



ORIGINAL ARTICLE

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An attempt to unify the Brinell, Janka and Monnin hardness of wood on the basis of Meyer law

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Abstract

This work concerns basic research on the concept of wood hardness in a sense similar to Brinell or in the modified sense of Monnin. The experimental part of this article is based on research carried out on beech wood with six indenters: three ball indenters of 10, 15 and 30 mm and three cylindrical indenters of 10, 15 and 30 mm. On the basis of measurements for a wide range of loads, relations analogous to Meyer power law of were obtained, with the exponent determined both for balls equal to 5/2 and for cylinders equal to 3/2. These exponents turned out to be exactly the arithmetic mean of exponents for ideally elastic and ideally plastic bodies. On this basis, new hardness formulae were proposed, taking into account Meyer law and the diameters of indenters. Therefore, the proposed two hardness formulae (for the ball and cylinder) are a generalization and development of Meyer hardness law.

Keywords: Wood hardness, Laws of hardness, Sphere and circular indenters, Contact mechanics, Beech wood

Introduction

Review of measures and hardness tests

The concept of hardness is included in the science of material strength, but it is also associated with a branch of mechanics called contact mechanics. Heinrich Hertz made pioneering progress in this area [1]. His achievements were continued, among others, by Maximilian Huber [2]. An outline of the history of hardness studies can be found in the study by Walley [3].

The hardness of a material is not as unambiguous as, for example, strength or the modulus of elasticity. There are, therefore, several types of hardness as well as many tests and hardness scales. There are basically three types of hardness as resistance: scratch resistance, static indentation and dynamic indentation [4, 5]. Moreover, permanent (plastic) indentation differs significantly from elastic indentation. Different shapes of indenters are used in the

case of static permanent indentation. For hard materials, indenters with a sharp end such as a cone (Rockwell test), a regular pyramid (Vickers test) or an extended pyramid (Knoop test) work well. However, rounded indenters, such as a ball (Brinell, Janka, Krippel and Meyer tests) and a cylinder (Monnin test), are much better suited for wood. Due to the specific structure of wood, we basically distinguish the hardness at the longitudinal sections and the significantly higher hardness at the cross section.

Regardless of the indenter's shape and the load applied, three basic indenter measures are used alternatively: indenter projection area, total indenter area or indenter depth [the width (diameter) of indentation d is the easiest to measure indentation parameter, but is not a direct measure of its hardness]. The first measure correctly reflecting the average stress was used by Meyer [6–8]:

$$H_M = \frac{F}{S}, \quad (1)$$

where F is the load force; S is the indentation projection area. Meyer referred this measure to the ball test, but it

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can also be tried on other indenters. The ball or cylinder projection area is determined by elementary formulae:

$$S = \pi d^2/4, S = d \cdot L, \quad (2)$$

where d is the average width of indentation; L is the length of the cylinder contact with the wood. However, it turns out that formula (1) is not widely used and, contrary to appearances, does not experimentally reflect the concept of hardness. In other words, this measure significantly depends on the load used and the size of the indenter. However, for wood, the traditional Janka hardness test [9–14] (the values in kg/cm^2 used by Janka need to be multiplied by $0.0980665 \approx 1/10$ to change to the value expressed in $\text{MPa} = \text{N}/\text{mm}^2$) is based on the above simple principle of determining hardness [15]:

$$H_J = \frac{F_{\max}}{S_{\max}}, \quad (3)$$

where H_J is the Janka hardness (in MPa) at a full indentation, F_{\max} is the force inducing full indentation and S_{\max} is the full indentation projection area. In the Janka method, there is no freedom in selecting the load, as it is measured when the ball enters the depth of its radius (diameter section). In standards [9, 10], the indenter speed is 6 mm/min (except for measuring the hardness module at a speed of 1.3 mm/min in standard [9]), which means that the measurement time is about 1 min (as in standard [16]). And the standards [15, 17, 18] recommend a measurement time of 1–2 min, which also effectively results from the recommendations of the standard [19]. A half-depth indentation is often allowed with the same definition of measure [15] (in this standard the hardness is expressed in $\text{N}/\text{mm}^2 = \text{MPa}$ and the formula (4) is implicitly written by the expansion of the area S , leading to a factor of 4/3):

$$H_{J1/2} = \frac{F}{S} \bigg|_{h=R/2}, \quad (4)$$

where $H_{J1/2}$ is the Janka hardness (in MPa) at half-depth indentation, h is the depth of indentation and R is the indenter radius. A ball with a projection area of $1 \text{ cm}^2 = 100 \text{ mm}^2$, less frequently $25\pi \text{ mm}^2$, is normally used. The standards [15, 18, 19] recommend minimum wood sample sizes of $50 \times 50 \times 50 \text{ mm}$. In modern Janka-type standards [17–19], the division of force by any surface area is abandoned. In this case, the hardness characteristics are the force itself (for indenter diameter $D = 11.28 \text{ mm}$):

$$\begin{aligned} F_J[\text{N}] &= 100 \cdot H_J[\text{MPa}], F_J^*[\text{N}] \\ &= 100 \cdot H_{J1/2}[\text{MPa}] = 4/3 \cdot F[\text{N}]. \end{aligned} \quad (5)$$

Janka hardness is very comfortable for machine use and has been continuously improved [20]. However, accurate measurement of indentation force is somehow problematic with respect to controlling indentation depth, and the use of a type (1) measure transfers its imperfections to (3) or (4). Another example of using the indentation projection area (i.e. the Meyer measure) is the Knoop test (hardness H_{KN}) [6]. The indenter in this test has the shape of a pyramid with a diamond base, but such indenters are not used for wood.

A very attractive concept of hardness measure was proposed by Brinell [21, 22], in which he used the total indentation area A :

$$H_B = \frac{F}{A}. \quad (6)$$

In the SI system, force F is expressed in newtons (N), so the hardness is consistently expressed in $\text{MPa} = \text{N}/\text{mm}^2$. Brinell (like Janka) gave the force in units of load mass, i.e. in kilograms (kg). This was later clarified by the introduction of a unit of kilogram-force (kgf or kgf). Unfortunately, the turbulence associated with changing units continues to lead to inaccuracies, and even to one formal error repeated in some standards. In the metal hardness standards [23, 24] and in the Brinell hardness standards for wood-based flooring materials [25, 26], a factor of 0.102 or $1/g$ appears in formula (6). The purpose of this coefficient is to refer to the traditional unit kg/mm^2 instead of $\text{N}/\text{mm}^2 = \text{MPa}$. However, calculating the hardness in kg/mm^2 and writing it in N/mm^2 is an error in the wooden standards [25, 26] because these units are incomparable, and using an N/mm^2 unit requires a conversion rate of 0.102 to kG/mm^2 (and not vice versa: $1 \text{ kG} = g \cdot 1 \text{ kg} = 9.80665 \text{ N}$, $1 \text{ N} \approx 0.102 \text{ kG}$). Metal hardness standards [23, 24] due to the use of dimensionless of the factor 0.102 do not contain any formal error, but only artificial normalization. Work [27] is proof that this is not just an academic problem. The author quotes the formula from standard [25] containing effectively the factor 0.102, and in the table of wood hardness does not take this factor into account (which results from the order of hardness values given in MPa). Also factor 0.102 does not take into account work [4], while different normalization is used in studies [5, 28]. A more correct and consistent use of the unit $\text{kG} = \text{kgf}$ is included in the standard for wood-based flooring [29] and in the work [30] on lignofol. Additional information on Brinell hardness can be found in numerous metal standards [31–34] and in conversion values of different scales [35].

Brinell referred his measure (6) to a ball (usually with a diameter of 10 mm), but the same hardness formula idea is used for a regular square pyramid in the Vickers test (hardness H_V) [6, 8]. The phenomenon of Brinell measure

for the sphere is based on a few interesting properties of a fragment of the sphere. First of all, the area of the sphere section is directly proportional to its height h (the depth of indentation):

$$A = \pi Dh = \pi D \left(D - \sqrt{D^2 - d^2} \right) / 2. \quad (7)$$

As a result, Brinell hardness is based on the depth of penetration and its total area at the same time, which makes this measure universal. It is worth noting at this point that there are measures based only on the depth of indentation. An example for conical and spherical indenters here is the Rockwell scale (hardness H_R) and for cylindrical indenters the Monnin measure (hardness H_{MO}) described below. The Rockwell scale is based on the arbitrary linear function of the depth of indentation h with a negative slope [6].

The surface area (7) in the form of a central expression may also be interpreted as a side area of a cylinder of ball diameter and height h . Moreover, field A can be treated as area of a circle (disk) with a radius l equal to the chord of half an arc of indentation (Fig. 1):

$$A = \pi l^2, \quad l = \sqrt{Dh}. \quad (8)$$

There is also an interesting approximate mathematical relationship between Brinell measure and Meyer measure for small indentations (it follows from the Laurent series):

$$H_B \approx H_M - F/A_D, \quad (9)$$

where $A_D = \pi D^2$ is the total sphere area. The Brinell () and (7) hardness test of wood is carried out using a ball

with a diameter of $D = 10$ mm loaded in with a force of $6F = 1$ kN (or in practice $F = 100$ kg = 980.7 N). For floor boards is recommended a square sample of 50×50 mm with the original thickness [25, 26]. The application of force should last about 15 s, and its sustaining about 25 s [25, 26]. The paper [36] compares the results of wood indentations made at different speeds of 5, 1 and 0.5 mm/min. In work [37], five different loads of 10, 20, 30, 40 and 50 kg were used for wood with oak hardness.

It turns out that the use of Brinell measure to full indentation gives formally 2 times less H_B hardness than H_J hardness. For a half-depth indentation, this ratio is reduced to 4/3 [see below (31), not (5)]—the total area of a half-depth indentation is equal to the cross-sectional area of the ball and both areas are 4/3 times greater than the projection area of the half-depth indentation. This shows that H_B measure is not easily proportional to H_J , $H_{J1/2}$, i.e. basically to H_M . This means that at least one of these measures is not a homogeneous measure, independent of the used load or size of the indentation. The wood standards [19] (strictly, this standard assumes (5), i.e. that for half-depth indentation $F = 3/4 F_{\max}$, because $S = 3/4 S_{\max}$), [18] assume uniformity of H_J and $H_{J1/2}$ measures for half-depth and full indentations. However, such an assumption does not have the status of confirmation in experiments or in theory [see power laws (14)–(18) and the implications of the Brinell measure (31)]. In turn, standards for wooden floors [25, 26] recommending a constant load of 1 kN in the Brinell method implicitly assume the homogeneity of the H_B measure. The point is that for softwood the indentations will be large and for hardwood the indentations will be small. For example, the research of Karl Huber [38] showed that none of

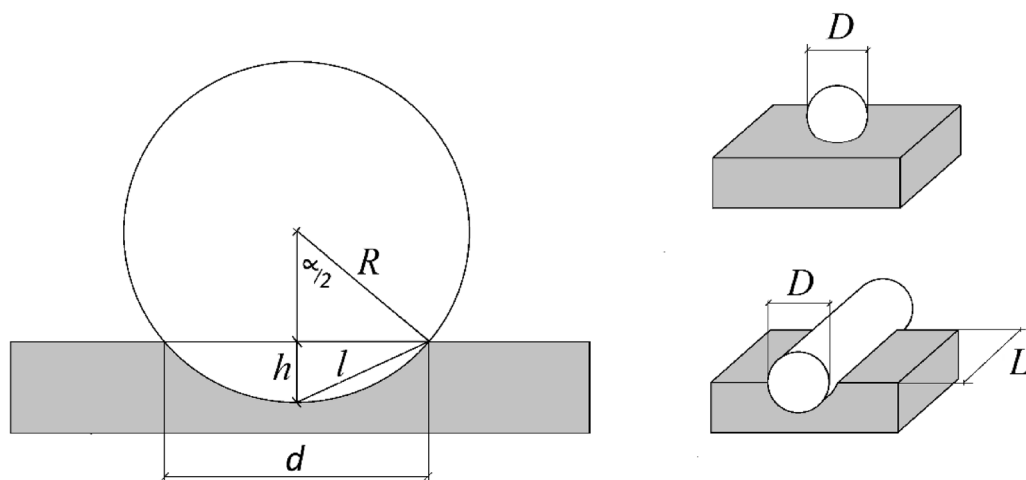


Fig. 1 Diagram of geometrical parameters of indentations in wood exerted by spherical (Brinell, Janka, Krippel and Meyer tests) and cylindrical (Monnin test) indenters

measures H_B and H_M is sufficiently homogeneous [30]. However, even a greater use of Brinell-type measures (for spheres and regular pyramids) provides that it is a measure that works better than the H_M measure. An example of this is the Krippel method (hardness H_{KR}), which combines the idea of Janka and Brinell, because it uses a measure of H_B type:

$$H_{KR} = \frac{F}{A} \Big|_{d \approx D/2 \leftarrow h \approx 2 \text{ mm}}. \quad (10)$$

However, Meyer measure for this method would only give a value of about 1.07 times greater. The Krippel method is based on a fixed-depth indentation. For the original values $h=2$ mm and $A=200$ mm², the indentations are approximately half diameter and the ball should have a diameter $D=100/\pi$ mm = 31.831 mm.

The use of cylindrical rather than spherical indenters [39–41] has become popular in France. Monnin [42, 43] proposed here a hardness measure H_{MO} inversely proportional to the depth of indentation h [44]:

$$H_{MO} = \frac{F}{100hL} \propto \frac{1}{h}. \quad (11)$$

For this reason such a definition gave some similarity to Brinell measure. Unfortunately, in the case of a cylinder, the depth of indentation is not proportional to any actual surface area, but to a certain conventional area A_{MO} adopted by Monnin:

$$A_{MO} = 100hL. \quad (12)$$

Despite the use of the L parameter in the definition, which formally gives a standard stress unit of 1 N/mm² = 1 MPa, the H_{MO} measure is not comparable to the above measures. The divisor 100 was selected here in such a way that the measure (11) calculated in N/mm² units for known species ranged from 1 to 10. Such values in order of magnitude corresponded to Brinell measure and Janka measure of wood expressed in kG/mm². Therefore, a slightly different standardization of Monnin's measurement (hardness H_{MO}^*) is also justified:

$$H_{MO}^* = \frac{F}{10hL}, \quad A_{MO}^* = 10hL, \quad (13)$$

where A_{MO}^* is the conventional area corrected by factor $0.1 \approx 0.102$ in Monnin hardness. According to sources [30, 45] Monnin used a cylinder with a diameter of $D=30$ mm and wooden samples with a cross section of 20×20 mm ($L=20$ mm). The force used was $F=1961$ N (200 kg), i.e. 100 kg of load per 1 cm of cylinder length [45]. For soft wood, the load was halved. The time of measurement was to exceed 5 s [45]. According to Ref. [44], the indenter had a speed of 0.5 mm/min for all

three used diameters of 10, 30 and 50 mm. More standards concerning Monnin test are given by Starecki [30]. A practical comparison between Monnin test and Janka test was made by Sunley [46].

It turns out that practically none of the measures (1)–(11) discussed above is strictly compliant with Meyer empirical power law (for n different from 2 and 1):

$$F \propto d^n. \quad (14)$$

Meyer referred to this law mainly to balls, but it can also refer to, at least, cylinders. In case of balls, conformity (14) with the Meyer measure occurs only for the exponent $n=2$ associated with Kick [28, 30, 47–50], true here only for perfectly plastic bodies. For this reason, Meyer law is sometimes incorrectly called the Kick law [51, 52]. In fact, Kick law says that similar stresses cause similar deformations and on this basis derives that for pyramidal shapes of indenters $n=2$ (or $n=1$ for elongated shapes). The Kick Law is also given in the equivalent form for these shapes in the following forms $F \propto h^2$ [28], and for balls a full square function of the depth of indentation h [28, 53] is considered. The generalization of Kick law to the square function of the d is called the PSR of Li and Bradt or Hays and Kendall model [48, 49, 51, 54]. It can therefore be concluded that the ordinary Kick law applies only to pyramids and cones ($n=2$) or wedge-shaped prisms ($n=1$), but not to balls or cylinders. On the other hand, the exponent in Meyer law (14) for balls is in the range of $2 \leq n \leq 3$ [50] (and for cylinders within the range $1 \leq n \leq 2$). Nevertheless, some sources give a narrower range of exponents $2 \leq n \leq 2.5$ [6]. Huber [38] determined $n \approx 1.5$ or $n \approx 3$ depending on the longitudinal or transverse cross section of wood. The case of $n < 2$ is called indentation size effect (ISE) and $n > 2$ reverse ISE [50, 54]. The maximum values correspond to a perfectly elastic materials. This fact is confirmed by the contact mechanics formulae for the diameter of the indentation from the ball [55, 56]:

$$d = \sqrt[3]{\frac{3FD}{E}}, \quad (15)$$

and the width of indentation from the cylinder

$$d = \sqrt{\frac{8FD}{\pi LE}}, \quad (16)$$

where E is the effective surface elasticity modulus. For comparison, the elastic indentation by a loading head in bending is expressed as follows [56]:

$$d = \sqrt[3]{\frac{8FDT}{\pi LE}}, \quad (17)$$

where T is the sample height. Plastic (or plastic–elastic) indentations from the loading head were tested at work [57]. Note that formulae (15, 16), contrary to Meyer law (14), also take into account the diameter of indenter D . The empirical equivalent (15) for non-elastic indentations is the formula needed to determine International Rubber Hardness Degree (IRHD) [6]:

$$\frac{F}{E} = 0.00242D^{0.65}h^{1.35}. \quad (18)$$

The original formula contains increments of forces and indentations, not absolute values, but this is not crucial for this work.

Justification and purpose of the work

The idea is to modify hardness formulae of Brinell H_B (6) and Monnin H_{MO} (11) in the direction of Meyer law (14) and relations taking into account the indenter diameter (15)–(18). The modification has the above theoretical basis, but is generally based on experimental research. Meyer law (14) should not be confused with Meyer hardness measure (1). In addition, the hardness measure (mathematical formula) should not be confused with the hardness measurement (experimental test).

The point is that the ideal measure of hardness should reflect some physical law or definition in relation to surface deformation (not just wood). Meanwhile, this criterion is met only by a simple and imperfect Meyer measure H_M (1) referring to the definition of pressure. The hardness of Janka is based on this measure. However, everything indicates that for the spherical indenter, the physics of partial indentation is better reflected in the Brinell H_B measure (6) than in the Meyer H_M measure. Physically, the effectiveness of the Brinell measure is not explained either by the use of the total surface area A of the indentation, nor even the use of the linearly related depth h of the indentation. In view of the above, it is justified to refer to the empirical power law of Meyer (14) with the power $n \neq 2$ for balls and $n \neq 1$ for cylinders. In contrast to the Brinell measure, Meyer law has a theoretical basis in the form of relationships (15), (16) and (17).

The search for a better measure of hardness is at the same time a search for a more universal and precise hardness law. The law of hardness is understood here as a mathematical formula associating indentation force with parameters describing the indentation size and indenter size. Meyer law still requires a generalization that will take into account the diameter D of the indenter as in formula (18). In addition, Meyer law (14) requires the introduction of a material coefficient that would describe the hardness of the material (and not just the type of material as Meyer index n).

The main unification aspect of the work is the search for similar formulae for measures of hardness for spherical and cylindrical indenters (in Brinell- and Monnin-type tests), which would additionally have similar values. The Brinell measure for half-width indentation was considered the reference measure (see methodological assumptions vii and viii). An additional aspect of unification is the possibility of using the hardness measure for full indentation (or for half depth) without obtaining a significantly different value relative to the hardness obtained with small indentations. Knowledge of a measure having this property would allow a rational conversion of Janka hardness into a new hardness measure or Brinell measure. Rational conversion of hardness measures is already a contribution to unification.

Materials and methods

Materials

Experimental hardness tests were carried out on two beech beams (C and D) measuring $45 \times 45 \times 750$ mm each (Fig. 2). The annual rings and medullary rays of wood in the beam ran diagonally across the cross section so that all 4 side walls of 45×750 mm had the same type of radial-tangential section. Thus, a uniform planed area for testing on one beam was as much as 1350 cm^2 here. Both samples had a moisture content of 9%. Sample C was narrowly ringed and had a density of 782 kg/m^3 , while wide-ringed sample D had a significantly lower density of 672 kg/m^3 .

When making indentations, care was taken to ensure that their distance was greater than the size of the larger of them. In the case of balls, the same rule applied to the edge of the specimen and in the case of cylinders to both ends of the specimen.

Additionally, surveys were carried out for several other wood species: Norway maple, European oak, American sweetgum wood and large-leaf linden. Maple and oak samples were made of parquet flooring and were $20 \times 70 \times 70$ mm and $20 \times 60 \times 60$ mm, respectively. The sweetgum and linden samples were $20 \times 20 \times 60$ mm. The moisture content of wood was in the range of 8–9%. These surveys helped formulate and test research hypotheses, but due to large amount of remaining data, only power and linear regressions results were presented for them in this article.

Types of experimental tests

This article is based on experimental hardness tests carried out on beech wood using six indenters: three ball and three cylindrical ones, in both cases with diameters of 10, 15 and 30 mm. The indenters were mounted in specially designed heads of the strength testing machine

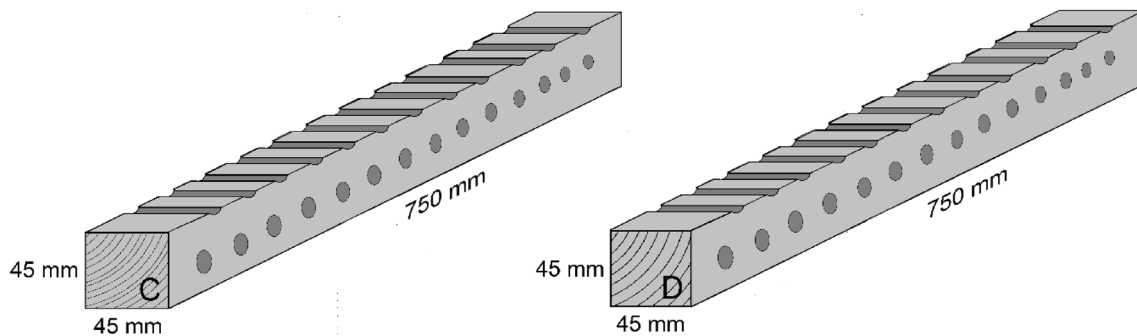


Fig. 2 Shape, dimensions and graining of beech specimens (samples C and D) with outline of ball and cylindrical indentations (samples A and B were cut into cubes and set aside for a completely different cross-section hardness test)

(Figs. 3 and 4). The ball indenters consisted of bearing balls made of bearing steel. Cylindrical indenters of 10 and 15 mm were made of hardened steel shock absorber rods with chromium plated coating. An exception was the 30 mm cylinder, which was the head of the testing machine (Fig. 4c).

Only the hardness on longitudinal sections, perpendicular to the direction of the fibres, was tested. The cylinders used were longer than the width of the beech beams and were perpendicular to them as in the Monnin test. The indentation methodology was essentially similar to that of the Brinell test, in the sense that for the maximum force measured by the testing machine the indentation width d was measured. The main difference was that not one pre-determined force was used but an entire spectrum of forces, which generated a measurable full range of indentations of $0 < d < D$.

Unfortunately, it was not possible to obtain the total indentations $d = D$, which would enable a precise determination of Janka hardness H_J (full indenters would be destructive for samples and limited by the attachment of indenters).

On average, for each indenter, 16 indentations were made with different force ranges. All tests used a constant speed rate of 1 mm/min. After reaching a given force, the feed rate of the machine was suspended for a time during which the force on the wood decreased by about 5%. This time allowed for the formation of a plastic indentation. Then the indentations were outlined with a pencil. The indentations from the sphere were inscribed in the shape of a rectangle $d_1 \times d_2$, whose sides were parallel and perpendicular to the fibres. Cylindrical indentations were inscribed in the shape of a trapezoid with bases d_1 and d_2 . In both cases, the sizes of indentations

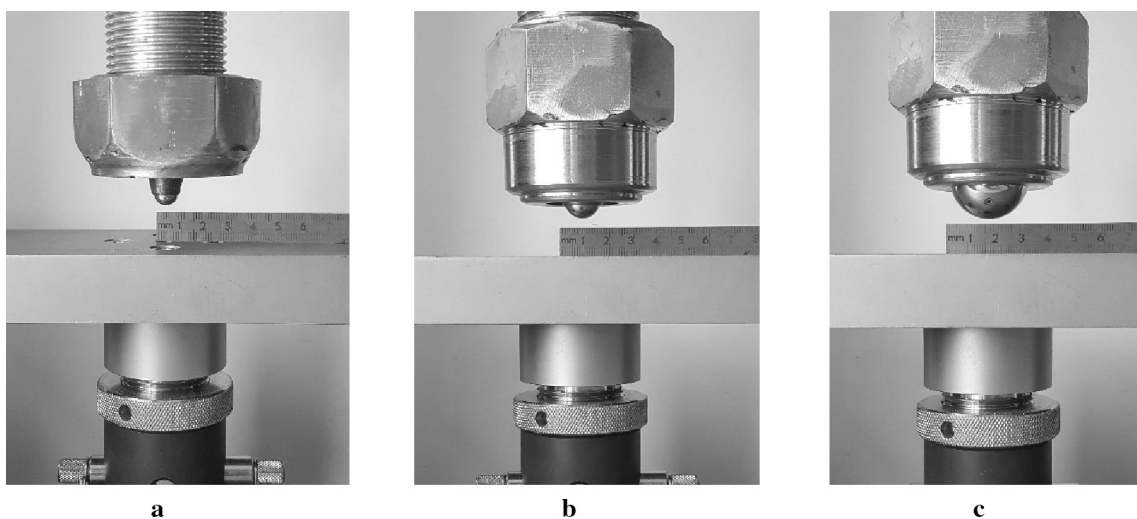


Fig. 3 Ball indenters with diameters, respectively: **a** 10 mm, **b** 15 mm, **c** 30 mm

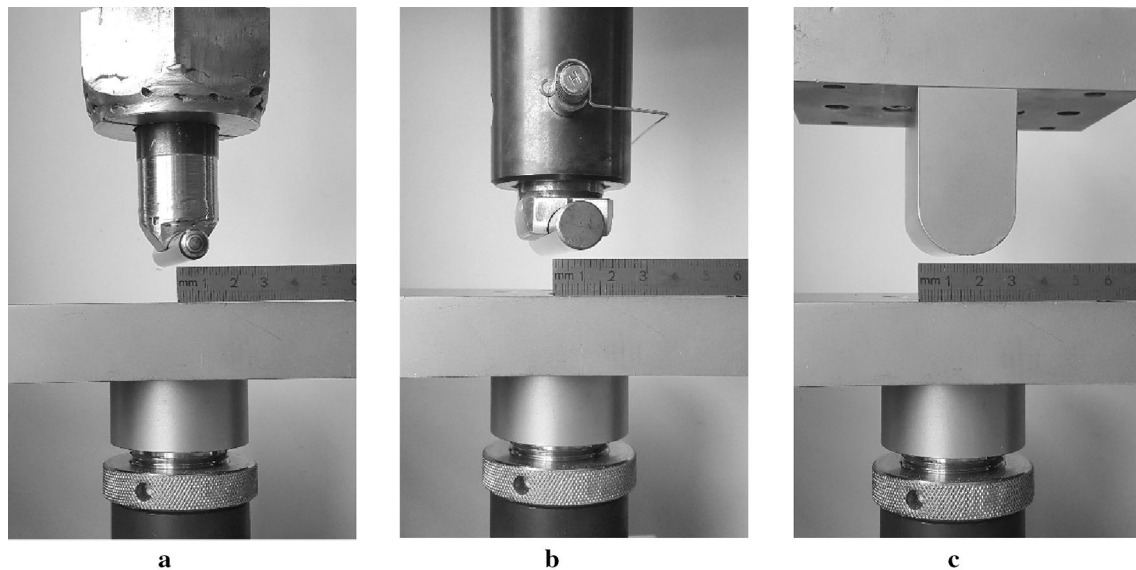


Fig. 4 Cylinder indenters with diameters, respectively: **a** 10 mm, **b** 15 mm, **c** 30 mm

were measured with a calliper and then averaged according to the geometric mean (for ball) and arithmetic mean (for cylinder), respectively:

$$d = \sqrt{d_1 d_2}, \quad (19)$$

$$d = (d_1 + d_2)/2. \quad (19a)$$

This averaging method is matched to the ellipse and trapezoid patterns. These tests were performed using 5-tonne and 10-tonne Instron testing machines.

In addition, survey and checking tests were carried out using a standard Brinell machine (with a 10-mm-diameter ball) for several types of wood described in the materials. Four force ranges, i.e. 306.5, 612.9, 980.7 and 1471 N, were used, with the preferred weight of 100 kg corresponding to 980.7 N. Due to the limited volume of work, this control scope of the study will be discussed only partially, even though it was significant for the creation of this article.

Theoretical assumptions and methodology

Assuming that the experimental part is performed correctly, this theoretical analysis requires further conceptual assumptions. These assumptions are important from the point of view of heuristic formulation of synthetic hardness laws for wood. Some assumptions are postulates and some will be explicitly confirmed experimentally. Some assumptions result from the analysis of the state of knowledge presented in the introduction, and some have already appeared in the course of experimental research. A hardness measure is here

understood as a specific mathematical formula for hardness tests. In order to increase the transparency of this work, postulates and assumptions for this measure are listed below:

- i. There is a measure of hardness which does not significantly depend on the size of indentations and load values.
- ii. There is a measure of hardness which does not significantly depend on the size of the indenter by including this parameter in the hardness formula.
- iii. There are similarly constructed hardness measures for ball and cylindrical indenters, which will give sufficiently consistent and commensurate values.
- iv. It is assumed that elastic–plastic relations for wood play a greater quantitative role than the friction forces between wood and the indenter (this assumption was initially positively verified by the friction model, which gave weaker results).
- v. It is assumed that the universal hardness measure is a generalization of Meyer law with additional power dependence on indenters diameters.
- vi. Preference is given to hardness measures expressed in MPa. This dimensional condition determines the exponent of power for indenter diameter at the fixed exponent for indenter size. Nevertheless, both exponents are examined independently in order to confirm this assumption.
- vii. The reference measure (coefficient normalization) for the ball indenter is the Brinell measure for half-width indentations ($d = D/2 = R$) (the reference to

- Brinell measure gives a value 7% less than the reference to Meyer measure).
- viii. The hardness measure for the cylindrical indenter is for indentations of half width ($d=D/2=R$) as much as the Meyer measure (about 7%), as the measure for balls according to vii (for cylinders the Brinell type measure would be only 5% less than the Meyer measure here).
 - ix. The criterion for choosing a better hardness measure is better power or linear regression of experimental points. A better fit means a higher determination factor and the smallest possible initial value (in the appropriate linear regression).

The next part of the article relies on these assumptions, but at the same time the results presented further experimentally prove that many of them are correct.

Results of research and discussion

The experimental and theoretical results are presented in three stages. First it is tested on the basis of measurements of Meyer law in opposition to Brinell measure. The experimental force dependence on the diameter of the indenter is also tested here. In the second stage, new hardness measures for ball and cylindrical indenters are developed on the basis of the collected measurement results and assumptions of i–ix. These new hardness measures are the result of theoretical work, although they are empirically based. The third stage of the results tests the new hardness measures used for the measurements taken and compares them with Brinell-type hardness.

Results of experimental tests

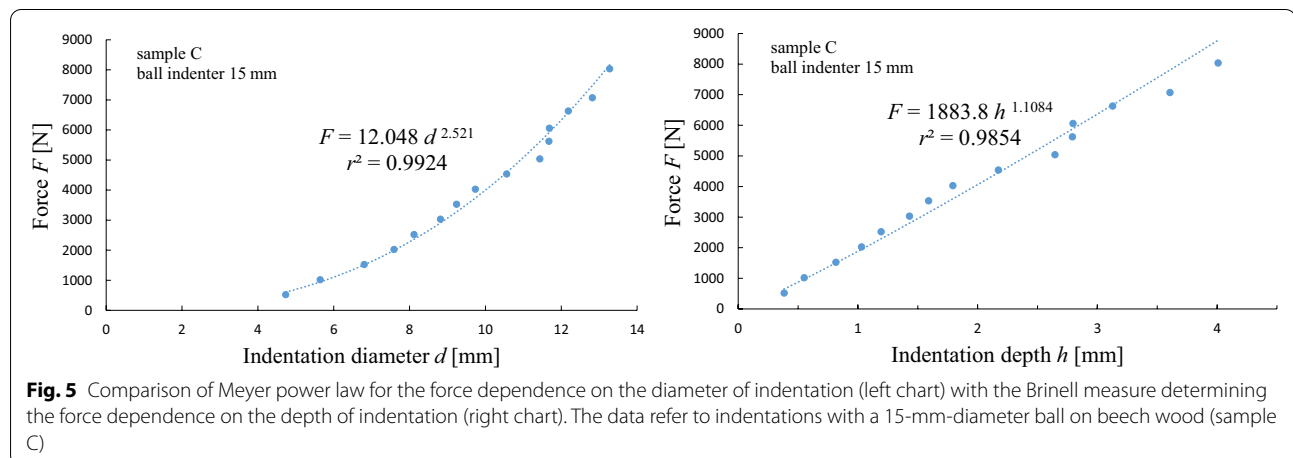
This part presents the development of direct results of force measurements and the obtained indentation sizes.

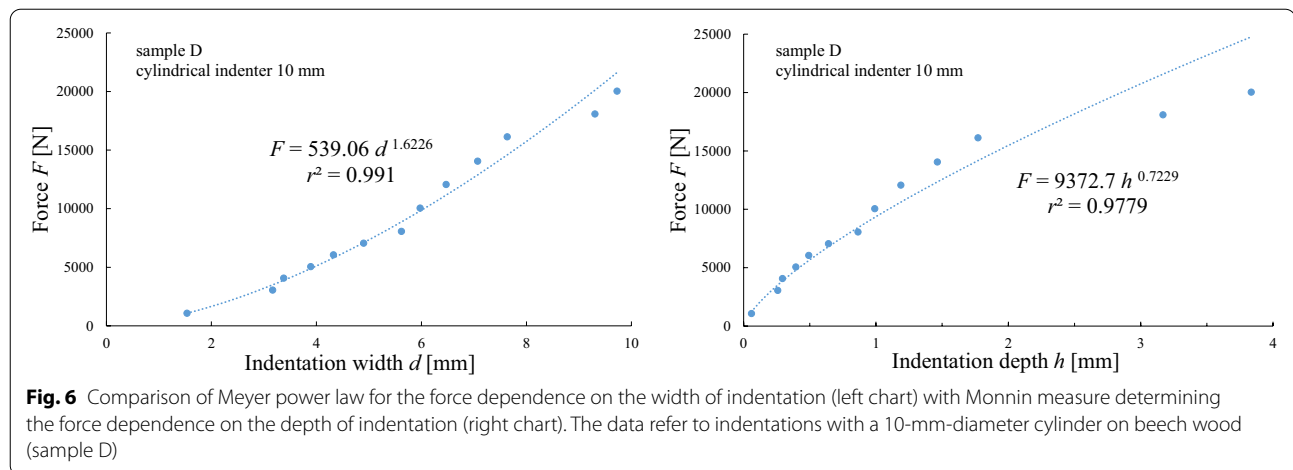
There are three main trends for matching measuring points in the charts. The first trend is compliance with the Meyer power law of (14) with an unknown exponent of power. The second trend is compliance with Meyer measure H_M (1), which from (2) for balls means the exponent $n=2$ (Kick law), and for cylinder the exponent $n=1$. Therefore, these two trends can be analysed in one graph $F(d)$ of the indentation force dependence on the indentation width. The diagrams on the left in Figs. 5 and 6 show this relationship for the spherical and cylindrical indenters, respectively.

The third possible trend is the dependence of the indentation force $F(h)$ on its depth, which reflects the Brinell measure H_B (6) and (7) for balls and the Monnin measure H_{MO} (11) for cylinders. The corresponding diagrams are presented on the right in Figs. 5 and 6 and can be compared with the previous diagrams.

As you can see, the trend based on Meyer law shows a determination coefficient (r^2) higher than Brinell or Monnin measure. In the case of cylinders, the adjustment to the trend conforming to Meyer law is clearly better, and in the case of balls, Meyer trend is slightly better than a good adjustment to the Brinell measure. In Table 1, the appointed exponents for Meyer law are given. The exponents $n=2.5$ for balls and $n=1.5$ for cylinders mean that the Meyer measure H_M (not to be confused with Meyer law) relating to the projection area S is poorly correlated with the experimental points. The Brinell-type measure based on the total area of indentation A shows a better correlation. Therefore, in the following part of the article, new measures based on Meyer law are compared only to Brinell-type measures.

Knowledge of exponents n in Meyer law allow for a more direct comparison of linear correlations $F(d^n)$ and $F(h)$. Table 2 contains all the determination factors for



**Table 1** Experimental values of exponents in Meyer law for balls and cylinders

Indenter	Wood sample	Exponent of power (Meyer index) n			Average n	$n \approx$
Ball	Norway Maple	2.6443				
	European Oak	2.1730			2.58	2.6
	American Sweetgum	2.6041			± 0.15	
	Large-leaf Linden	2.9130				
Ball	Beech (sample C)	2.3616	2.5210	2.7289	2.478	2.5 = 5/2
	Beech (sample D)	2.4208	2.3875	2.4452	± 0.055	
Cylinder	Beech (sample C)	1.5120	1.3280	1.6968	1.483	1.5 = 3/2
	Beech (sample D)	1.6226	1.3634	1.3756	± 0.062	
Indenter diameter D (mm)		10	15	30	\pm Standard error	

Table 2 Comparison of linear correlations for Meyer law and Brinell and Monnin measures

Information about the hardness test			$F = a \cdot (d^n) + b$		$F = a \cdot h + b$	
Indenter	D (mm)	Wood sample	r^2	b (N)	r^2	b (N)
Ball ($n = 2.5$)	10	Norway Maple	0.9964	-53	0.9935	-203
Ball ($n = 2.5$)	10	European Oak	0.9986	55	0.9949	-70
Ball ($n = 2.5$)	10	American Sweetgum	0.9964	4	0.9989	-146
Ball ($n = 2.5$)	10	Large-leaf Linden	0.9057	-182	0.8999	-324
Ball ($n = 2.5$)	10	Beech (sample C)	0.9929	38	0.9917	-48
Ball ($n = 2.5$)	15	Beech (sample C)	0.9916	136	0.9897	5
Ball ($n = 2.5$)	30	Beech (sample C)	0.9841	10	0.9790	-893
Ball ($n = 2.5$)	10	Beech (sample D)	0.9868	7	0.9763	45
Ball ($n = 2.5$)	15	Beech (sample D)	0.9920	-25	0.9941	-53
Ball ($n = 2.5$)	30	Beech (sample D)	0.9965	-291	0.9987	-668
Cylinder ($n = 1.5$)	10	Beech (sample C)	0.9694	-207	0.8838	6005
Cylinder ($n = 1.5$)	15	Beech (sample C)	0.9964	1719	0.8831	9718
Cylinder ($n = 1.5$)	30	Beech (sample C)	0.9967	-2200	0.9906	3301
Cylinder ($n = 1.5$)	10	Beech (sample D)	0.9792	-143	0.8771	3769
Cylinder ($n = 1.5$)	15	Beech (sample D)	0.9882	1735	0.9249	6624
Cylinder ($n = 1.5$)	30	Beech (sample D)	0.9876	2864	0.9170	10,004

these correlations and its absolute terms b . In most cases, the correlation based on Meyer law has a higher determination factor and a lower absolute value of b .

The form of force dependence on indenter diameters is still to be determined. For this purpose, a_0 coefficients in simple proportionality of $F = a_0 \cdot d^n$ for each diameter of 10, 15 and 30 mm have been used. Charts of the $a_0(D)$ dependence of these coefficients on D diameters for balls and cylinders are shown in Fig. 7.

In both cases the relation $a_0(D)$ is the inverse of the square root of D . All the measured exponents for this relation together with the averages are presented in Table 3.

Although the dependencies on which Table 3 were three point (10, 15 and 30 mm), behind each of these points stood an average of 16 elementary measurement points. The values of the a_0 coefficient resulted from these elementary points. Therefore, the value of the exponent $k = -1/2$ is reliable, the more so that for $n=5/2$ or $n=3/2$ this value means that the assumption vi is met.

Resultant mathematical formulae

The experimental results presented in the above charts and tables allowed to determine the power dependence of the indenter force F on the indenter width d as well as on the indenter diameter D . For ball and cylindrical indenters these relationships are as follows:

$$F \propto d^{2.5} D^{-0.5} = \sqrt{\frac{d^5}{D}}, \quad (20)$$

$$F \propto d^{1.5} D^{-0.5} = \sqrt{\frac{d^3}{D}}. \quad (20a)$$

The power dependence on d is in accordance with Meyer law (14) for exponents with values equal to the

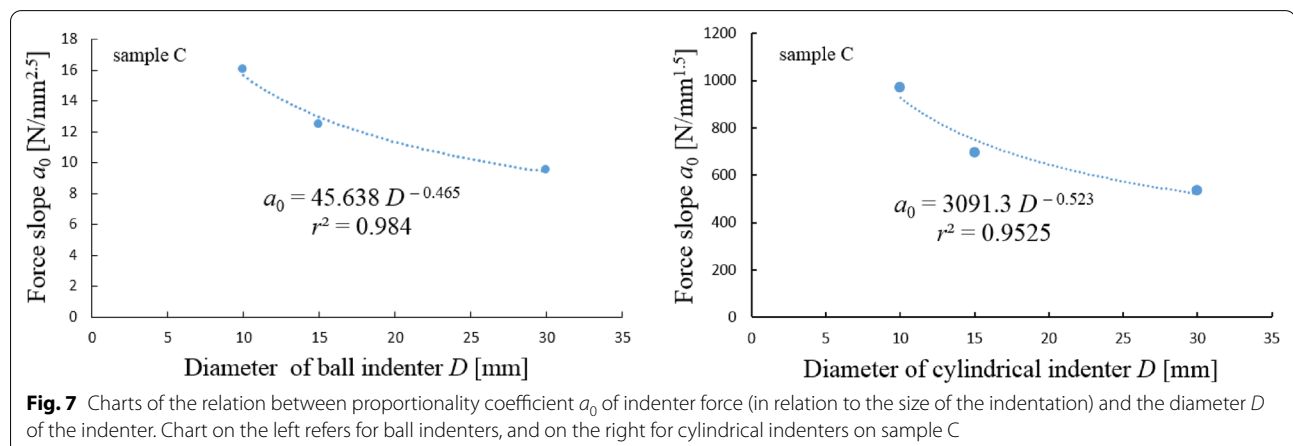
Table 3 Experimental values of the exponents of power k for the diameters D of indenters

Indenter	Wood sample	Exponent of power k	Average k	$k \approx$
Ball	Beech (sample C)	-0.465	-0.510	-0.5 = -1/2
	Beech (sample D)	-0.554	± 0.045	
Cylinder	Beech (sample C)	-0.523	-0.553	-0.5 = -1/2
	Beech (sample D)	-0.583	± 0.030	

centre of the considered ranges. These exponents correspond to the elastic-plastic material, which wood clearly is, in particular beech wood. On the other hand, the dependence on D in formulae (20, 20a) is a new element. A slight similarity to (18) is apparent, since in (20, 20a) diameter D appears in the denominator and not in the numerator. Thus, (18) reflects in some way the Brinell formula (7). Note that the exponent of the power of relationship (20, 20a) is significantly different than those of formulae (15), (16), (17), (18). The D dependence is also found in Brinell H_B measure (6) and Monnin H_{MO} measure (11), both of which depend on the depth of indentation h , which is a function of D (7). However, this function is more complex and also does not resemble relation (20, 20a). Note that the proportionality coefficients in these relationships with an accuracy to dimensionless constants define a new hardness measure, but for cylinders the contact length L (beam width or cylinder length) must also be taken into account:

$$F = N_b \cdot H_{Kb} \cdot \sqrt{\frac{d^5}{D}}, \quad (21)$$

$$F = N_c \cdot H_{Kc} \cdot \sqrt{\frac{d^3}{D}} \cdot L. \quad (21a)$$



New hardness measures H_{Kb} and H_{Kc} are marked according to the common initials of the authors names and the indenter shape index. Condition vii allows the first normalization factor to be calculated unequivocally:

$$\begin{aligned} H_{Kb}|_{d=R} = H_B|_{d=R} \rightarrow N_b &= \frac{\pi D(D - \sqrt{D^2 - R^2})}{2\sqrt{\frac{R^5}{D}}} \\ &= \frac{\pi}{\frac{(\sqrt{6} + \sqrt{8})}{2}} \cong \frac{\pi}{\sqrt{7}}, \end{aligned} \quad (22)$$

where the approximation used has an accuracy of 0.26%. The condition viii results in the second normalization constant:

$$\begin{aligned} \frac{H_{Kc}}{H_{Mc}} \Big|_{d=R} &= \frac{H_{Kb}}{H_{Mb}} \Big|_{d=R} \cong \frac{\sqrt{14}}{4} \rightarrow N_c \\ &\cong \frac{4}{\sqrt{14}} \cdot \frac{L \cdot R}{L \cdot \sqrt{R^3/D}} = \frac{4}{\sqrt{7}}. \end{aligned} \quad (22a)$$

Note that formulae (21, 21a) and (22, 22a) contain formulae for certain conventional surface areas (Fig. 8):

$$A_b = \pi \sqrt{\frac{d^5}{7D}}, \quad (23)$$

$$A_c = 4L \cdot \sqrt{\frac{d^3}{7D}}. \quad (23a)$$

Brinell noted that the total indentation area A is a better measure of the area than the area of the projection S , although this was not directly related to the definition of (1) stress depending on the area of the projection. Similarly, areas (23, 23a) should be a better measure of indentation than the projection area, and perhaps even better than the total indentation area or formula (12) for the

area $100hL$ used by Monnin for the cylinder. Note that the total cylindrical indentation area is expressed by the formula:

$$A = L \cdot D \cdot \arcsin \frac{d}{D}. \quad (24)$$

Referring this formula to (6) we get the equivalent of the Brinell measure for the cylinder:

$$H_{Bc} = \frac{F}{A} = \frac{F}{L \cdot D \cdot \arcsin(d/D)}. \quad (25)$$

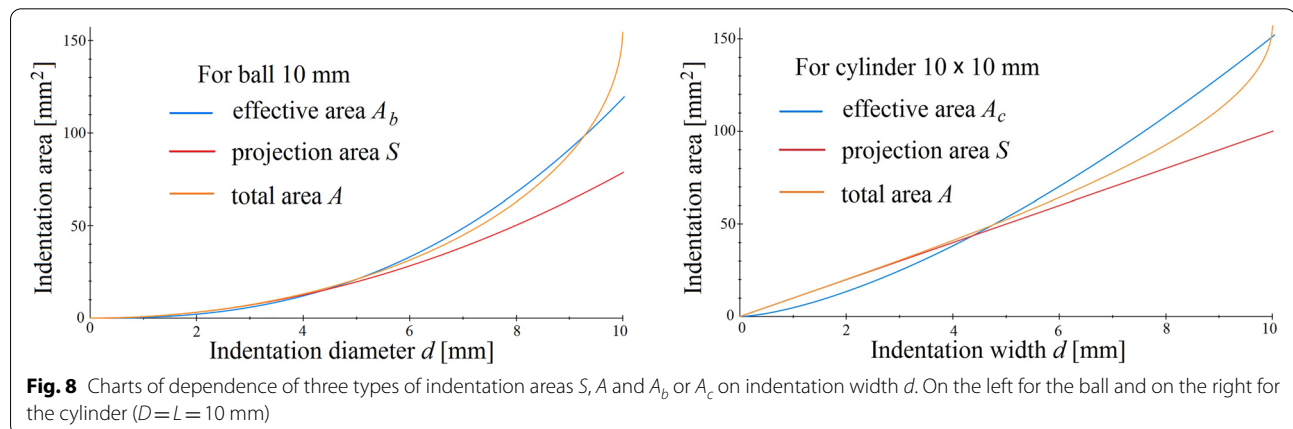
New simple and similar formulae can now be finally given to measure the hardness of ball and cylindrical indenters:

$$H_{Kb} = \frac{F}{A_b} = \frac{F}{\pi \sqrt{\frac{7D}{d^5}}}, \quad (26)$$

$$H_{Kc} = \frac{F}{A_c} = \frac{F}{4L \sqrt{\frac{7D}{d^3}}}. \quad (26a)$$

Results of new hardness formulae for measurements

Once the new hardness formulae H_{Kb} and H_{Kc} (26, 26a) based on fields (23, 23a) have been derived, it should be checked whether they give a better correlation of force F to the indentation area A given by (7) for balls or (24) for cylinders. At the same time, the slope (the directional coefficient) of this linear correlation will be a measure of the mean hardness ($H_{Kb,c} \cong a_0 \approx a$) for different indenter diameters for a given hardness measure. This correlation can even be analysed simultaneously for balls and cylinders. A comparison of these correlations for all three balls (10, 15 and 30 mm) is shown in Fig. 9. Although the correlation arguments on the left and right charts are modified, the determination coefficients (r^2) and absolute terms (b) of these correlations are identical



to those given in Table 2. At the same time, these values for sample C are more favourable for the new indentation area measure than for the total indentation area (Brinell measure).

An analogous correlation comparison for all cylinders is shown in Fig. 10. Similarly as for balls, there is a compliance with Table 2 in case of the new area measure. On the other hand, the correlation for the total area is clearly better than the correlation for the Monnin measure. However, the correlation for the new measure is by far the best.

A total analysis of all 85 indentations of sample C leads to similar conclusions (Fig. 11). In the case of sample D, there were as 100 indentations and the charts looked similar. In all correlations relatively small absolute terms (b) and better determination coefficients (r^2) were obtained for left charts (i.e. new measures of area indentation of balls and cylinders).

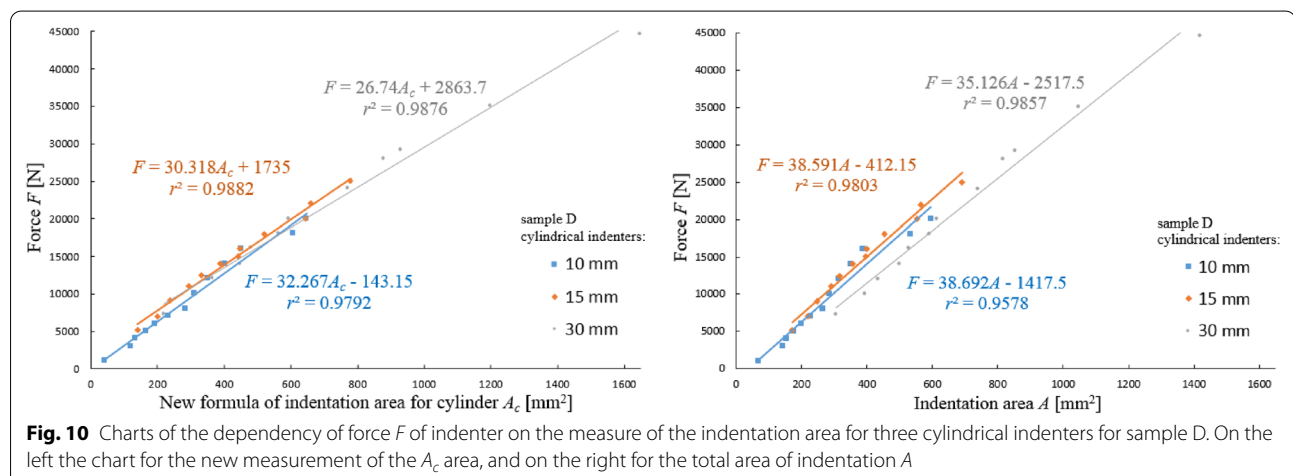
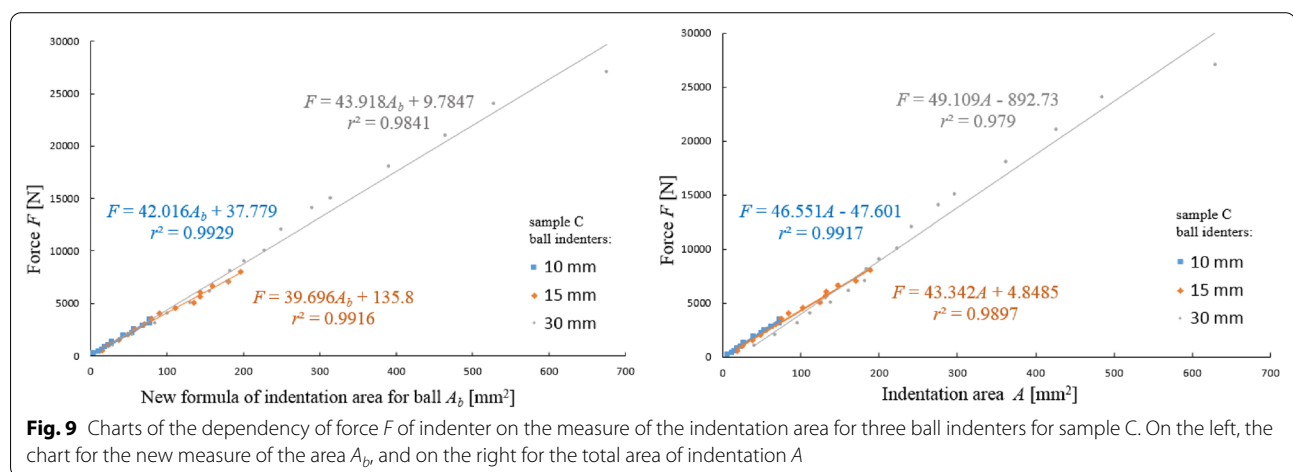
The final testing of the new hardness measures H_{Kb} and H_{Kc} (26, 26a) used directly for beech wood measurements

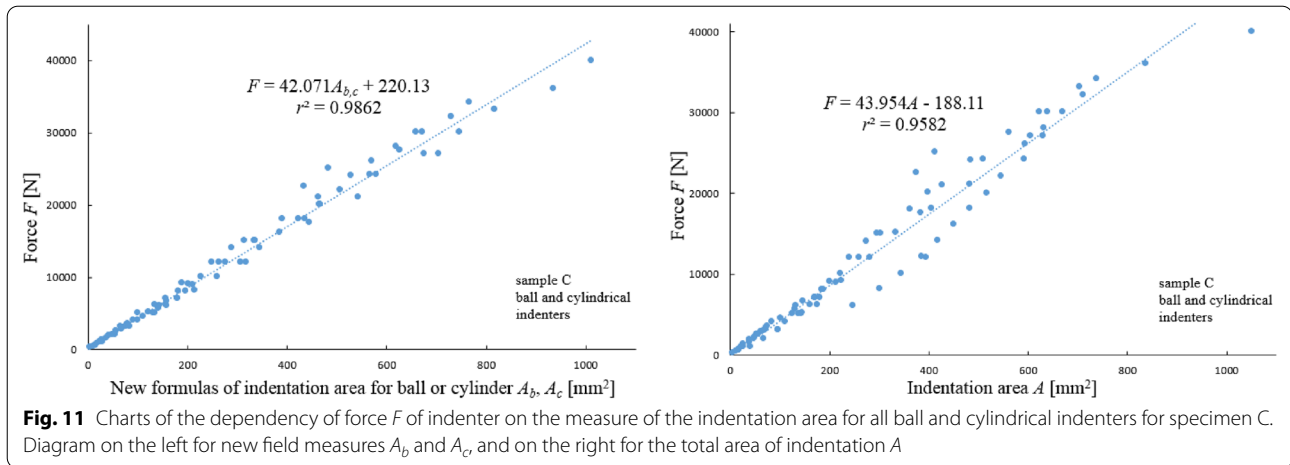
is still to be tested. Tables 4 and 5 show the mean values obtained for sample C and sample D, respectively. They are compared to the Brinell-type measure based on the total indentation area.

For both samples C and D (Fig. 12) we can see a high compliance of all four hardness measures H_{Kb} , H_B , H_{Kc} and H_{Bc} . However, the newly introduced hardness measures H_{Kb} and H_{Kc} are characterized by the lowest variability. Even if it had not appeared for the arithmetic mean for sample C, it is visible for individual diameters for this sample (with the exception of the standard deviation for 10 mm). These tables, therefore, confirm the conclusions of the correlations previously analysed.

Discussion of Janka hardness

The introduced hardness measure for H_{Kb} balls gives the possibility to predict the measure of Janka type referring to extreme indentations. If H_{Kb} is a homogeneous measure for each indentation range (assumption i), the following relations should occur (for $d = D$ or $d = \sqrt{3}R$):



**Table 4** Results of measurements of mean hardness values for sample C of beech wood

Beech 782 kg/m ³ Sample C (h 9%)	Hardness measures (mean value \pm standard error) (MPa)			
	Spherical (ball)		Cylindrical	
Indenter diameter D (mm)	New measure H_{Kb}	Brinell H_B	New measure H_{Kc}	Brinell type H_{Bc}
10	44.0 \pm 1.5	44.1 \pm 1.3	45.2 \pm 1.1	46.8 \pm 2.4
15	41.3 \pm 0.7	42.5 \pm 1.2	42.7 \pm 1.1	42.7 \pm 1.2
30	42.5 \pm 1.0	42.2 \pm 1.9	42.8 \pm 0.6	37.2 \pm 1.8
Arithmetic mean	42.6 \pm 1.4*	42.9 \pm 1.0*	43.6 \pm 1.4*	42.2 \pm 4.8*
Average means	42.83 \pm 0.57*			

*Standard deviation of the type s_{m-1} for $m=3$ or $m=4$ without the t -Student factor, but also without divisor \sqrt{m} for the standard error of the mean value.

Table 5 Results of measurements of mean hardness values for sample D of beech wood

Beech 672 kg/m ³ Sample D (h 9%)	Hardness measures (mean value \pm standard error) (MPa)			
	Spherical (ball)		Cylindrical	
Indenter diameter D (mm)	New measure H_{Kb}	Brinell H_B	New measure H_{Kc}	Brinell type H_{Bc}
10	31.19 \pm 0.70	31.37 \pm 0.74	30.99 \pm 0.85	31.27 \pm 2.03
15	29.07 \pm 0.55	29.22 \pm 0.49	35.20 \pm 0.65	37.03 \pm 0.93
30	28.41 \pm 0.49	27.01 \pm 0.83	31.74 \pm 0.56	30.49 \pm 0.99
Arithmetic mean	29.6 \pm 1.5*	29.2 \pm 2.2*	32.6 \pm 2.2*	32.9 \pm 3.6*
Average means	31.1 \pm 2.0*			

*Standard deviation of the type s_{m-1} for $m=3$ or $m=4$ without the t -Student factor, but also without divisor \sqrt{m} for the standard error of the mean value

$$H_I = 1.51H_{Kb}, H_{I1/2} = 1.41H_{Kb} \rightarrow H_I = 1.07H_{I1/2}. \quad (27)$$

For comparison, these relations would look as follows if the Brinell measure were more homogeneous:

$$H_I = 2H_B, H_{I1/2} = 1.33H_B \rightarrow H_I = 1.5H_{I1/2}. \quad (28)$$

Relations (27) and (28) do not contradict that $H_{Kb} \cong H_B$, because they refer to extreme indentations in which only a significant difference between the new measure and Brinell measure is revealed. Source materials [27, 58] show that $H_I/H_B = 1.37 \pm 0.13$, 1.48 ± 0.27 , respectively. It can therefore be provisionally concluded that the relationships (27) for the new measure are closer to reality. After all, as



Fig. 12 Photos of indentations in beech wood. At the top, sample C with part of the alternating indentations of $D=10$ mm and $D=15$ mm cylindrical indenters. At the bottom, sample D with indentations from the $D=30$ mm ball. For each indentation its average measured width d and calculated depth h are given

in (28), Janka H_J hardness is usually not 100% greater than Brinell H_B hardness and the hardness values of H_J and $H_{J1/2}$ do not differ by as much as 50%, which would contradict the sense of the standards [15, 18, 19, 30].

For beech wood samples C and D it was possible to calculate Janka hardness $H_{J1/2}$ for all ball diameters (10, 15 and 30 mm). In sample C, this corresponded to indentations with widths of 8.4, 13.3, 25.0 mm and sample D with widths of 8.7, 13.3 and 25.9 mm. On average, Janka hardness $H_{J1/2}$ turned out to be 40% higher than H_{Kb} and H_B hardness:

$$\begin{aligned} (H_{J1/2}/(H_{Kb})_{\text{mean}})_{\text{mean}} &= 1.389 \pm 0.026, (H_{J1/2}/(H_B)_{\text{mean}})_{\text{mean}} \\ &= 1.407 \pm 0.032. \end{aligned} \quad (29)$$

Thus, the H_{Kb} measure gives a slightly better prediction (27) of Janka hardness $H_{J1/2}$ than the Brinell measure (28). Calculation of H_J hardness for full indentations was not possible, because the largest indentations had slightly too small widths of 9.7, 14.2 and 27.1 mm. However, according to the H_{Kb} measure, the Janka hardness H_J should be greater than the Janka hardness $H_{J1/2}$ by only 7%.

Conclusions

On the basis of experimental results for beech wood were introduced hardness measures H_{Kb} (26) and H_{Kc} (26a) meeting the i–ix assumptions.

In case of cylindrical indenters, the introduced H_{Kc} hardness measure showed undoubtedly better correlation with experimental points than Brinell-type H_{Bc} measure, and the more the Meyer H_{Mc} measure and Monnin H_{MO} measures (H_{MO}^*). Despite significantly lower variability of the new H_{Kc} (26a) measure, it is well in accordance with the Brinell-type H_{Bc} (25) measure for average values:

$$H_{Kc} = \frac{F}{4L} \sqrt{\frac{7D}{d^3}} \cong H_{Bc}. \quad (30)$$

The new H_{Kb} hardness measure for spherical indenters showed a slight correlation advantage over Brinell H_B and higher than Meyer H_{Mb} measure. Condition vii. despite slightly less variability of H_{Kb} (26) new measure ensured good compliance of average values with Brinell H_B (6) measure:

$$H_{Kb} = \frac{F}{\pi} \sqrt{\frac{7D}{d^5}} \cong H_B. \quad (31)$$

The hardness measures for H_{Kb} balls and H_{Kc} cylinders have very similar (almost identical) and very simple formulae (26) or (30) and (26a) or (31). These formulae are significantly simpler than Brinell formulae (6), (7) and (25). The new hardness formulae reflect Meyer law with an exponent of power $n=2.5$ for balls and $n=1.5$ for cylinders. Moreover, these formulae resemble theoretical

relationships (15) and (16), but about different exponents than for perfectly elastic bodies. At the same time Meyer law was extended here to the power dependence on the diameter of the indenter (20, 20a), the exponent of which for both shapes resulted at $k = -1/2$. A somewhat similar empirical formula can be found in Annex A of the standards [59, 60]. Among other things, thanks to condition viii, the average hardness measures H_{kb} and H_{kc} were found to be satisfactory compatible for the tests carried out:

$$H_{kb} \approx H_{kc}. \quad (32)$$

It can, therefore, be said that these new measures are competitive with Brinell measures, but that they are consistent with the average values and make it possible to reduce the variability of the developed measurement results.

Thanks to the simple geometrical relationship $d = 2\sqrt{Dh - h^2}$, the new H_{kb} measure can be implemented in industrial hardness testers measuring the depth h of indentation in wood. In addition, the H_{kb} measure gives a reliable prediction of Janka hardness:

$$H_{J1/2} = 1.41H_{kb} \cong 1.41H_B, H_J = 1.07H_{J1/2}. \quad (33)$$

Therefore, there is every reason to believe that the title objective of the work for progress in the unification of wood hardness measures has been achieved. Unification is understood here as a proposition of hardness measures H_{kb} and H_{kc} aspiring to the role of hardness laws for wood (generalizing Meyer law). These measures combine qualitatively or numerically the hardness of Brinell, Janka and Monnin.

Abbreviations

α : A measure of the central angle based on the indentation arc; a, b : Slope and initial value in the linear correlation equation; a_0 : Slope in the linear correlation equation without the initial value ($b=0$); A : Total indentation area; $A_D = \pi D^2$: The total sphere area; A_b, A_c : Conventional indentation areas according to new hardness measures (for ball and cylindrical indenters); A_{MO} : Conventional indentation area according to Monnin hardness measure; A_{MO}^* : Conventional indentation area according to Monnin hardness measure (corrected by factor $0.1 \approx 0.102$); C, D : Denotation of wood samples; D : Indenter diameter (ball or cylindrical); d : Average width of indentation (for ball equal to the diameter of the indentation); d_1, d_2 : Width of indentations, diameters along and across the fibres for the sphere, trapezoidal bases of indentations from the cylinder; E : Effective modulus of elasticity in compression; F : Load force (indentation); F_J : Force (in N) as a characteristic of hardness in the Janka method; F_J^* : Force (in N) in the Janka method scaled from the half-depth indentation; F_{max} : The force inducing full indentation; $g = 9.80665 \text{ m/s}^2$: Acceleration of the Earth; h : The depth of indentation; H_B : Brinell hardness and general measure of Brinell hardness (w MPa = N/mm² without factor 0.102); H_{bc} : A measure of Brinell hardness in relation to a cylindrical indenter; H_J : Hardness (in MPa) in the Janka method (at a full indentation); $H_{J1/2}$: Hardness (in MPa) in the Janka method at half-depth indentation; H_{kb}, H_{kc} : New hardness measures from the first common letter of the names of the three authors (for ball and cylindrical indenters); H_{KN} : Hardness in the Knoop test (usually referred to as HK); H_{KR} : Hardness in the Krippl method; H_M : Meyer hardness measure; H_{Mb}, H_{Mc} : Meyer hardness measures for ball and cylindrical indenters; H_{MO} : Hardness and hardness measure according to Monnin in MPa, but with normalization

closer to kG/mm²; H_{MO}^* : Hardness and hardness measure according to Monnin with units and value normalization in MPa; H_R : Rockwell hardness scale; H_V : Hardness in the Vickers test; k : Exponent of power for the diameters D of indenters; l : Chord of half an arc of indentation, otherwise the generalized radius of indentation counted from the bottom of indentation (slant height of the cone inscribed in the spherical cap of indentation); L : The length of the cylinder contact with the wood (the width of the wooden beam or the length of the sharp-edged cylinder); m : Number of averaged values; n : Power exponent in Meyer law (Meyer index) for spherical or cylindrical indenters; N_b, N_c : Dimensionless normalization factors for ball and cylindrical indenters; R : Indenter radius; r^2 : Coefficient of determination of linear or power correlation; S : Indentation projection area; S_{max} : Indenter projection area (full indentation projection area); T : Sample height.

Acknowledgements

Not applicable.

Authors' contributions

GK designed experiment of the study and was responsible for the data collection and wrote the manuscript; ZK preformed wood samples and preformed experiment and helped in interpretation of data; PK helped design the work and was major contributor in writing the manuscript and revised the manuscript. All authors read and approved the final manuscript.

Funding

This work was made as part of own research in Warsaw University of Life Sciences–SGGW [Szkoła Główna Gospodarstwa Wiejskiego w Warszawie (PL)], Institute of Wood Sciences and Furniture, Department of Wood Sciences and Wood Preservation (using hardware resources of this institution).

Availability of data and materials

The data sets used for analysed during the current study are available from the corresponding author on reasonable requests.

Competing interests

The authors declare that they have no competing interests.

Received: 22 January 2020 Accepted: 25 December 2020

Published online: 13 January 2021

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