J Wood Sci (2002) 48:277–282

ORIGINAL ARTICLE

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Theoretical analysis of bandsaw roll-tensioning

Received: April 27, 2001 / Accepted: September 19, 2001

Abstract Bandsaw roll-tensioning was theoretically analyzed by amplifying Aoyama's fundamental equations. In the analysis the accumulated amount of tension after several roll-stretching passes was expressed as the linear accumulation of the amount of tension at each roll-stretching pass. The same condition was applied for the expression of the accumulated amount of crown back. The relations between the distance from the gullet in the transverse direction of the bandsaw blade to the roll-stretching point and the distance from the gullet in the transverse direction of the bandsaw blade at which the amount of tension becomes maximum, minimum, or zero were clarified. To meet the requisites to adjust the amount of tension at an arbitrary distance in the transverse direction of the bandsaw blade or the amount of crown back, an adjusting procedure was developed. The process of bandsaw roll-tensioning can theoretically be controlled by this procedure.

Key words Bandsaw roll-tensioning · Theoretical analysis · Amount of tension · Amount of crown back

Introduction

The manufacturing and maintenance of bandsaw blades depends on the skills of saw filers, but the shortage of successors to experienced saw filers has become an acute problem in recent years. The most promising measure to solve this problem is to develop a computer-controlled automatic bandsaw stretching machine. There are almost no quantitative data concerning the bandsaw roll-tensioning technique necessary to control this machine by computer. To quantify this technique, therefore, I carried out a theoretical analysis of bandsaw roll-tensioning.

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Basic theory of bandsaw roll-tensioning

Basic equation of tension

The amount of tension (deflection to the transverse deflected bandsaw surface) T (mm) in the transverse direction of a bandsaw blade after a roll-stretching pass is shown in Fig. 1 and is given by the following equation.[1]

For the range $0 \le x \le x_R$:

$$T = \frac{P_A L^3}{60DR}\left\{\left(-3 + 21\alpha - 30\alpha^2 + 10\alpha^3\right)\beta - (10 - 15\alpha)\beta^4 + (3 - 6\alpha)\beta^5\right\} \tag{1}$$

For the range $x_R \le x \le L$:

$$T = \frac{P_A L^3}{60DR}\left\{-10\alpha^3 - \left(3 - 21\alpha - 10\alpha^3\right)\beta - 30\alpha\beta^2 + 10\beta^3 - (10 - 15\alpha)\beta^4 + (3 - 6\alpha)\beta^5\right\} \tag{2}$$

where x is the distance from the gullet in the transverse direction of the bandsaw blade (mm); x_R is the distance from the gullet in the transverse direction of the bandsaw blade to the roll-stretching point (mm); P_A is the compression force parallel to the bandsaw surface at the roll-stretching position (kgf); L is the width of the bandsaw blade between the gullet and the back (mm); D is the flexural rigidity of the bandsaw blade (kgf·mm) $[=Et^3/12\ (1 - \mu^2)]$; E is Young's modulus (kgf/mm^2); t is the thickness of the bandsaw blade (mm); μ is Poisson's ratio; R is the radius of curvature of the bandsaw blade (mm); α is the roll-stretching position ratio (x_R/L); and β is the distance ratio (x/L).

For the actual roll-tensioning, an appropriate tensioning performance for sawing operations was achieved after several roll-stretching passes. Therefore, the accumulated amount of tension T_S (mm) is:

$$T_S = \sum T_j \tag{3}$$

where: j is 1,2,3 ... N; and N is the number of roll-stretching passes.

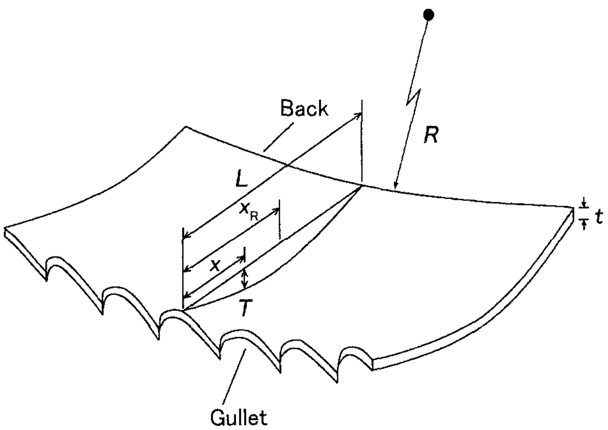


Fig. 1. Transverse deflection of tensioned bandsaw blade bent over radius R

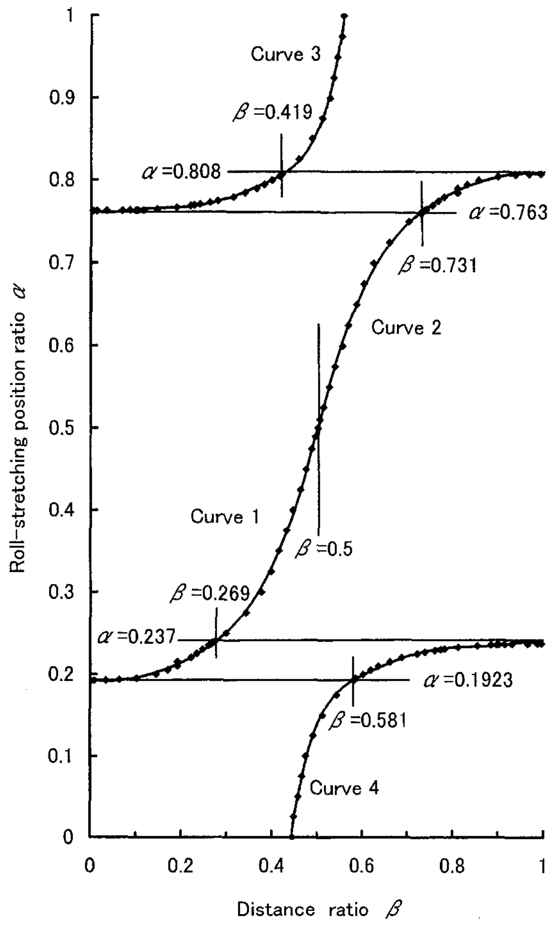


Fig. 2. Relation between β [at which T in Eqs. (1) and (2) becomes maximum or minimum] and α

Occurrence of maximum or minimum value of tension

The value of α that maximizes or minimizes T in Eqs. (1) and (2) at an arbitrary value of β can be calculated. Using several pairs of α and β, the relation between β and α is shown in Fig. 2. Curve 1, related to the maximum value (positive), is effective for the range $0 \leq \beta \leq 0.5$ and is given by:

Table 1. Relation between range of α and decrease or increase range of tension

Range of α	Range of β	
	Decrease of tension	Increase of tension
0–0.192	0.444–0.581	
0.192–0.237	0.581–1	0–0.269
0.237–0.500		0.269–0.500
0.500–0.763		0.500–0.721
0.763–0.808	0–0.419	0.731–1
0.808–1	0.419–0.556	

$$\alpha = 9.1354\,\beta^4 - 5.0272\,\beta^3 + 1.6781\,\beta^2 - 0.1048\,\beta + 0.1934 \tag{4}$$

Curve 2, related to the maximum value (positive), is effective for the range $0.5 \leq \beta \leq 1$ and is given by:

$$\alpha = -9.1355\,\beta^4 + 31.515\,\beta^3 - 41.409\,\beta^2 + 24.712\,\beta - 4.875 \tag{5}$$

Curve 3, related to the minimum value (negative), is effective for the range $0 \leq \beta \leq 0.556$ and is given by:

$$\alpha = 300.29\,\beta^6 - 431.91\,\beta^5 + 236.66\,\beta^4 - 60.417\,\beta^3 + 7.324\,\beta^2 - 0.3389\,\beta + 0.7659 \tag{6}$$

Curve 4, related to the minimum value (negative), is effective for the range $0.444 \leq \beta \leq 1$ and is given by:

$$\alpha = -300.29\,\beta^6 + 1369.8\,\beta^5 - 2581.5\,\beta^4 + 2573\,\beta^3 - 1431.3\,\beta^2 + 421.9\,\beta - 51.376 \tag{7}$$

The relations between the ranges of α and β in which the amount of tension increases or decreases can be obtained from Eqs. (4) through (7) and are shown in Table 1.

Calculation of maximum or minimum value of tension

For the convenience of analysis, it is assumed that $P_A L^3/60DR$ in Eqs. (1) and (2) is equal to 1. Accordingly, for the case of $R = 550$ mm, $L = 117$ mm, $E = 21\,000$ kgf/mm^2, $t = 0.9$ mm, and $\mu = 0.3$, P_A becomes 28.9 kgf. According to Aoyama,[2] P_A increases with increasing roll-stretching force F_R (kgf). Then,

$$P_A = k_A F_R \tag{8}$$

where k_A is the roll-stretching force transmission coefficient.

It has been shown that k_A is affected by the thickness of the bandsaw blade, the radius of the curvature of the bandsaw blade at the measurement of tension, and so forth.[2] Therefore, it is practical to determine k_A based on the experiments of tensioning. Assuming that k_A is equal to 0.015, taking into consideration Aoyama's data,[2] F_R becomes 1925.7 kgf.

Under the assumption that $P_A L^3/60DR$ is equal to 1, the maximum value of tension T_{max} or the minimum one T_{min} at a pair of α and β can be calculated by Eqs. (1) and (2). Using several pairs of T_{max} or T_{min} and β, the relation between β and T_{max} or T_{min} is shown in Fig. 3. Curve 5, related to pairs of α

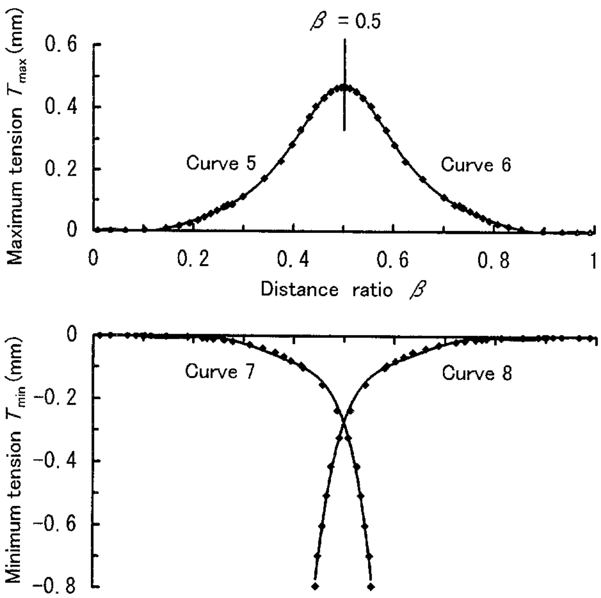


Fig. 3. Relation between maximum or minimum value of tension and β

and β from Eq. (4), is effective for the range $0 \leq \beta \leq 0.5$ and is given by:

$$T_{max} = -1059\,\beta^6 + 1432.6\,\beta^5 - 729.54\,\beta^4 + 177.55\,\beta^3 - 18.905\,\beta^2 + 0.7627\,\beta - 0.0068 \tag{9}$$

Curve 6, related to pairs of α and β from Eq. (5), is effective for the range $0.5 \leq \beta \leq 1$ and is given by:

$$T_{max} = -1059\,\beta^6 + 4921.7\,\beta^5 - 9452.2\,\beta^4 + 9595.6\,\beta^3 - 5423.2\,\beta^2 + 1613.9\,\beta - 196.59 \tag{10}$$

Curve 7, related to pairs of α and β from Eq. (6), is effective for the range $0 \leq \beta \leq 0.556$ and is given by:

$$T_{min} = -1366.4\,\beta^6 + 2020.4\,\beta^5 - 1144.9\,\beta^4 + 306.06\,\beta^3 - 39.022\,\beta^2 + 2.0353\,\beta - 0.0265 \tag{11}$$

Curve 8, related to pairs of α and β from Eq. (7), is effective for the range $0.444 \leq \beta \leq 1$ and is given by:

$$T_{min} = -1358.8\,\beta^6 + 6144.4\,\beta^5 - 11478\,\beta^4 + 11340\,\beta^3 - 6251.9\,\beta^2 + 1825.4\,\beta - 220.89 \tag{12}$$

The relations between the range of α and the range of T_{max} or T_{min} obtained from Eqs. (9) through (12) are shown in Table 2.

Neutral line of tension

As shown in Table 2, positive and negative values of tension coexist in the ranges $0.192 \leq \alpha \leq 0.237$ and $0.763 \leq \alpha \leq 0.808$. Therefore, the value of α at which T in Eqs. (1) and (2) becomes zero at an arbitrary value of β can be calculated. Using several pairs of α and β, the relation between β and α is shown in Fig. 4. Curves 9 and 10 indicate the neutral lines of tension. Curve 9, related to the range $0.192 \leq \alpha \leq 0.237$, is given by:

$$\alpha = -0.7631\,\beta^6 + 2.2278\,\beta^5 - 2.2108\,\beta^4 + 0.6617\,\beta^3 + 0.1434\,\beta^2 - 0.0148\,\beta + 0.1924 \tag{13}$$

Table 2. Relation between range of α and maximum or minimum value of tension

Range of α	Change of maximum or minimum value of tension	
	T_{min} (mm)	T_{max} (mm)
0–0.192	−0.794–−0.099	
0.192–0.237	−0.099–0	0–0.087
0.237–0.500		0.087–0.469
0.500–0.763		0.469–0.087
0.763–0.808	0–−0.099	0.087–0
0.808–1	−0.099–−0.794	

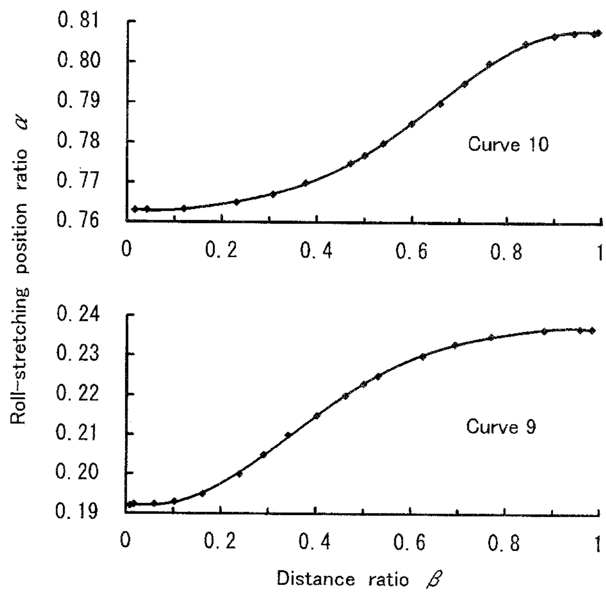


Fig. 4. Relation between β [at which T in Eqs. (1) and (2) is zero] and α

Curve 10, related to the range $0.763 \leq \alpha \leq 0.808$, is given by:

$$\alpha = 0.7631\,\beta^6 - 2.3505\,\beta^5 + 2.5176\,\beta^4 - 1.1646\,\beta^3 + 0.3043\,\beta^2 - 0.0256\,\beta + 0.7635 \tag{14}$$

Basic equation of crown back

The amount of crown back C (mm) of the bandsaw blade after a roll-stretching pass is given by[3]:

$$C = \frac{3b^2 P_B}{2L^3 tE} x_{RC} \tag{15}$$

where b is the length of back gauge (mm); P_B is the compression force parallel to the bandsaw surface at the roll-stretching position (kgf); x_{RC} is the roll-stretching distance from the center of the bandsaw blade (mm); and x_{RC} can be expressed as follows.

$$x_{RC} = x_R - 0.5L \tag{16}$$

From Eqs. (15) and (16):

$$C = \frac{1.5b^2 P_B}{L^2 tE}(\alpha - 0.5) \tag{17}$$

where C is a negative value in the range $\alpha \leq 0.5$ and positive in $0.5 \leq \alpha$.

For the actual roll-tensioning, an appropriate tensioning performance for sawing operations was achieved after several roll-stretching passes. Therefore, the accumulated amount of crown back C_S (mm) is:

$$C_S = \sum C_j \tag{18}$$

where j is 1,2,3 . . . N, and N is the number of roll-stretching passes.

It is possible to increase or decrease the amount of crown back C without changing the amount of tension at an arbitrary position in the transverse direction of the bandsaw blade. From Eq. (13), the range of roll-stretching position ratio α in which C decreases is $0.192 \leq \alpha \leq 0.237$. From Eq. (14), the range of α in which C increases is $0.763 \leq \alpha \leq 0.808$.

Calculation of crown back

According to Aoyama,[2] P_B in Eq. (15) increases with increasing roll-stretching force F_R (kgf). Thus,

$$P_B = k_B F_R \tag{19}$$

where k_B is the roll-stretching force transmission coefficient.

It has been proven that k_B is affected by the thickness of the bandsaw blade, the straightness of the bandsaw blade at the measurement of crown back, and so forth.[2] Therefore, it is practical to determine k_B based on the experiments of tensioning. Assuming that k_B is equal to 0.050, taking into consideration Aoyama's data,[2] P_B becomes 96.3 kgf under F_R = 1925.7 kgf. The amount of crown back can be calculated by Eqs. (17) and (19).

Calculating tension and crown back: example. The sample data used for the calculations of tension and crown back are shown in Table 3. The accumulation curves of tension drawn using the values calculated by Eqs. (1) through (3) and the accumulated amount of crown back by Eq. (18) are shown in Fig. 5. The accumulation curve of tension after 12 roll-stretching passes is expressed by:

$$T_S = 5.1075\beta^6 - 18.273\beta^5 + 24.592\beta^4 - 14.682\beta^3 + 2.072\beta^2 + 1.1821\beta + 0.0008 \tag{20}$$

From Eq. (20) the maximum value of $T_{S.M}$ is 0.330 mm at $\beta = 0.427$. From Eq. (18) C_S is 0.190 mm.

Adjusting the amount of tension and crown back. For the actual tensioning, there is a continual need to adjust the amount of tension or crown back (or both). This need is classified into nine alteration patterns as listed in Table 4. As an example of adjustment, it is possible to show the calculation procedure according to alteration pattern 7 in Table 4.

PROBLEM. After 12 roll-stretching passes in Fig. 5, the maximum value of T_S is 0.330 mm at $\beta = 0.427$, and C_S is 0.190 mm. Decrease the amount of tension from 0.330 mm to 0.280 mm without changing the amount of crown back of 0.190 mm.

To *adjust the tension*:

Step 1: The roll-stretching position ratio α to decrease the amount of tension at $\beta = 0.427$ effectively can be calculated by Eq. (6), and α is 0.808.

Step 2: From Eq. (11) the amount of tension T_{min} at $\beta = 0.427$ under the roll-stretching force F_R of 1925.7 kgf is -0.105 mm. The decrease target of tension T_D is -0.050 mm. Thus, the effective roll-stretching force ratio γ is defined as follows:

$$\gamma = \frac{T_D}{T_{min}} = \frac{-0.050}{-0.105} = 0.476$$

Therefore, the effective roll-stretching force $F_{R \cdot E}$ is given by:

$$F_{R \cdot E} = F_R \gamma$$

Finally, the value of $F_{R \cdot E}$ for adjusting the tension is 896.7 kgf at $\alpha = 0.809$.

The accumulation curve of tension after the adjustment of tension is shown in Fig. 6 as curve 11 and is expressed by:

$$T_S = 7.603\beta^6 - 25.429\beta^5 + 31.522\beta^4 - 17.257\beta^3 + 2.577\beta^2 + 0.9808\beta + 0.0012 \tag{21}$$

From Eq. (21) the maximum accumulated amount of T_S is 0.280 mm at $\beta = 0.427$. In addition, the accumulated amount of crown back C_S increases to 0.236 mm.

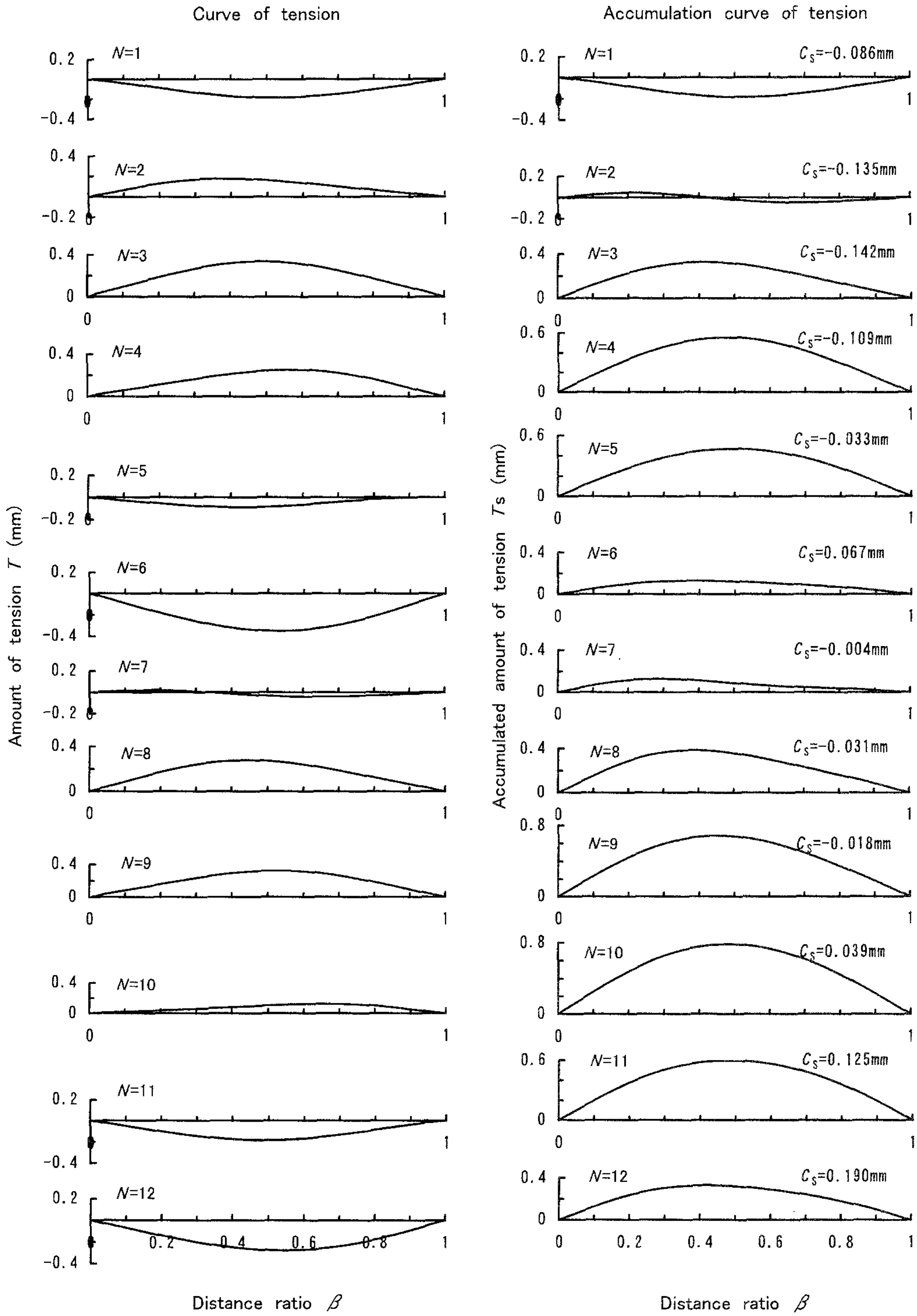
Table 3. Sample data used to calculate tension and crown back

Parameters	N	X_R	F_R
L = 117 min	1	17.5	1500
E = 21 000 kgf/mm^2	2	35.0	1500
t = 0.9 mm	3	55.0	1400
μ = 0.3	4	75.0	1400
R = 550 mm	5	95.0	1500
b = 750 mm	6	106.5	1500
k_A = 0.015	7	24.5	1500
k_B = 0.050	8	44.5	1400
	9	65.5	1400
	10	85.5	1500
	11	99.5	1500
	12	111.0	900

Table 4. Alteration patterns of tension and crown back

Alteration pattern	Tension	Crown back
1	Unchanged	Unchanged
2	Unchanged	Increased
3	Unchanged	Decreased
4	Increased	Unchanged
5	Increased	Increased
6	Increased	Decreased
7	Decreased	Unchanged
8	Decreased	Increased
9	Decreased	Decreased

Fig. 5. Curve of tension, accumulation curve of tension, and accumulated amount of crown back C_S



To *adjust the crown back*:

Step 3: From Eq. (17) the amount of crown back C at $\alpha = 0.809$ under the effective roll-stretching force $F_{R \cdot E}$ of 896.7 kgf is 0.046 mm. To keep the accumulated amount of crown back of 0.190 mm, it is necessary to decrease by 0.046 mm.

Step 4: The roll-stretching position ratio α to decrease the amount of crown back effectively without changing the adjusted amount of tension at $\beta = 0.427$ can be calculated by Eq. (13), and α is 0.217.

Step 5: The roll-stretching force F_R is calculated by Eqs. (17) and (19). Finally, the value of F_R for the adjustment of crown back is 981.4 kgf at $\alpha = 0.217$.

The accumulation curve of tension after adjusting the crown back is shown in Fig. 6 as curve 12 and is expressed by:

$$T_S = 10.665\,\beta^6 - 35.456\,\beta^5 + 43.455\,\beta^4 - 23.075\,\beta^3 + 3.3285\,\beta^2 + 1.0808\,\beta + 0.0013 \tag{22}$$

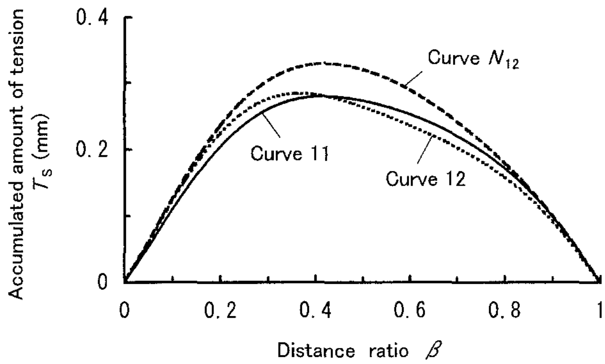


Fig. 6. Adjustment of the amount of tension after 12 roll-stretching passes. Curve N_{12}, accumulation curve of tension at $N = 12$ in Fig. 5

From Eq. (22) T_S is 0.279 mm at $\beta = 0.427$, and the maximum value of T_S is 0.284 mm at $\beta = 0.385$. The accumulated amount of crown back C_S is 0.190 mm.

Conclusions

Bandsaw roll-tensioning was theoretically analyzed by amplifying Aoyama's fundamental equations. The following findings were relevant. (1) The value of the roll-stretching position ratio that maximizes or minimizes the amount of tension at an arbitrary value of the distance ratio can be calculated by equations. (2) The value of the roll-stretching position ratio at which the amount of tension becomes zero at an arbitrary value of the distance ratio can be calculated by equations. (3) Using the theoretical equations developed, it is possible to adjust the amount of tension at an arbitrary distance in the transverse direction of the bandsaw blade, the amount of crown back, or both.

References

1. Aoyama T (1970) Tensioning of band saw blade by rolls. II. Mokuzai Gakkaishi 16:376–381
2. Aoyama T (1971) Tensioning of band saw blade by rolls. III. Mokuzai Gakkaishi 17:188–195
3. Aoyama T (1970) Tensioning of band saw blade by rolls. I. Mokuzai Gakkaishi 16:370–375