electron microscope (WET-SEM). The cross section of the specimen was continuously observed in this

WET-SEM chamber under radial compression. The crosshead speed was 0.3 mm/min.

The SEM images were analyzed as in the previous study.[11] The shape and size of the first broken cells were measured up to just before the first break. One row of earlywood tracheids arranged in a tangential direction suffered the first break at the same time. Eight to ten cells were selected for each specimen at random and measured.

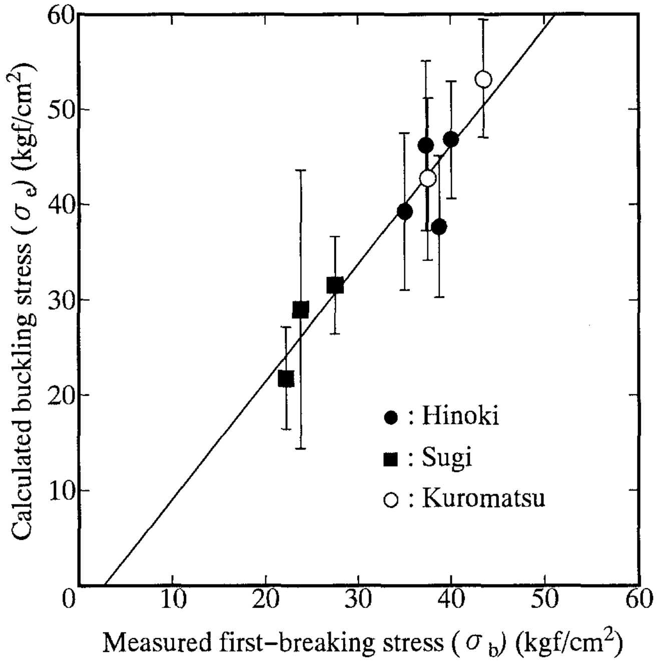

Fig. 2. Relations between the calculated buckling stress (σ_e) and the measured first-breaking stress (σ_b) of the radial walls of tracheids. Each symbol represents the average value within one test specimen. Horizontal lines are the standard deviations. Slanted line is the regression line $\sigma_e = 1.24\sigma_b - 3.21$; $r = 0.946$, which is significant at the 1% level

Results and discussion

The shape parameters of the first broken cells were measured with image analysis, and the results are shown in Table 1. Figure 2 shows the relation between the theoretical nominal stress at the buckling of the radial cell wall obtained by Eq. (4) and the measured nominal stress at the first break. The points show the average buckling stress of cells within one specimen; and the line extending from each point, parallel to the x-axis, is the range of their standard deviation. It should be kept in mind that the theoretical values vary according to the shape and size of each cell, and that only one value for the measured first breaking stress was obtained because the first break occurred on one tangential row of earlywood tracheids at the same time.

The theoretical buckling stress and the measured first breaking stress of the radial wall of the tracheids were found to have a high correlation. This suggests that the first break that occurs in wood under radial compression can be mainly attributed to the buckling of radial cell walls.

Because deformation of cells by bending was not found prior to the first break in this study, the effects of bending stress on radial walls were not taken into consideration, nor were the effects of rays arranged in the direction in which the load was applied. Therefore, these problems remain to be solved in the devised model. Further improvement in this model is needed to consider the deformation behavior of cells in the next stratification stage[11] after the first break.

Figure 3 shows the deformation process of each cell from when the load starts to the first break under compression in a radial direction. We think that such a change of cell shape is a important factor in relaxing the stress concentration of wood.

Fig. 3. Deformation model of an earlywood tracheid under radial compression up to the first break

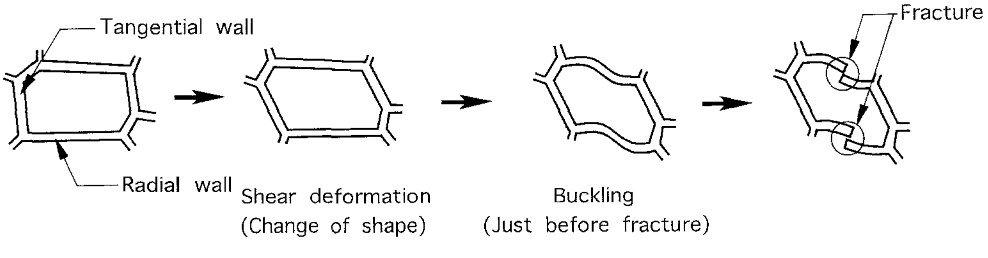

Table 1. Shape parameters of each species

Species	l (μm)	h (μm)	t (μm)	θ (°)	n	E_S (kgf/cm^2)
Hinoki (*Chamaecyparis obtusa* Endl.)	26.7 (4.6)	18.0 (4.1)	3.5 (0.3)	11.0 (7.4)	0.757 (0.047)	3.26×10^4
Sugi (*Cryptomeria japonica* D. Don)	23.4 (3.6)	20.5 (3.7)	3.0 (0.1)	14.5 (4.4)	0.712 (0.039)	3.26×10^4
Kuromatsu (*Pinus thunbergii* Parl.)	28.2 (2.3)	22.4 (2.8)	4.3 (0.3)	13.9 (3.4)	0.728 (0.023)	3.26×10^4

Numbers in parentheses are standard deviations

n, coefficient of fixity; E_S, modulus of elasticity of cell wall substances in a radial direction, the value noted by Ohgama and Yamada.[3] See Fig. 1 for definitions of l, h, t, and θ

Conclusion

Coniferous woods were modeled as honeycomb cellular solids consisting of hexagonal-prism tracheids to examine the mechanism for radial compression. It was assumed that the first break occurred because of the abrupt breaks of the radial cell walls, following Euler's equation of buckling. The nominal stress at the buckling of radial cell walls was theoretically obtained based on this assumption, and the actual nominal stress was obtained by experiment. The theoretical stress was found to correspond almost completely to the actual stress. This finding suggests that the abrupt first break that occurs in wood under radial compression can be mainly attributed to the buckling of radial cell walls.

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